145

NASA CR-54101

N64-31715
(ACCESSION NUMBER)
(THRU)
270
(PAGES)
1
(CODE)
CR-54101
(NASA CR OR TMX OR AD NUMBER)
06
(CATEGORY)
FACILITY FORM 602

VOLUME 3

BRUSHLESS ROTATING ELECTRICAL GENERATORS FOR SPACE AUXILIARY POWER SYSTEMS

by

J. N. Ellis and F. A. Collins

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

CONTRACT NO. NAS 3-2783

OTS PRICE

XEROX $ 6.00 FS
MICROFILM $ 1.25 mf.

LEAR SIEGLER, INC.

POWER EQUIPMENT DIVISION
CLEVELAND 1, OHIO

CASE FILE COPY

Requests for copies of this report should be referred to:

National Aeronautics and Space Administration
Office of Scientific and Technical Information
Washington 25, D.C.
Attention: AFSS-A

NASA CR - 54101

THIRD QUARTERLY
REPORT

July 15, 1964

BRUSHLESS ROTATING ELECTRICAL GENERATORS FOR SPACE AUXILIARY POWER SYSTEMS

Contract No. NAS 3-2783

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

by

J. N. Ellis and F. A. Collins

Technical Management
Howard A. Shumaker
NASA Lewis Research Center
Space Power System Division
Solar and Chemical Power Branch

LEAR SIEGLER, INC.
Power Equipment Division
Cleveland, Ohio

TABLE OF CONTENTS

SUMMARY

3175

In this quarterly report, design manuals and Fortran computer programs are presented for the following AC generators:

1. Two-Coil, Inside-Coil Lundell or Becky-Robinson Generator
2. Two-Coil, Outside-Coil Lundell
3. Single-Coil, Outside Coil Lundell
4. Rotating-Coil Lundell (Automotive Type)
5. Inside-Coil, Stationary-Coil Lundell.

Design manuals without computer programs are presented for:

6. Permanent-Magnet AC Generators
7. Homopolar Inductor AC Generators
8. Disk-Type or Axial Air-Gap Lundell Generator

An equivalent circuit representation of synchronous AC generators is published with a discussion of its development.

THE NEXT REPORT

The next report is the final report, to be issued in October, and it will consist of two parts. The first part is generator selection criteria and the second part is the electrical design section.

In the first section, the selection criteria, comparison data will be published. Such data will be weight and physical size comparison, evaluations of rotor dynamics, suitability of the various types of generator rotors for use with gas or liquid bearings. Thermal equivalent circuits are to be published in the selection section also.

The second section of the final report will contain the generator design manuals with the Fortran computer programs and the synchronous generator equivalent circuits.

An appendix will be published containing the small studies, discussions and derivations that support the rest of the study.

For each generator design manual, a general approach to the start of a generator design will be provided. It will be similar to that provided for permanent magnet generators in this third quarterly report. Beyond this general approach, the user must select the various design parameters himself. The user of these programs should have some familiarity with AC machine design.

In this, the third quarterly report, Fortran computer programs and design manuals are published for the following AC generators:

1. Two-Coil, Inside-Coil Lundell or Becky-Robinson Generator
2. Two-Coil, Outside-Coil Lundell Generator
3. Single-Coil, Outside-Coil Lundell Generator
4. Rotating-Coil Lundell (Automotive-Type) Generator
5. Inside-Coil, Stationary-Coil, Lundell Generator

Design manuals without computer programs are published for:

6. Permanent-Magnet AC Generators
7. Homopolar Inductor AC Generators
8. Axial Air-Gap Lundell Generators

The last three design manuals are to be programmed in Fortran for the final report. And, in addition, a program for Induction generators will be included if time on this contract permits.

Because of the general and widely understood use of the term Lundell, all of the generators discussed in this study that have claw-type or interlocking, finger-type poles are called Lundell generators. To most engineers, the name Lundell describes the rotor pole arrangement. In this report, there is no other basis for the use of the name.

NOTE ON WINDAGE CALCULATIONS

In each design manual there is a statement to the effect that there is no known satisfactory method of calculating windage. That, of course, is open to challenge and probably should read "we know of no . . . . ". The formula given is crude and is only intended for use in standard air.

For gases or fluids other than standard air, the fluid density and viscosity must be considered. The formula given in the manual can be modified by the factors

$$\left(\frac{\rho}{.0765}\right)^{.8}\left(\frac{u}{.0435}\right)^{.2}$$

where

ρ = density - Lbs FT^{-3}

μ = viscosity LBS FT^{-1} HR^{-1}

.0765 = density std. air

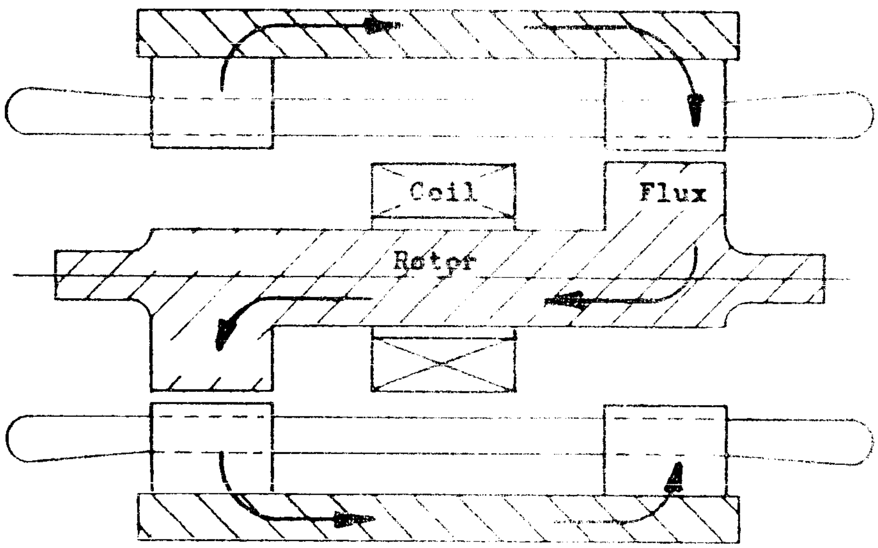
.0435 = viscosity Std air

The above relationship can be arrived at by referring to Shepherd "Principles of Turbomachinery", Macmillan Pub. Company. See page 152. The friction factor for turbulent flow is a function of $\frac{1}{(R_e)^{.2}}$ and the loss is a function of $\frac{\rho}{(R_e)^{.2}}$ times a constant for a fixed velocity and fixed dimensions. The correction for a gas other than standard air, since $R_e = \frac{DV\rho}{u}$, would be

$$\frac{\rho}{\rho_{air}} \cdot \left(\frac{\rho_{air}}{\rho}\right)^{.2} \cdot \left(\frac{u}{u_{air}}\right)^{.2} \quad \text{or} \quad \left(\frac{\rho}{.0765}\right)^{.8}\left(\frac{u}{.0435}\right)^{.2}$$

A person unfamiliar with electrical machine design would benefit from a synthesis program that selected the design parameters for the design program inputs, but the time available on this contract does not allow its development.

HOMOPOLAR-INDUCTOR AC GENERATORS



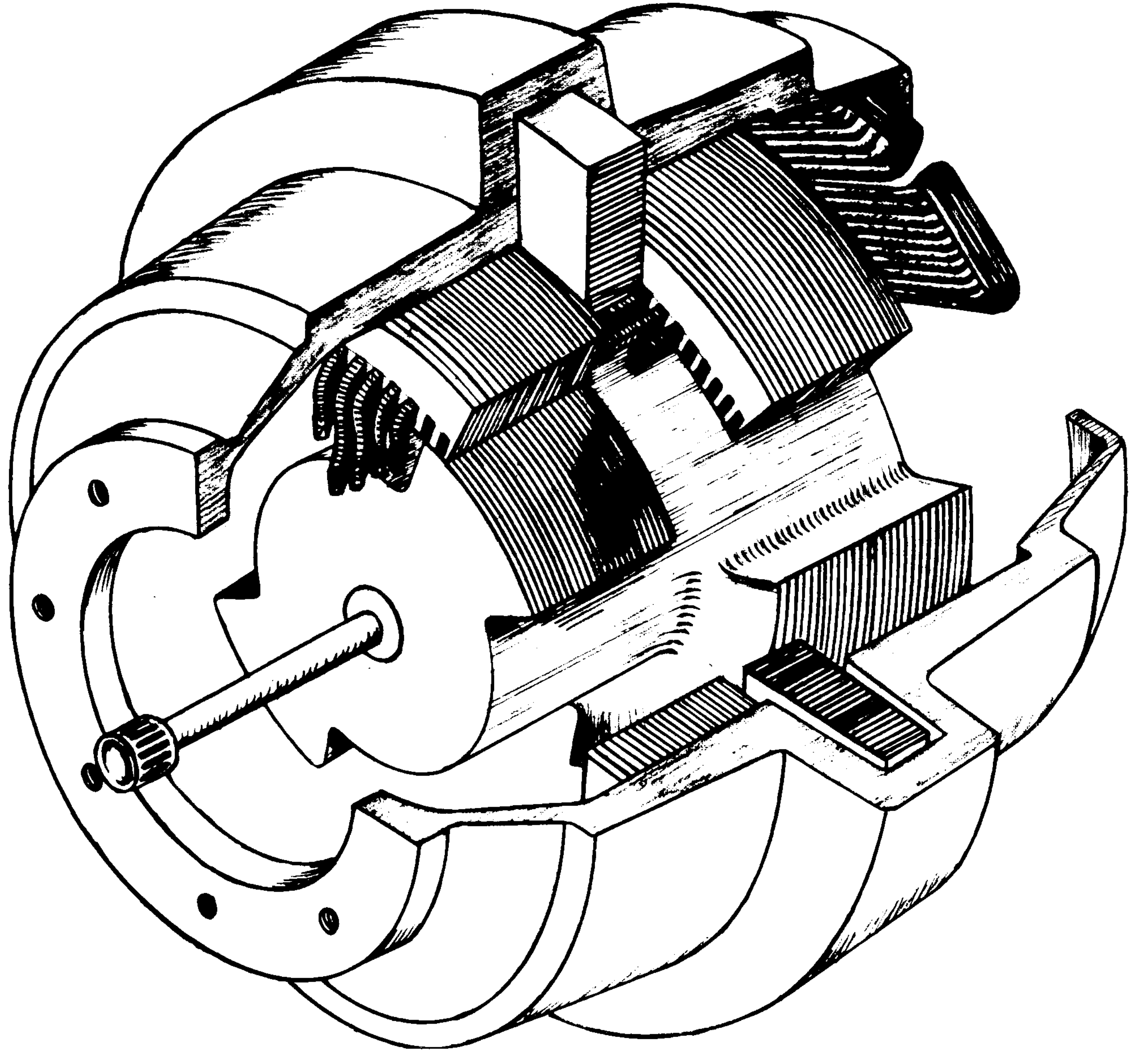
G

HOMOPOLAR INDUCTOR, AC GENERATOR

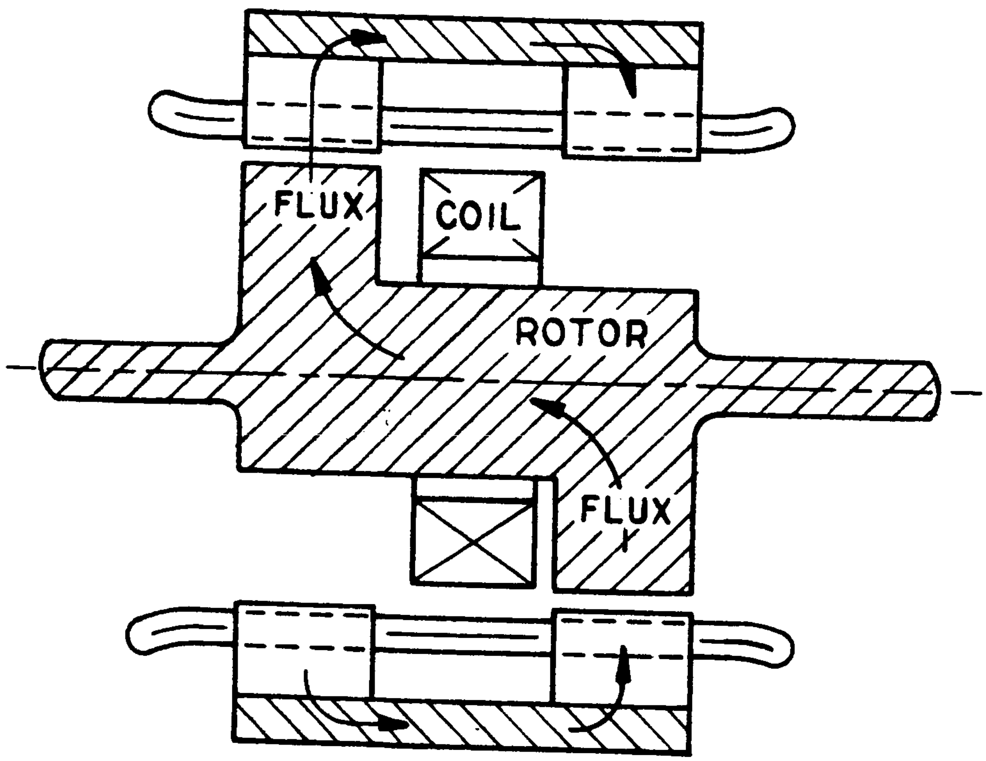
Before 1900 in the young age of electrical power engineering, many different generator designs were proposed and patented. One of those old designs, widely used since its conception, is described in U.S. Patent No. 499446 issued to William Stanley, Jr. and John F. Kelly in 1893.

The same configuration is now made by every company building homopolar inductor AC generators.

The AC generator known as the homopolar inductor is confused in the literature with a DC generator that is also called a homopolar inductor. The DC generator is called both a unipolar generator and an acyclic generator to distinguish it from the AC machine. A paper given by B. G. Lamme, AIEE Transactions 1912, PP 1811-1835, describes the development problems of a 2000 KW acyclic (DC) generator. The acyclic generators are of interest for generating the high direct currents needed for pumping liquid metals but are not discussed in this study.



HOMOPOLAR INDUCTOR GENERATOR



SMALL HOMOPOLAR INDUCTOR GENERATOR

The homopolar inductor, AC generator uses two identical wound stators and two identical rotor sections to produce AC electrical power.

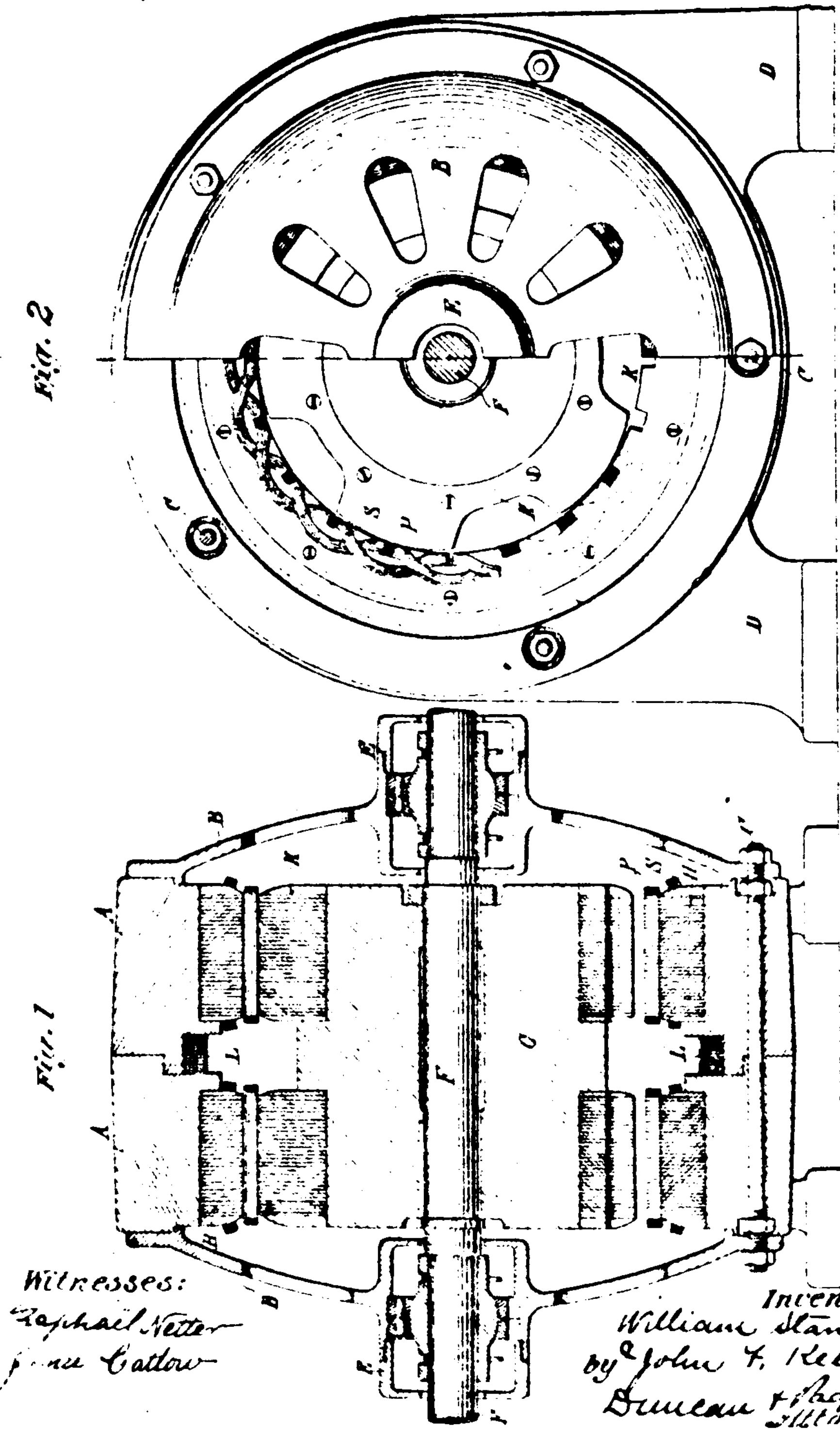
The magnetic flux from the rotor poles passing through each stator section and linking the output windings, is unidirectional and pulsating. Since the magnetic flux never changes direction in a stator and the poles of a rotor section are of one polarity, the generator has been called a homopolar generator (or alike-pole generator).

(No Model.)

W. STANLY, Jr. & J. F. KELLY.

ALTERNATING CURRENT GENERATOR.

No. 499,446. Patented June 13, 1893.



Witnesses:

Raphael Netter

Jnne Catlow

Inventors

William Stanly, Jr.

John F. Kelly

by Duncan & Page

Attorneys.

The usual homopolar inductor consists of two identical stators wound with a common winding, a double rotor having all north poles on one end and all south poles on the other end, and a field coil enclosed in the magnetic path formed by the outer shell or yoke, the stators, and the rotor.

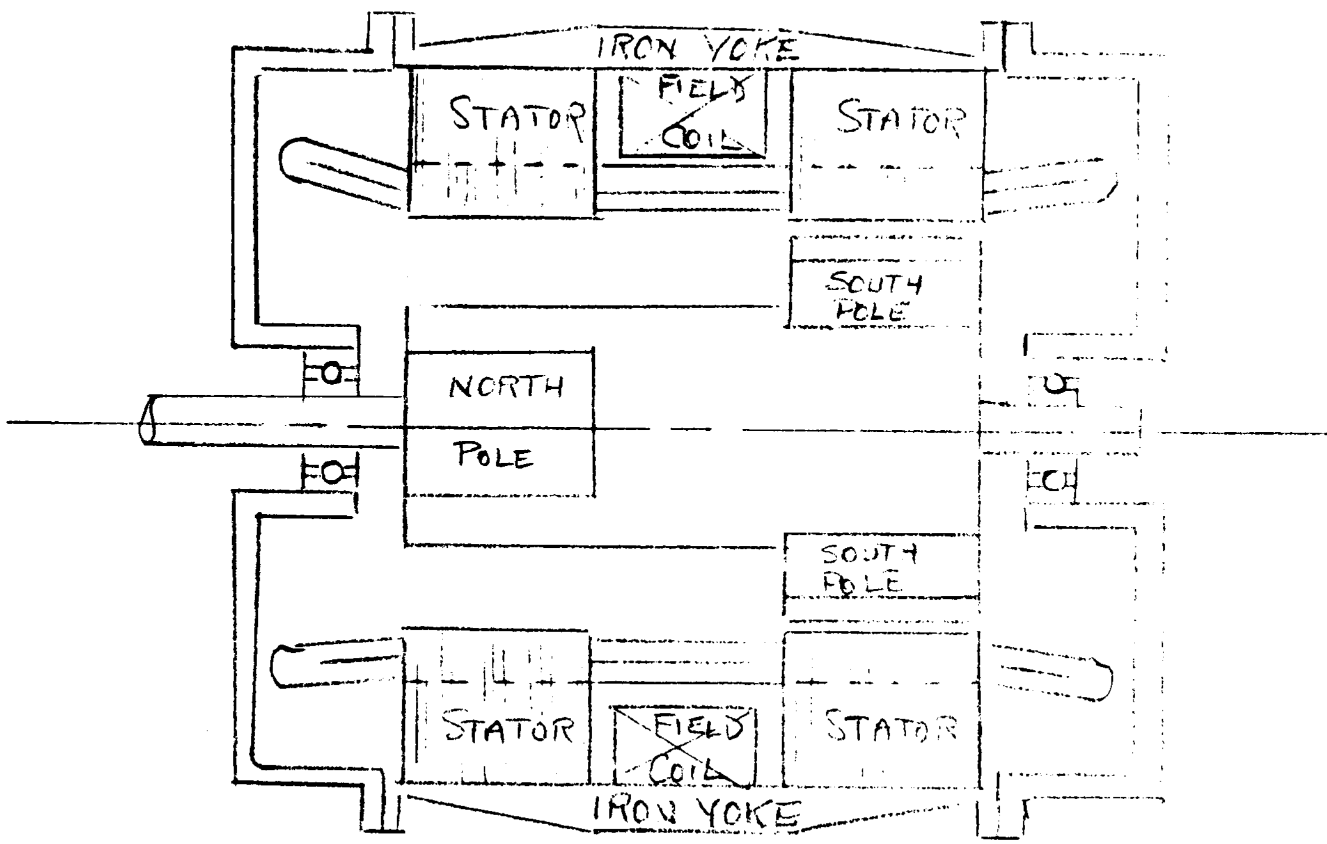
When the field coil is excited and the rotor is rotating, unidirectional fields of flux cut the windings of each stator in such a manner that approximately the same voltage is generated in the two stator combined as would be generated in one stator by a single rotor having both the north and south poles of the two ends of the homopolar inductor rotor. In other words, two stators and two rotor ends are electrically and magnetically accomplishing what one stator and its corresponding rotor would do in a conventional salient-pole, synchronous, wound-field generator.

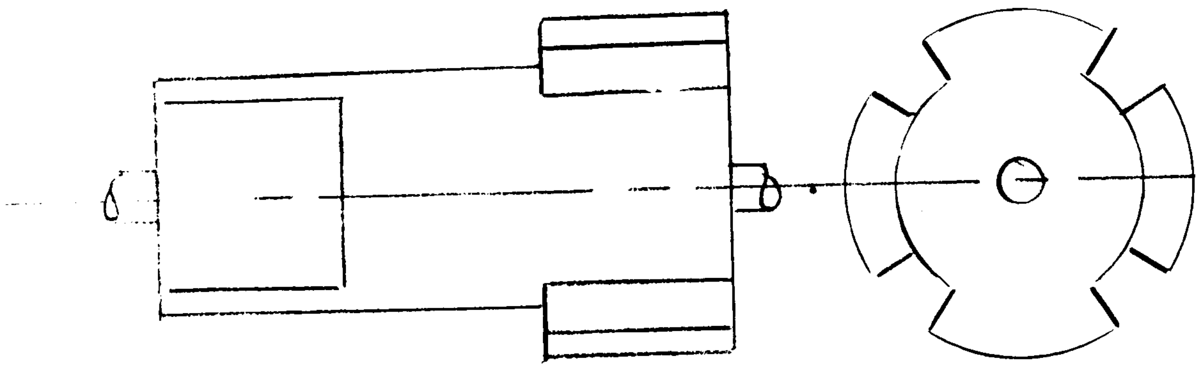
The homopolar inductor alternator has been known and used for approximately seventy (70) years. During this time it has been used mainly in industrial applications where size and weight were of little consequence. One of its uses has been to supply high frequency electrical power for induction heating of steel products.

Homopolar inductor designs used in industrial applications have poles, or rotor teeth as they are often called, protruding far out of the shaft so that only a very small amount of unwanted flux passes from the shaft to the stator between the poles of a single polarity (on one end of the stator.)

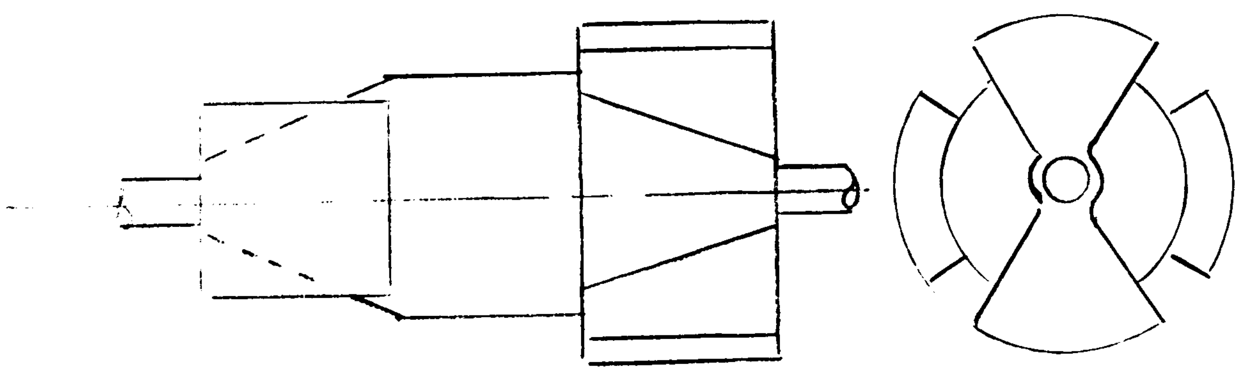
Recently, the homopolar inductor is being used in airborne and space applications where size and weight are of primary importance. In these applications the area of the shaft between the two groups of poles of opposite polarities directly limits the maximum output of the machine. In these minimum weight designs, the shaft is the largest diameter practicable and the poles or rotor teeth do not protrude far from the shaft. The unwanted flux passing from the shaft to the stator in the region between poles of like polarity is significant. It is of the order of several percent in a practical, useable design. This unwanted flux generates a voltage opposite to the output voltage in the output windings and reduces the output of the machine.

VIEW OF A 4-POLE HOMOPOLAR INDUCTOR





View showing a conventional four-pole, homopolar inductor rotor with approximately the proportions that might be used for maximum output



View of the rotor shown at the top of the page after removal of excess metal to improve the output of the generator

Advantages of the homopolar inductor generator for use in space power systems are:

1. It is simple in design and inherently reliable.

2. The homopolar rotor has high strength and can be used for high rotational speeds if bearing problems permit.

3. At lower speeds the rotors can be laminated to remove the output limits imposed by pole-face losses.

Disadvantages of the machine for the same applications are:

1. It is a heavy machine -- the heaviest of all of the AC generators if compared at the same rpm.

2. Stator protection problems are compounded by the two stators when used in a hostile environment.

3. The solid pole faces limit the output unless the poles are treated to reduce the pole-face losses.

4. The long, double rotor is sometimes not as stiff as desirable for high-speed applications where fluid or gas bearings are used.

NOTICE

The design procedure given here has not been checked yet. For the final report a design calculation will be compared against test data and the errors in procedure, etc., will be corrected.

The numbers and general arrangement for a computer program are used and the program will be in the final report.

HOMOPOLAR INDUCTOR A-C GENERATOR

______ KVA ______ % PF ______ Volts ______ Amps. ______ Ph. ______ Cycles ______ RPM

______________________________ Date ______________

STATOR

PUNCHING I. D. ______

PUNCHING O. D. ______

CORE LENGTH ______

DBS x 2 ______

SLOTS ______

SIZE SLOTS ______

CARTER COEFF. ______

TYPE WDG. ______

THROW ______

SKEW & DIST. FACT. ______

CHORD FACT ______

COND. PER SLOT ______

TOTAL EFF. COND. ______

COND. SIZE ______

COND. AREA ______

CURRENT DENSITY ______

WDG. CONST. ______ C1 ______

TOTAL FLUX ______

GAP AREA ______

GAP DENSITY ______

POLE CONST. ______

FLUX PER POLE ______

TOOTH PITCH ______

TOOTH DENSITY ______

CORE DENSITY ______

GRADE OF IRON ______

½ MEAN TURN ______

RES. PER PH. • ______ 0 ______

EDDY FACT. TOP ______

EDDY FACT. BOT. ______

DEMAG. FACT. Cm ______ Cq ______

AMP. COND. PER IN. ______

REACT. FACTOR ______

COND. PERM. ______

END PERM. ______

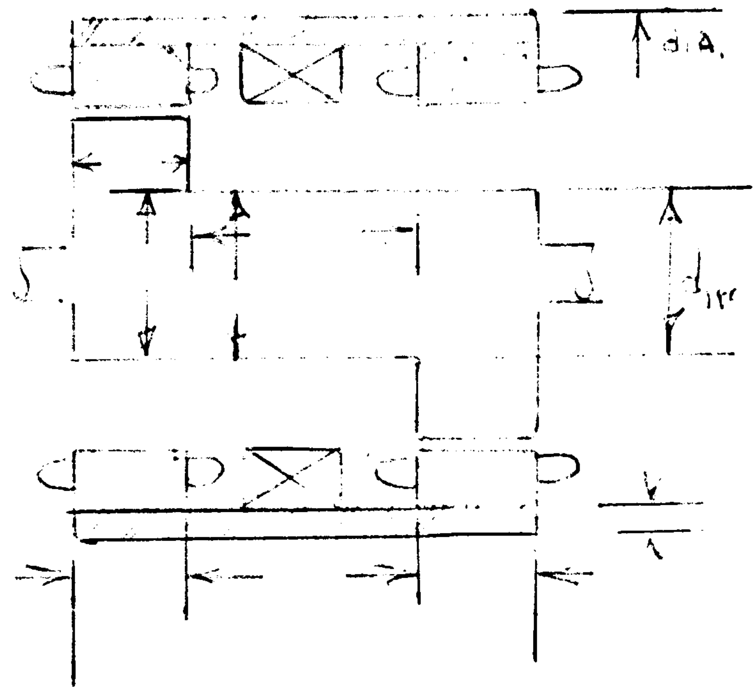
LEAKAGE REACT. ______

AIR GAP PERM. ______

REACT. OF ARM Xad ______ Xaq ______

WT. OF COPPER ______

WT. OF IRON ______



SLOT

Field turns ______

COND. SIZE ______

COND. AREA ______

MEAN TURN ______

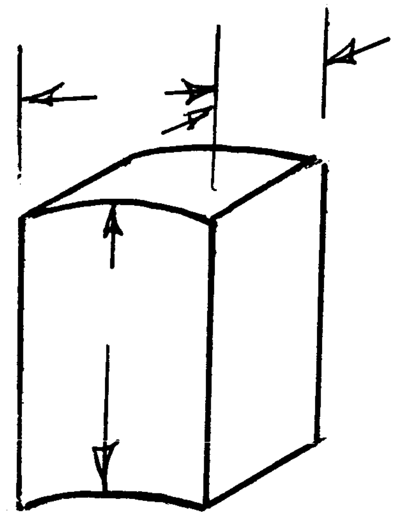
RES. • 0 ______

% LOAD			
AMPS.			
VOLTS			
AMPS /IN. 2			

FIELD LEAK. REACT. ______

FIELD SELF INDUCT. ______

DAMP. LEAK. XDd ______ XDq ______



ROTOR	
SINGLE GAP ______ Ge ______	
ROTOR DIAMETER ______	
PERIPHERAL SPEED ______	
POLE PITCH ______ ∝ ______	
POLE AREA ______	
POLE DENSITY ______	
GRADE OF IRON	

LOSSES-EFFICIENCY

% LOAD			
F & W			
STA. TEETH			
STA. CORE			
POLE FACE			
DAMPER			
STA I^2R			
EDDY			
ROT. I^2R			
Σ LOSSES			
RATING			
RTG+LOSS			
% LOSS			
% EFF			

REACT-TIME CONST.

SYNCH. Xd ______ Xq ______

UNSAT TRANS. ______

SAT. TRANS. ______

SUBTRANS. X″d ______ X″q ______

NEG. SEQUENCE ______

ZERO SEQUENCE ______

OPEN CIRC. TIME CON. ______

ARM TIME CON. ______

TRANS. TIME CON. ______

SUBTRANS. TIME CON. ______

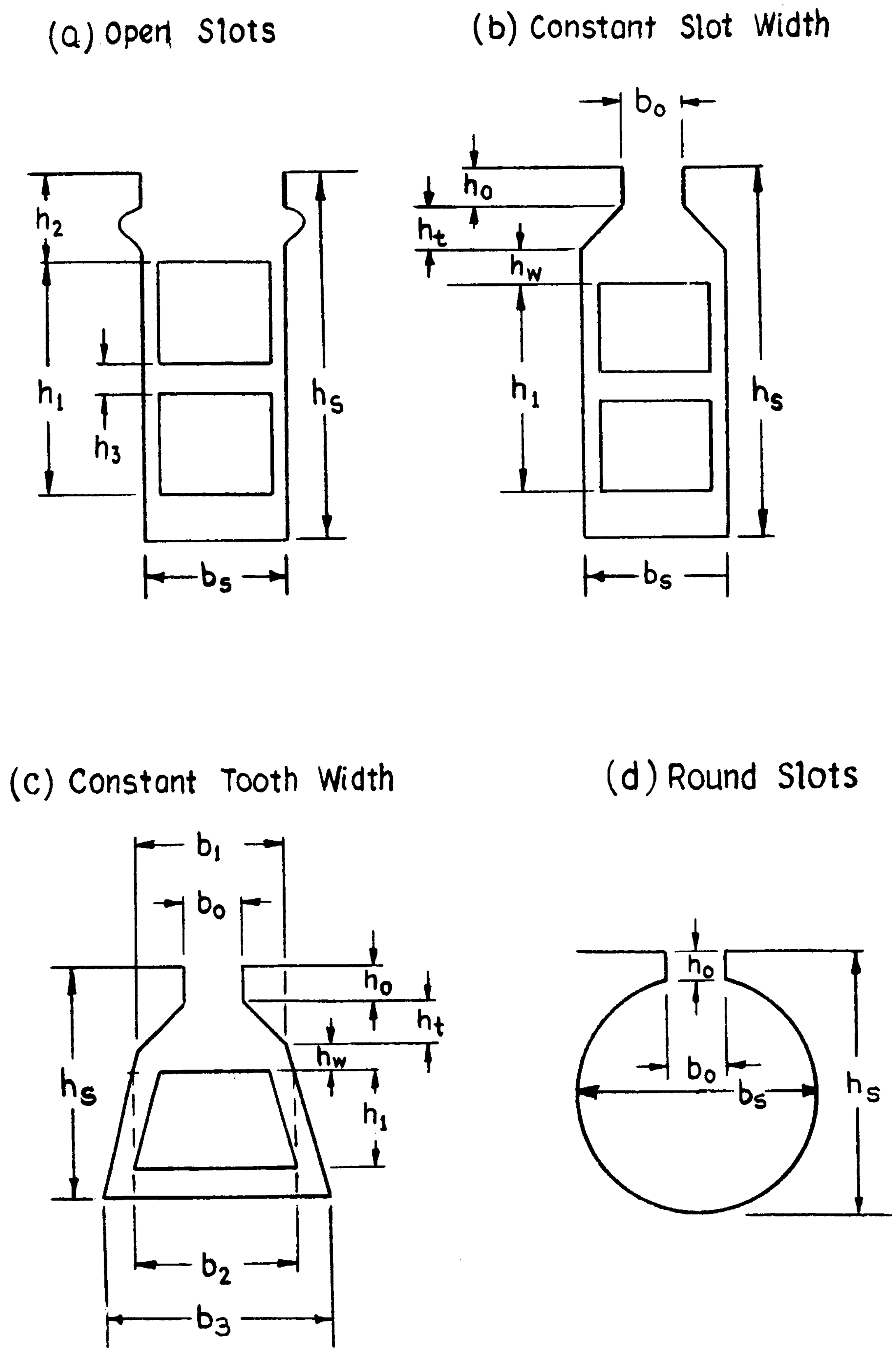

(a) Open Slots
(b) Constant Slot Width
(c) Constant Tooth Width
(d) Round Slots
h2
h1
h3
hs
bs
bo
ho
ht
hw
b1
b2
b3

FIG 1

TABLE - 1

CHORD FACTORS K_p FOR HARMONICS AT DIFFERENT PITCHES

q_1														y/mg																									
1	3/3																																2/3						
2	6/6																5/6																4/6						
3	9/9											8/9										7/9											6/9						
4	12/12								11/12								10/12								9/12								8/12						7/12
5	15/15						14/15							13/15						12/15							11/15						10/15				9/15		
6	18/18					17/18						16/18					15/18					14/18						13/18					12/18						
7	21/21				20/21					19/21					18/21				17/21					16/21					15/21				14/21						
8	24/24			23/24					22/24				21/24				20/24				19/24				18/24					17/24			16/24			15/24			14/24
9	27/27		26/27					25/27				24/27				23/27		22/27				21/27				20/27					19/27		18/27		17/27			16/27	
10	30/30	29/30					28/30				27/30			26/30			25/30			24/30			23/30				22/30					21/30	20/30	19/30			18/30		
y/mg	1.0	.967	964	959	952	945	934	926	917	905	906	889	875	867	857	852	833	815	810	800	792	779	767	762	750	741	734	722	712	709	705	700	667	634	630	.625	600	592	584
1	1.0	999	998	998	997	996	995	993	991	989	988	985	981	978	975	973	966	958	956	951	947	940	934	931	924	918	914	906	901	897	894	891	866	839	835	831	809	800	793
3	-1.0	-988	-985	-981	-975	-966	-951	-940	-924	-901	-891	-865	-831	-809	-782	-766	-707	-643	-623	-588	-556	-500	-454	-434	-383	-342	-309	-259	-222	-195	-174	-156	0.0	156	174	195	309	342	383
5	1.0	966	958	947	931	906	866	835	793	733	707	643	556	500	434	396	259	116	075	0.0	-065	-174	-259	-295	-383	-447	-500	-574	-623	-660	-690	-707	-866	-966	-974	-982	-1.0	-998	-993
7	-1.0	-943	-918	-897	-866	-819	-743	-686	-609	-500	-454	-342	-177	-105	0.0	058	259	449	500	588	659	766	839	866	924	958	978	996	1.0	998	993	988	866	629	588	559	309	225	139
9	1.0	897	866	831	782	707	588	500	383	223	156	0.0	-191	-304	-434	-500	-707	-866	-901	-951	-981	-1.0	-998	-975	-924	-866	-804	-707	-623	-556	-500	-454	0.0	358	407	462	500	755	875
11	-1.0	-834	-802	-750	-680	-574	-407	-287	-131	075	156	342	556	660	782	795	966	998	989	951	897	766	629	583	383	231	105	-087	-222	-321	-346	-454	-866	-999	-994	-981	-809	-721	-618
13	1.0	777	727	659	563	423	208	058	-131	-365	-454	-643	-831	-914	-975	-993	-966	-802	-733	-588	-442	-174	052	148	383	550	669	818	901	947	973	988	866	371	294	195	-309	-459	-598
15	-1.0	-707	-643	-556	-434	-259	0.0	174	383	624	707	866	981	1.0	975	940	707	342	223	0.0	-195	-500	-707	-782	-924	-981	-1.0	-966	-907	-831	-766	-707	0.0	696	760	831	1.0	982	930
17	1.0	629	550	442	295	087	-208	-346	-609	-826	-891	-985	-981	-914	-782	-686	-259	231	365	588	752	940	999	997	924	802	669	423	222	065	-058	-156	-866	-940	-898	-831	-309	-100	113
19	-1.0	-545	-449	-321	-149	087	407	597	793	956	988	985	831	669	434	287	-259	-727	-826	-951	-998	-940	-777	-680	-363	-116	105	423	623	752	835	891	866	072	-047	-194	-809	-952	-989
21	1.0	454	342	195	0.0	-259	-588	-766	-924	-1.0	-988	-866	-556	-309	0.0	174	707	985	1.0	951	831	500	156	0.0	-383	-643	-804	-966	-1.0	-981	-940	-891	0.0	881	935	981	809	628	402
23	-1.0	-358	-231	-064	149	423	743	894	991	956	891	643	195	-105	-434	-597	-966	-918	-826	-588	-321	174	545	680	924	998	978	819	623	442	287	156	-866	-792	-696	-555	309	566	778
25	1.0	259	116	-064	-295	-574	-866	-973	-991	-826	-707	-342	195	500	782	894	966	550	365	0.0	-321	-766	-966	-997	-924	-727	-500	-087	222	442	597	707	866	-061	-383	-555	-1.0	-951	-809
27	-1.0	-170	0.0	195	434	707	951	1.0	924	624	454	0.0	-556	-809	-975	-1.0	-707	0.0	223	558	831	1.0	897	782	383	0.0	-305	-707	-901	-981	-1.0	-988	0.0	983	1.0	981	309	-025	-356
29	1.0	052	-116	-321	-563	-819	-995	-973	-793	-365	-156	342	831	978	975	894	259	-550	-733	-951	-998	-766	-358	-148	383	727	914	996	901	752	597	454	-866	-569	-412	-194	809	965	995
31	-1.0	052	231	442	680	906	995	894	109	075	-156	-643	-981	-978	-782	-597	259	918	989	951	752	174	-358	-583	-924	-998	-914	-574	-222	065	287	454	866	-517	-673	-831	-809	-525	-162
33	1.0	-156	-342	-556	-782	-966	-951	-766	-383	223	454	866	981	809	434	174	-707	-985	-901	-588	-195	500	891	975	924	643	309	-259	-623	-831	-940	-988	0.0	992	946	831	-309	-666	-910
35	-1.0	259	449	659	866	996	866	597	131	-500	-707	-985	-831	-500	0.0	287	966	727	500	0.0	-442	-940	-966	-866	-383	116	500	906	1.0	947	835	707	866	294	079	194	1.0	905	637
37	1.0	-358	-550	-750	-931	-996	-743	-396	131	793	891	985	556	105	-434	-686	-966	-231	075	588	897	940	545	295	383	802	-978	-906	-623	-321	-058	156	866	855	-884	-981	-309	150	577
39	-1.0	454	643	831	975	966	588	174	-383	-901	-988	-866	-195	309	782	940	707	-342	-623	-951	-981	-500	156	434	924	985	809	259	-222	-556	-766	-891	0.0	909	780	555	-805	-990	939
41	1.0	-545	-727	-847	-997	-906	-407	058	609	989	988	643	-195	-669	-975	-933	-259	802	956	951	659	-174	-777	-931	-924	-550	-105	574	901	998	973	891	-866	010	264	555	809	414	-088
43	-1.0	629	802	947	997	819	208	-287	-793	-989	-891	-342	556	914	975	795	-259	-998	-956	-588	-065	766	999	931	383	-231	-669	-996	-901	660	346	-156	866	-916	-990	981	309	754	984
45	1.0	-707	-866	-987	-925	-707	0.0	500	924	901	707	0.0	-831	-1.0	-782	-500	707	866	623	0.0	-556	-1.0	-707	-434	383	866	1.0	707	222	-145	-500	-707	0.0	739	523	194	1.0	-844	-262
47	-1.0	777	918	998	931	574	-208	-686	-991	-733	-454	342	981	914	434	058	-966	-449	-075	588	947	766	052	295	-924	-958	669	087	623	897	993	988	866	312	575	831	305	273	762
49	1.0	-839	-958	-998	-866	-423	407	835	991	500	156	-643	-981	-669	0.0	396	966	-116	-500	-951	-947	-174	629	866	224	597	-105	-819	-1.0	-897	690	454	866	-994	-979	-831	809	1.0	823
51	-1.0	891	985	981	782	259	-588	-940	-924	-223	156	866	831	309	-434	-766	-707	643	901	951	556	-500	-988	-975	-383	342	809	966	623	195	-174	-454	0.0	501	203	-194	-805	-297	333
53	1.0	-934	-998	-947	-680	-087	743	993	793	-075	-454	-985	-556	105	782	973	259	-958	-989	-588	065	940	839	583	-383	-918	-978	-423	222	660	894	988	-866	585	818	981	-309	-721	-997
55	-1.0	966	998	897	563	-087	-866	-993	-605	365	707	985	195	-500	-975	973	259	958	733	0.0	-659	-940	-259	148	924	918	500	-423	-901	-998	-894	-707	866	-979	-853	-555	1.0	771	187
57	1.0	-988	-985	-831	-434	255	951	940	383	-624	-891	-866	195	809	975	766	-707	-643	-223	588	981	500	-454	-782	-924	-342	309	966	901	556	174	-156	0.0	215	-141	-555	-309	407	899
59	-1.0	999	958	756	295	-423	-995	-835	131	826	988	643	-556	-978	-782	-396	966	116	-365	-951	-897	174	934	997	383	-597	-914	-819	-222	321	690	891	-866	803	965	981	-809	994	-777
61	1.0	-999	-978	-659	-149	574	995	686	-131	-956	-988	-342	831	978	434	-058	-966	-449	826	951	442	-766	-934	-680	383	958	914	087	-623	-947	-993	891	866	-872	-625	-194	809	175	-557
63	-1.0	988	866	556	0.0	-707	-051	-500	383	1.0	891	0.0	-981	-809	0.0	500	707	-866	-1.0	-588	195	1.0	454	0.0	-924	-866	-309	707	1.0	831	500	156	0.0	-091	-468	-831	309	894	947
65	1.0	-966	-807	-442	149	814	866	287	-609	-956	-707	342	981	500	-434	-745	-299	998	826	0.0	-752	-766	259	680	924	231	-500	-996	-623	-065	396	707	-966	946	997	831	-1.0	-689	063

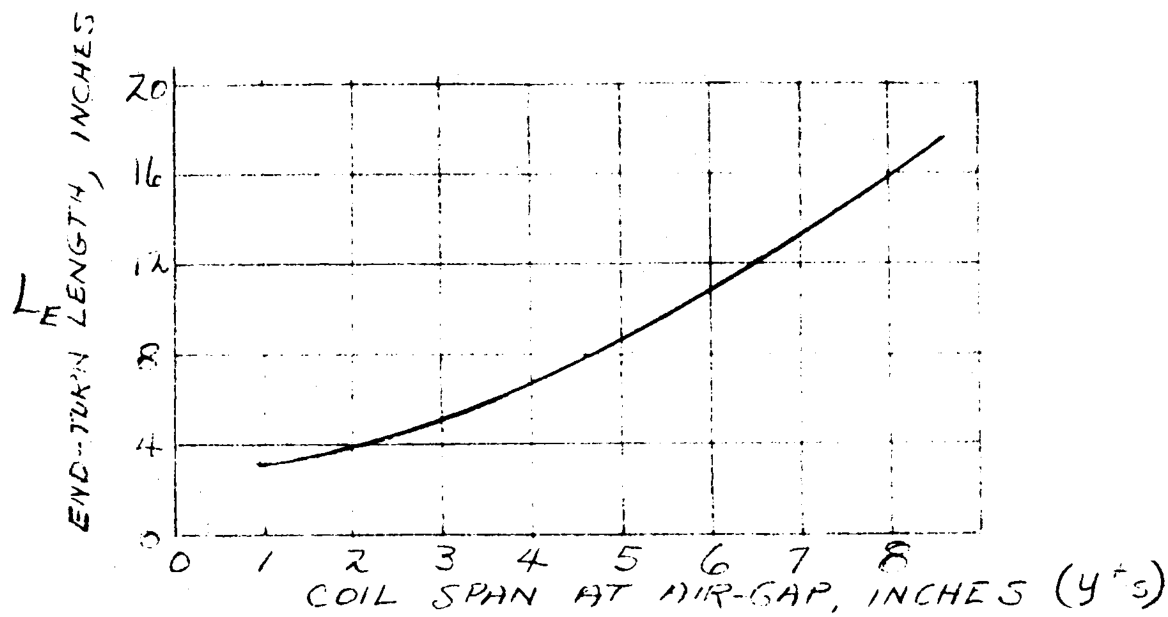
VALUES OF K_{dn} FOR INTEGRAL-SLOT, 3ϕ WINDINGS - TABLE 2

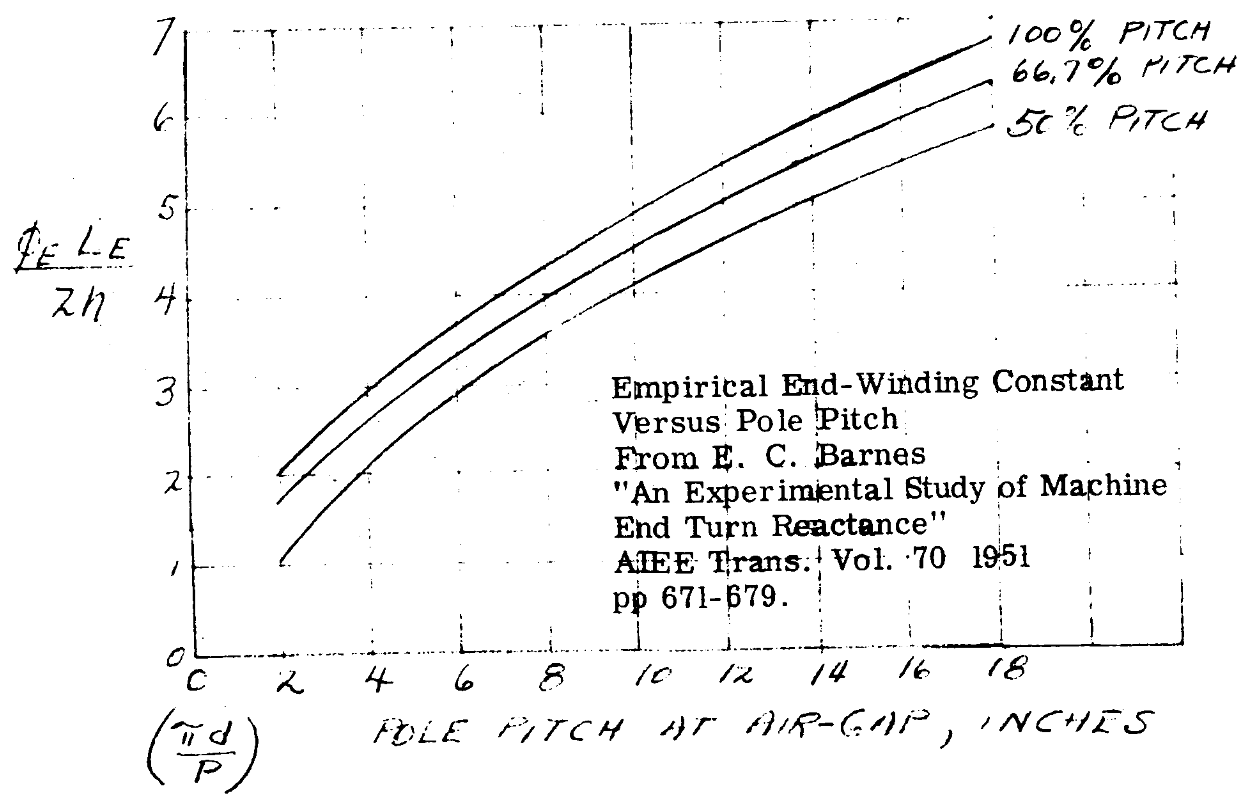
n	K_{dn} – HARMONIC DISTRIBUTION FACTORS								
q=	2	3	4	5	6	7	8	9	10	∞
1	.966	.960	.958	.957	.957	.957	.956	.955	.955	.955
3	.707	.667	.654	.646	.644	.642	.641	.640	.639	.636
5	.259	.217	.205	.200	.197	.195	.194	.194	.193	.191
7	-.259	-.177	-.158	-.149	-.145	-.143	-.141	-.140	-.140	-.136
9	-.707	-.333	-.270	-.247	-.236	-.229	-.225	-.222	-.220	-.212
11	-.966	-.177	-.126	-.110	-.102	-.097	-.095	-.093	-.092	-.087
13	-.966	.217	.126	.102	.092	.086	.083	.081	.079	.073
15	-.707	.667	.270	.200	.172	.158	.150	.145	.141	.127
17	-.259	.960	.158	.102	.084	.075	.070	.066	.064	.056
19	.259	.960	-.205	-.110	-.084	-.072	-.066	-.062	-.060	-.059
21	.707	.667	-.654	-.247	-.172	-.143	-.127	-.118	-.112	-.091
23	.966	.217	-.958	-.149	-.092	-.072	-.063	-.057	-.054	-.041
25	.966	-.177	-.958	.200	.102	.075	.063	.056	.052	.038
27	.707	-.333	-.654	.646	.236	.158	.127	.111	.101	.071
29	.259	-.177	-.205	.957	.145	.086	.066	.056	.050	.033
31	-.259	.217	.158	.957	-.197	-.097	-.070	-.057	-.050	-.031
33	-.709	.667	.270	.646	-.644	-.229	-.150	-.118	-.101	-.058
35	-.966	.960	.126	.200	-.957	-.143	-.083	-.062	-.052	-.027
37	-.966	.960	-.126	-.149	-.957	.195	.095	.066	.054	.026
39	-.707	.667	-.270	-.247	-.644	.642	.225	.145	.112	.049
41	-.259	.217	-.158	-.110	-.197	.957	.141	.081	-.060	.023
43	.259	-.177	.205	.102	.145	.957	-.194	-.093	-.064	-.022
45	.707	-.333	.654	.200	.236	.642	-.641	-.222	-.141	-.042
47	.966	-.177	.958	.102	.102	.195	-.956	-.140	-.079	-.020
49	.966	.217	.958	-.110	-.092	-.143	-.956	.194	.092	.019
51	.707	.667	.654	-.247	-.172	-.229	-.641	.640	.220	.038
53	.259	.960	.205	-.149	-.084	-.097	-.194	.955	.140	.018
55	-.259	.960	-.158	.200	.084	.086	.141	.955	-.193	-.017
57	-.707	.667	-.270	.646	.172	.158	.225	.640	-.639	-.033
59	-.966	.217	-.126	.957	.092	.075	.095	.194	-.955	-.016
61	-.966	-.177	.126	.957	-.102	-.072	-.083	-.140	-.955	.016
63	-.707	-.333	.270	.646	-.236	-.143	-.150	-.222	-.639	.030
65	-.259	-.177	.158	.200	-.145	-.072	-.070	-.093	-.193	.015

ROUND COPPER WIRE TABLE 3

SIZE AWG	BARE DIAMETER	AREA □"	Ω/1000' @25°C	SINGLE FORMVAR	HEAVY FORMVAR	SINGLE GLASS FORMVAR	BARE WT. #/1000'	SINGLE GLASS SILICONE	DOUBLE GLASS SILICONE
36	.0050	.0000196	424	.0056	.0060		.0757		
35	.0056	.0000246	338	.0062	.0066		.0949		
34	.0063	.0000312	266	.0070	.0074		.1201		
33	.0071	.0000396	210	.0079	.0084		.1526		
32	.0080	.0000503	165	.0088	.0094	.0121	.1937		
31	.0089	.0000622	134	.0097	.0104	.0130	.2398		
30	.0100	.0000785	106	.0108	.0116	.0142	.3025	.0132	.0152
29	.0113	.000100	83.1	.0122	.0130	.0156	.3866	.0145	.0165
28	.0126	.000125	66.4	.0135	.0144	.0169	.4806	.0158	.0178
27	.0142	.000158	52.6	.0152	.0161	.0186	.6101	.0174	.0194
26	.0159	.000199	41.7	.0169	.0179	.0203	.7650	.0191	.0211
25	.0179	.000252	33.0	.0190	.0200	.0224	.970	.0211	.0231
24	.0201	.000317	26.2	.0213	.0223	.0263	1.223	.0251	.0276
23	.0226	.000401	20.7	.0238	.0249	.0289	1.546	.0276	.0301
22	.0254	.000507	16.4	.0266	.0277	.0317	1.937	.0303	.0328
21	.0285	.000638	13.0	.0299	.0310	.0349	2.459	.0335	.0360
20	.0320	.000804	10.3	.0334	.0346	.0384	3.099	.0370	.0395
19	.0360	.00102	8.14	.0374	.0386	.0424	3.900	.0409	.0434
18	.0403	.00126	6.59	.0418	.0431	.0468	4.914	.0453	.0478
17	.0453	.00159	5.22	.0469	.0482	.0519	6.213	.0503	.0528
16	.0508	.00204	4.07	.0524	.0538	.0575	7.812	.0558	.0583
15	.0571	.00255	3.26	.0588	.0602	.0639	9.87	.0621	.0646
14	.0641	.00322	2.58	.0659	.0673	.0710	12.44	.0691	.0716
13	.072	.00407	2.04	.0738	.0753	.0789	15.69	.0770	.0795
12	.0808	.00515	1.61	.0827	.0842	.0877	19.76	.0858	.0883
11	.0907	.00650	1.28	.0927	.0942	.0977	24.90	.0957	.0982
10	.102	.00817	1.02	.1039	.1055	.1089	31.43	.1069	.1094
9	.114	.0102	.814	.1165	.1181	.1225	39.62	.1204	.1254
8	.129	.0131	.634	.1306	.1323	.1366	49.98	.1345	.1395
7	.144	.0163	.510	.1465	.1482	.1525	63.03	.1503	.1553
6	.162	.0206	.403	.1643	.1661	.1703	79.44	.1680	.1730
5	.182	.0260	.319	.1842	.1861	.1902	100.2	.1879	.1929
4	.204	.0327	.254				126.3	.2103	.2153
3	.229	.0412	.202				159.3		
2	.258	.0523	.159				200.9		
0	.325	.0830	.100						
2/0	.365	.105	.0791						
4/0	.460	.166	.0500						

CURVE 1



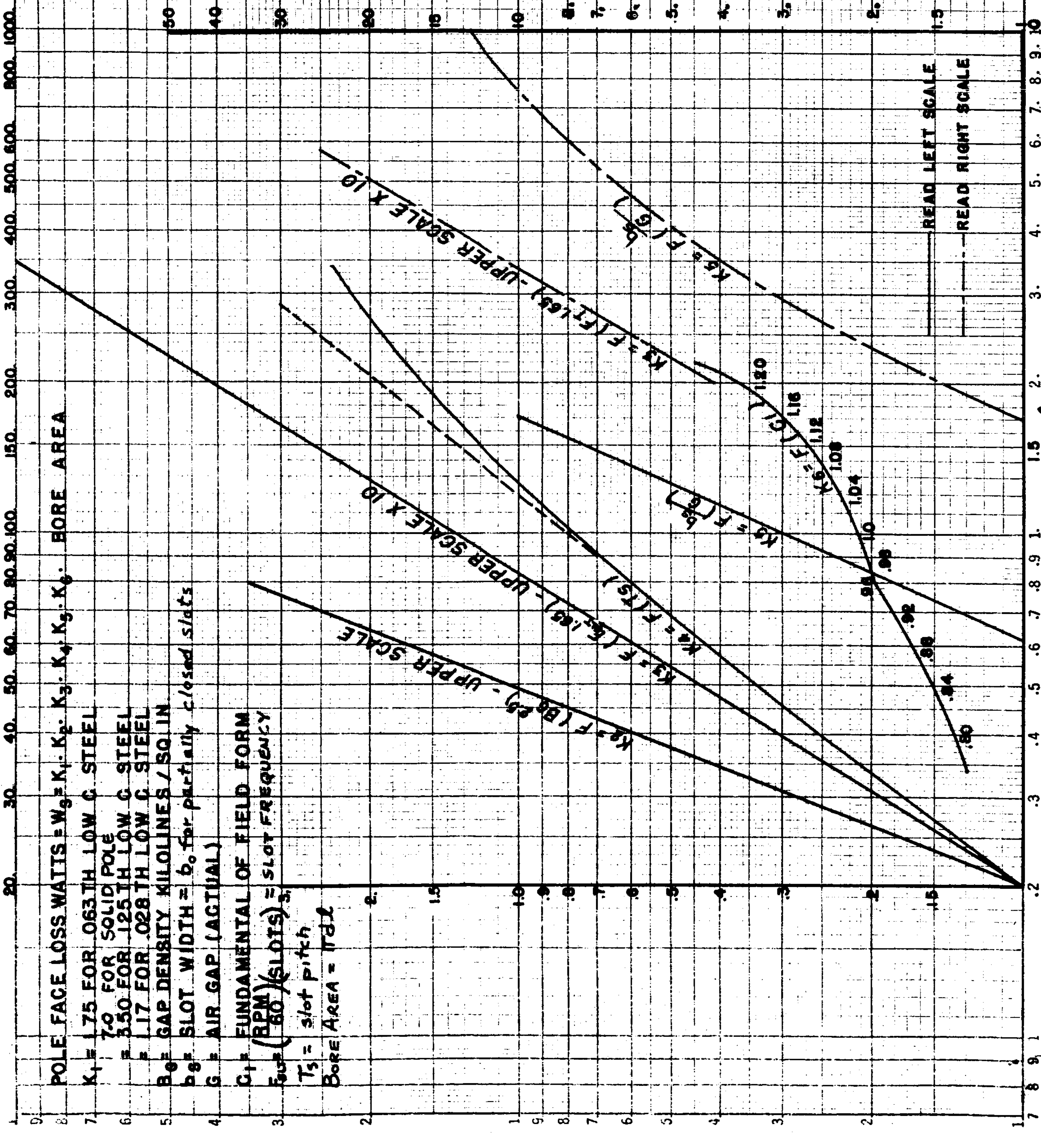


Empirical End-Winding Constant
Versus Pole Pitch
From E. C. Barnes
"An Experimental Study of Machine
End Turn Reactance"
AIEE Trans. Vol. 70 1951
pp 671-679.

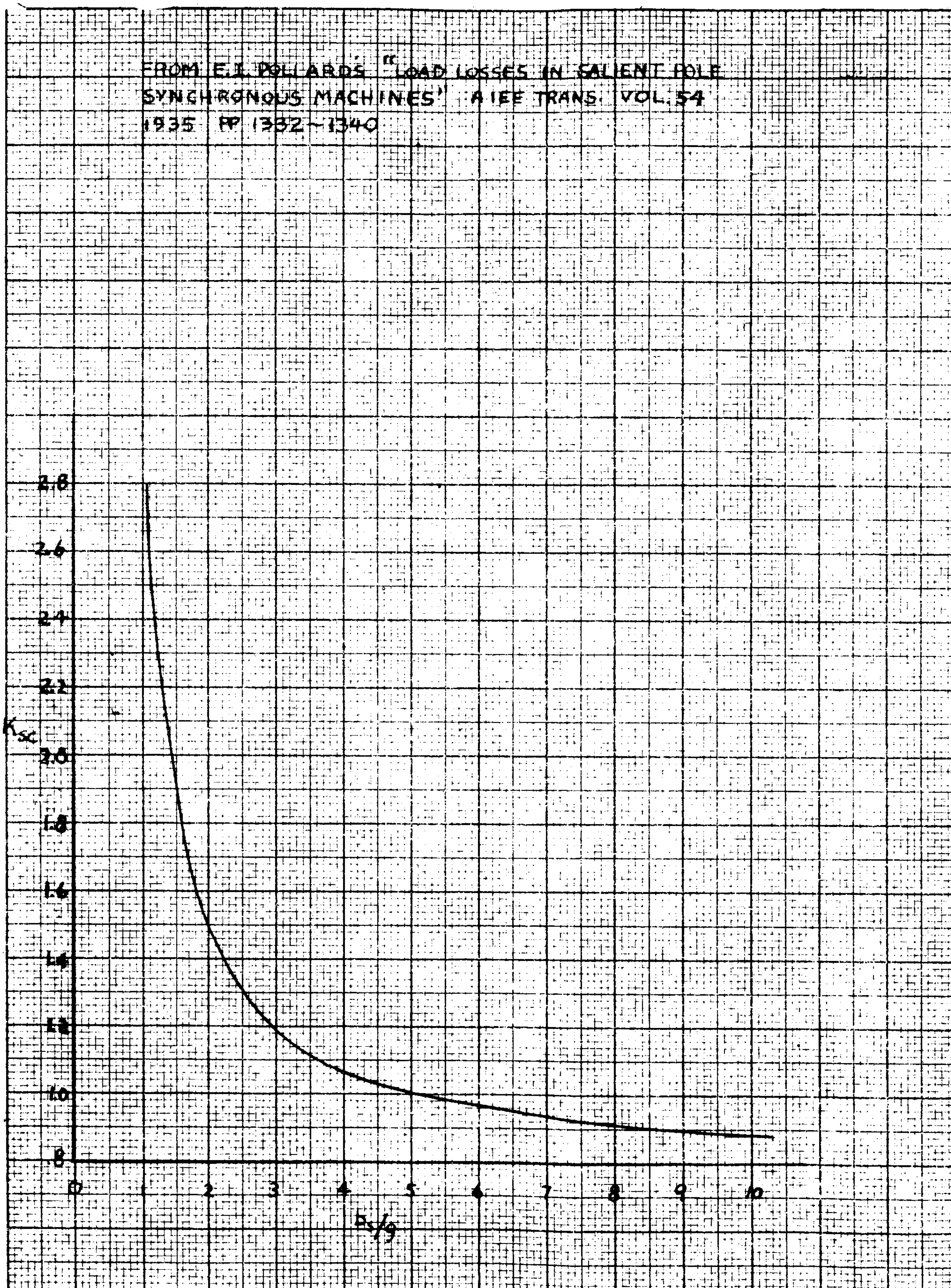
CURVE 2

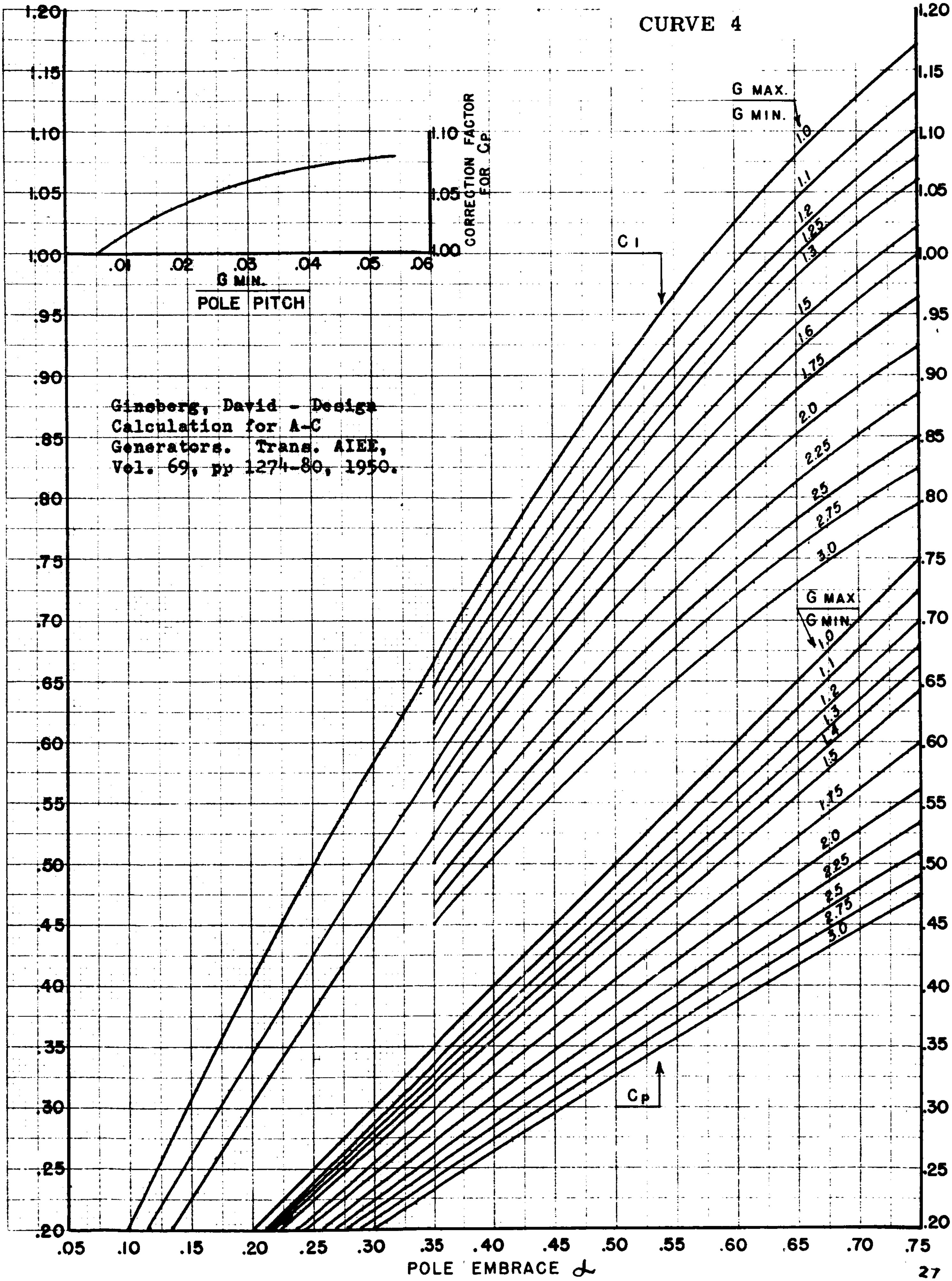
From Kennard and Spooner "Surface Iron Losses with Respect to Laminated Materials", Trans. AIEE, Vol. 43, 1924, pp 262-281.

REFER TO ITEM (150) IN SALIENT POLE DESIGN MANUAL FOR SAMPLE USE OF THIS CURVE

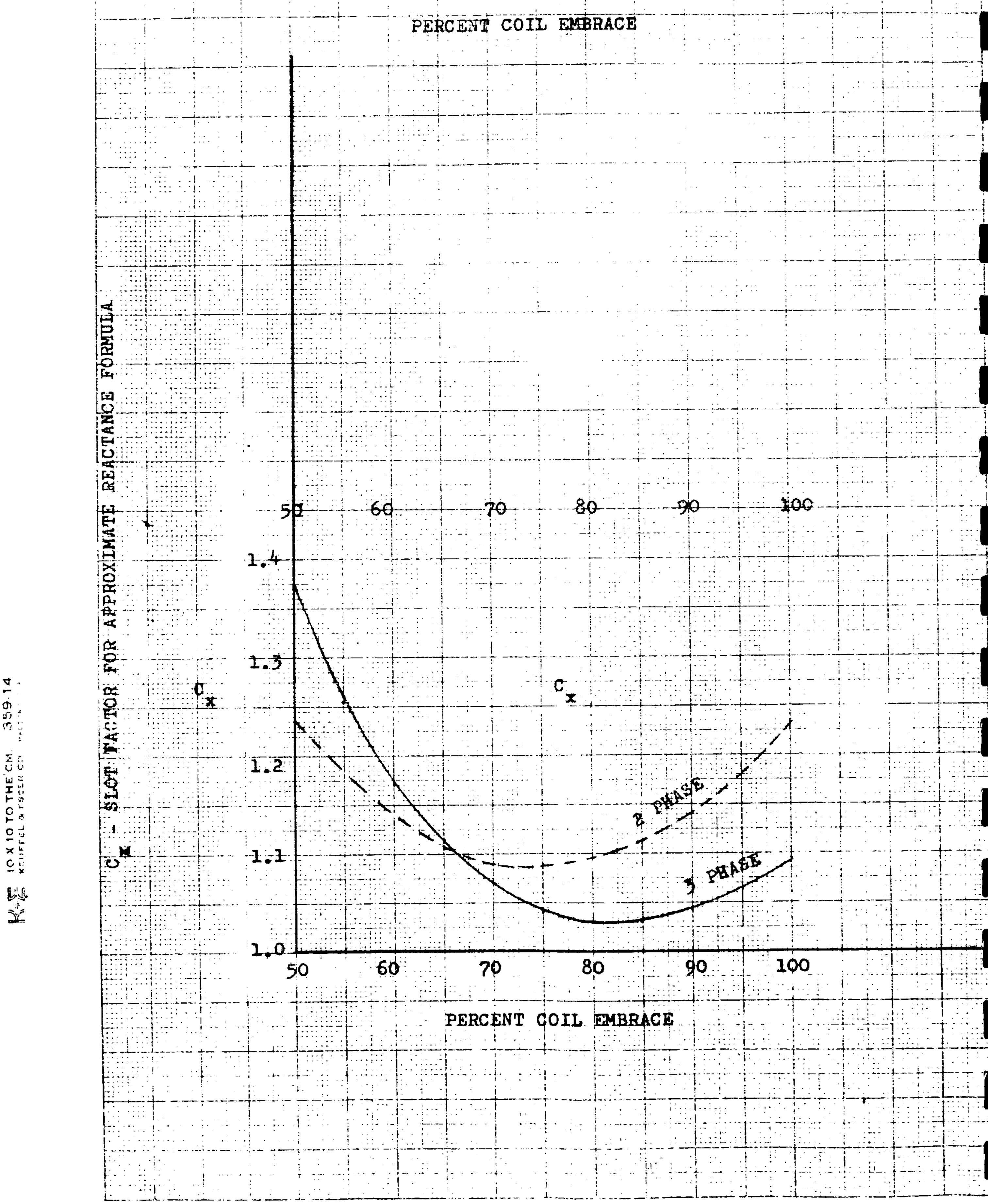


CURVE-3


FROM E.I. POLLARD'S "LOAD LOSSES IN SALIENT POLE SYNCHRONOUS MACHINES" AIEE TRANS. VOL. 54 1935 PP 1332-1340
K_{sc}
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
0
2
3
4
5
6
7
8
9
10
P_s/g


CURVE 4
Ginsberg, David - Design Calculation for A-C Generators. Trans. AIEE, Vol. 69, pp 1274-80, 1950.
G MAX. / G MIN.
1.0
1.1
1.2
1.25
1.3
1.5
1.6
1.75
2.0
2.25
2.5
2.75
3.0
C I
G MAX / G MIN.
1.0
1.1
1.2
1.3
1.4
1.5
1.75
2.0
2.25
2.5
2.75
3.0
Cp
CORRECTION FACTOR FOR Cp
G MIN. / POLE PITCH
.01
.02
.03
.04
.05
.06
1.00
1.05
1.10
POLE EMBRACE α
.05
.10
.15
.20
.25
.30
.35
.40
.45
.50
.55
.60
.65
.70
.75
.20
.25
.30
.35
.40
.45
.50
.55
.60
.65
.70
.75
.80
.85
.90
.95
1.00
1.05
1.10
1.15
1.20

CURVE 5


PERCENT COIL EMBRACE
50
60
70
80
90
100
1.4
1.3
1.2
1.1
1.0
C_x
C_x
C_M - SLOT FACTOR FOR APPROXIMATE REACTANCE FORMULA
2 PHASE
3 PHASE
PERCENT COIL EMBRACE
10 X 10 TO THE CM. 359-14

NO-LOAD DAMPER LOSS CURVE 7

$$D.L. = \frac{1.246\, p\, n_b\, l_b\, \rho}{10^6\, a_b}\left[\tau_s B_g K_p K_g\right]^2 \left[K_{f_1}\left(\frac{K_{w1}}{2\lambda_s + \frac{\lambda_g}{K_{\phi 1}}}\right)^2 + K_{f_2}\left(\frac{K_{w2}}{2\lambda_s + \frac{\lambda_g}{K_{\phi 2}}}\right)^2\right]$$

= LOSS IN KW

$$\lambda_s = \frac{h_r}{b_r} + \lambda_t + \lambda_e$$

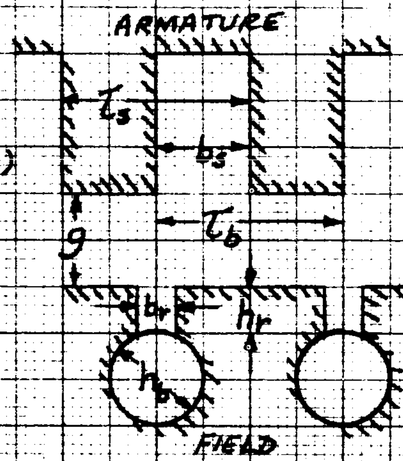
$$\lambda_g = \frac{\tau_b}{K_g g} = \frac{\tau_b}{g'}$$

a_b = BAR AREA IN SQ IN.

n_b = BARS/POLE

p = NO. POLES

l_b = LENGTH BAR IN.



K_g = CARTER'S COEFFICIENT (TOTAL)

$K_p = fn(b_s/g)$, CURVE (a.) ($b_s = b_o$ for partially closed slots)

K_{f_1} AND $K_{f_2} = fn(f_s/\rho)$ CURVE (b)

ρ = DAMPER BAR RESISTIVITY (MICROHMS PER CU. IN.)

K_{W1} AND $K_{W2} = fn(b_s/\tau_s)$, CURVE (C_1) AND (C_2)

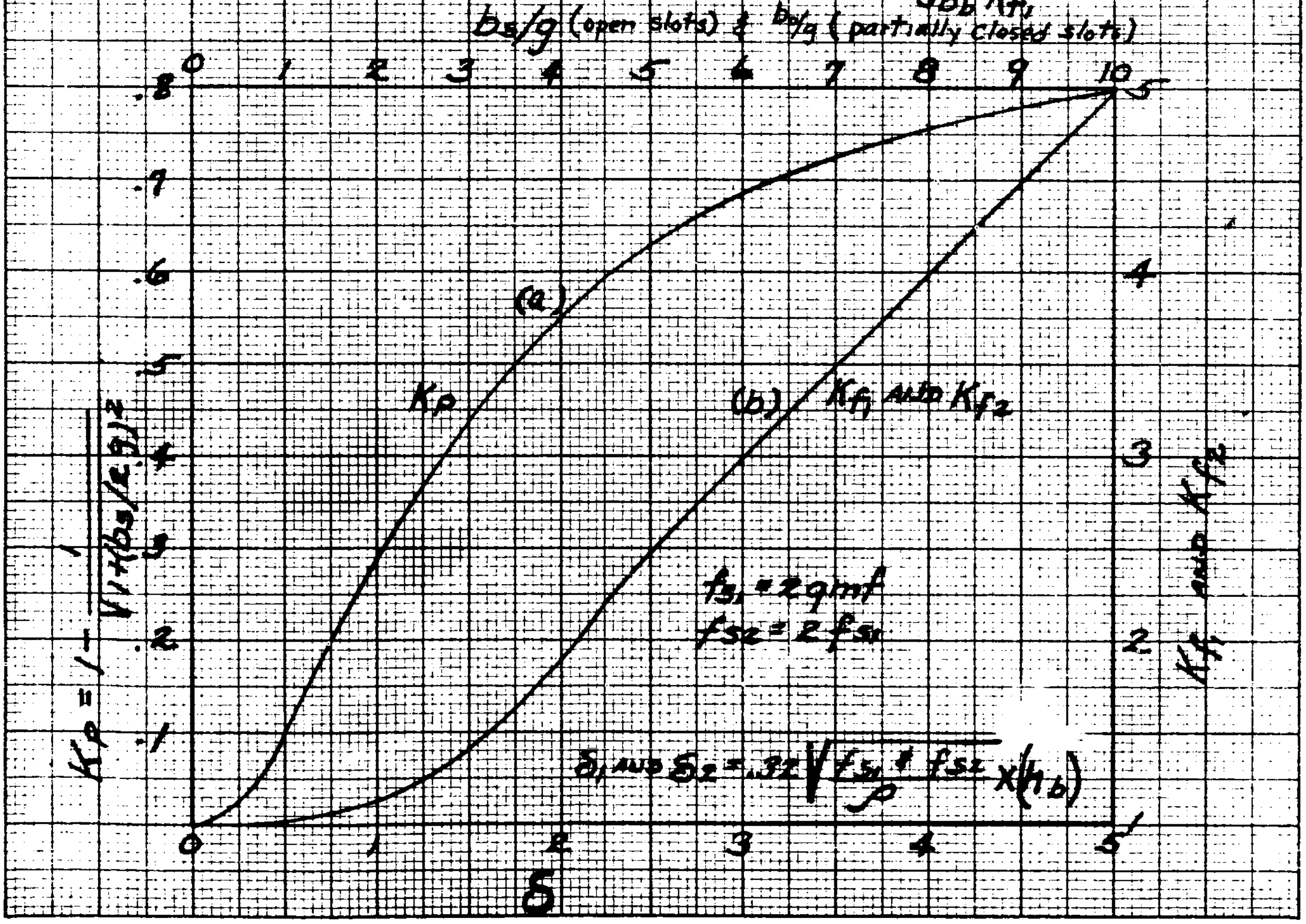
$K_{\phi 1}$ AND $K_{\phi 2} = fn(\tau_b/\tau_s)$, CURVE ($d_1$) AND ($d_2$)

$\lambda_t = fn(b_r/gK_g)$ CURVE (e)

B_g IS IN KILOLINES PER SQ INCH

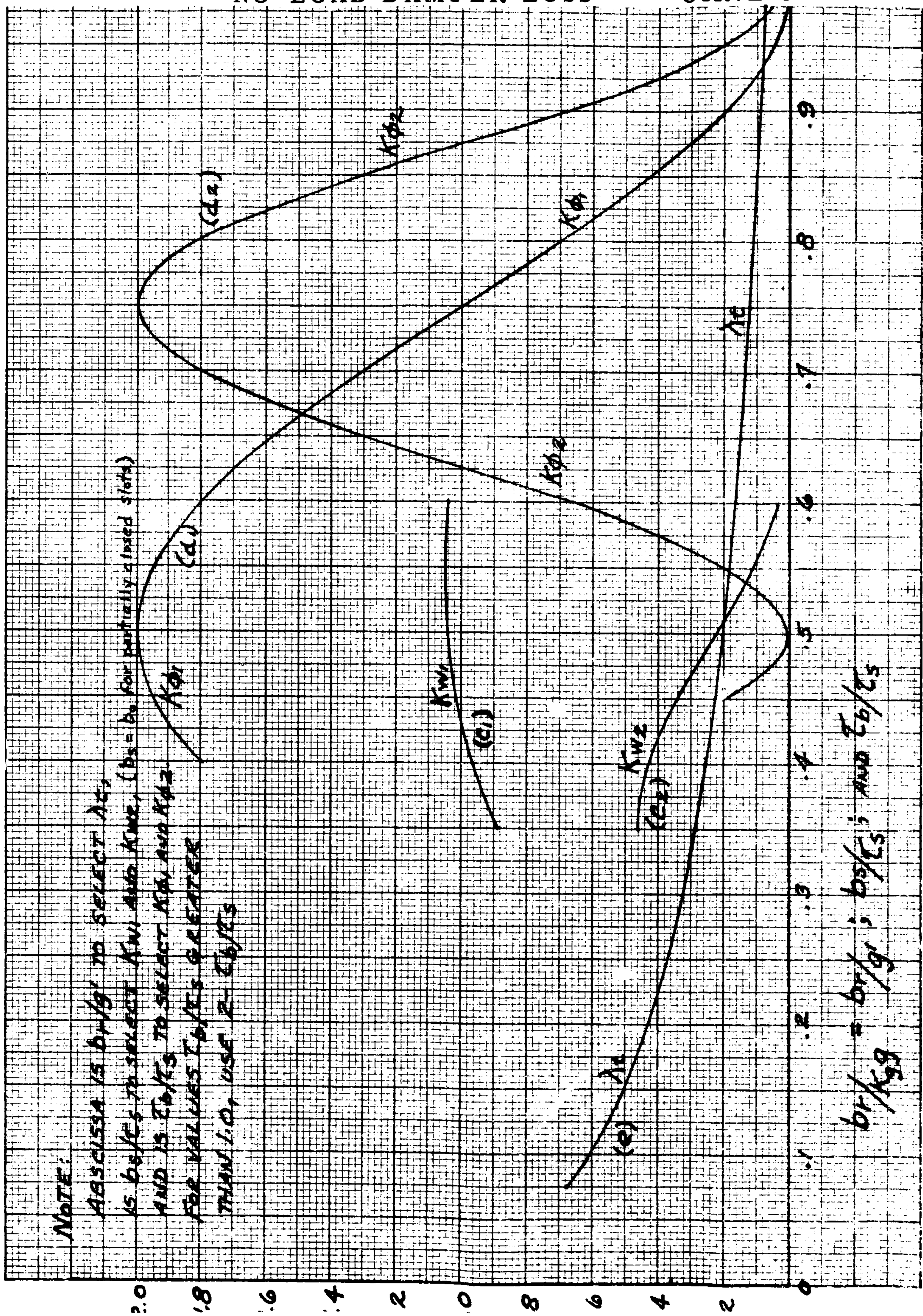
$$\lambda_e = \frac{.75}{K_{f_1}} \text{ (FOR ROUND OR SQ. BARS)}$$

$$\lambda_e = \frac{h_b}{3 b_b K_{f_1}} \text{ (FOR RECT. BARS)}$$

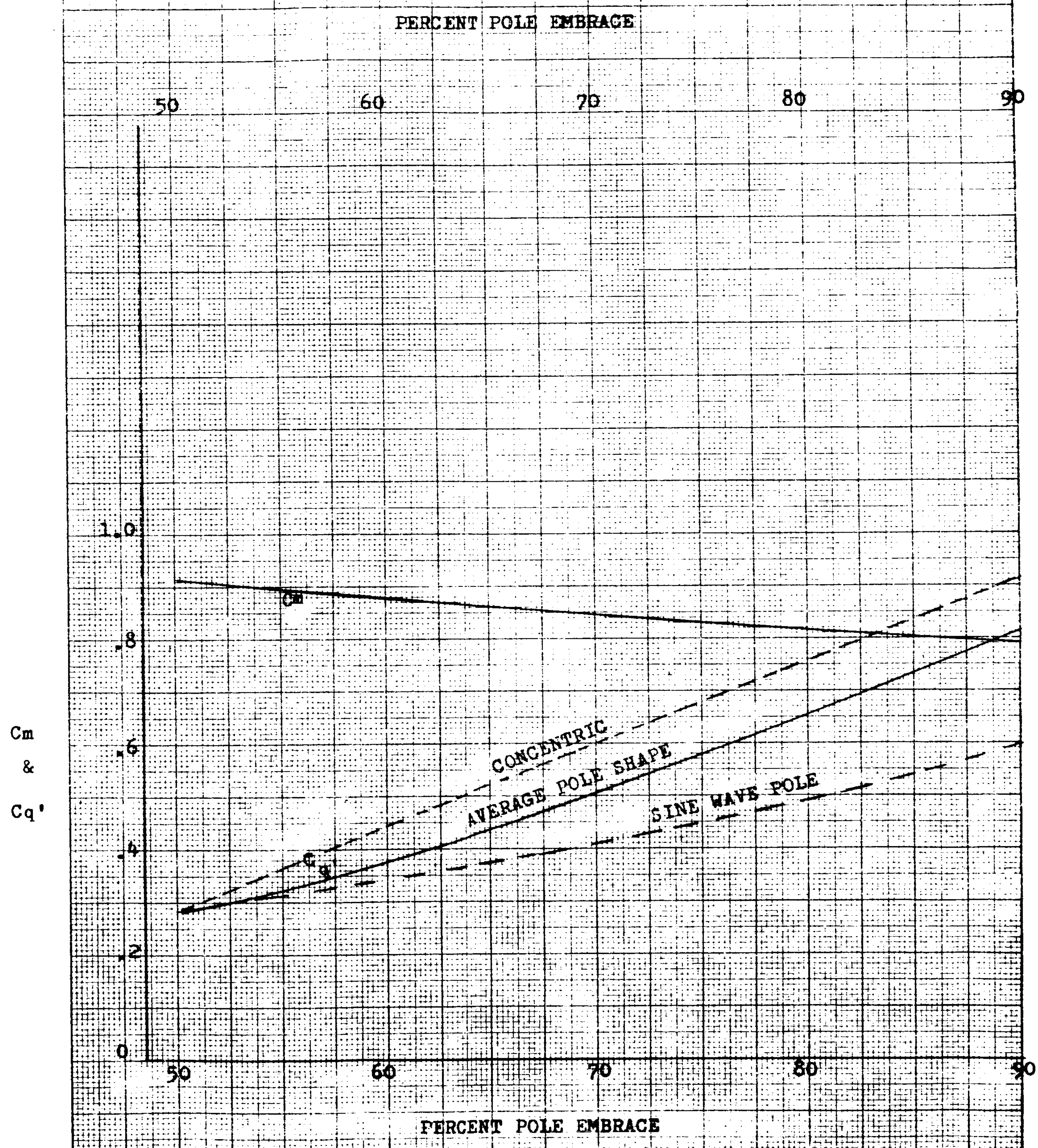


From E. I. Pollard "Calculation of No-Load Damper Winding Loss in Synchronous Machines", AIEE Vol. 51, 1932, pp 477-81.

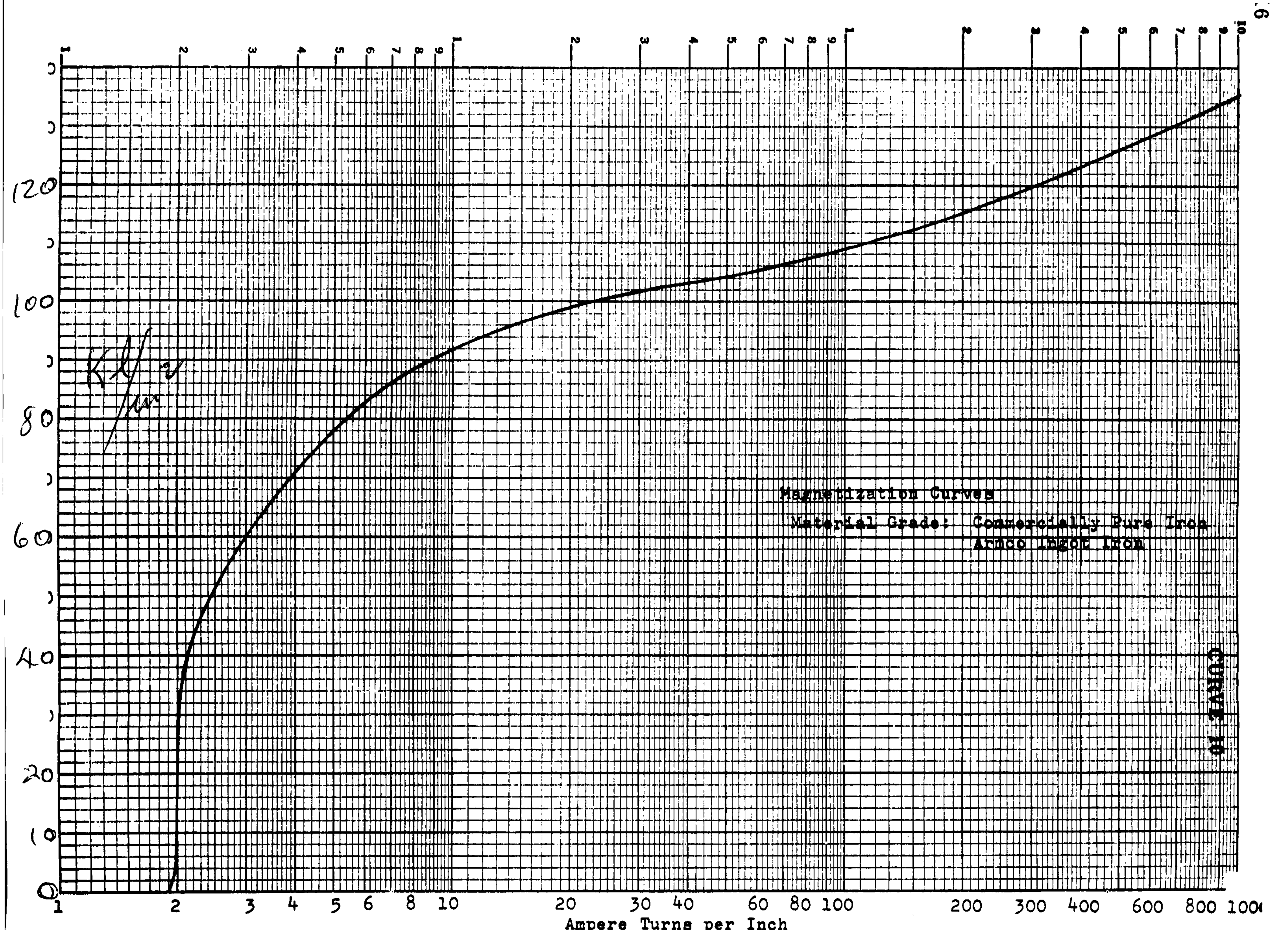
NO-LOAD DAMPER LOSS CURVE 8



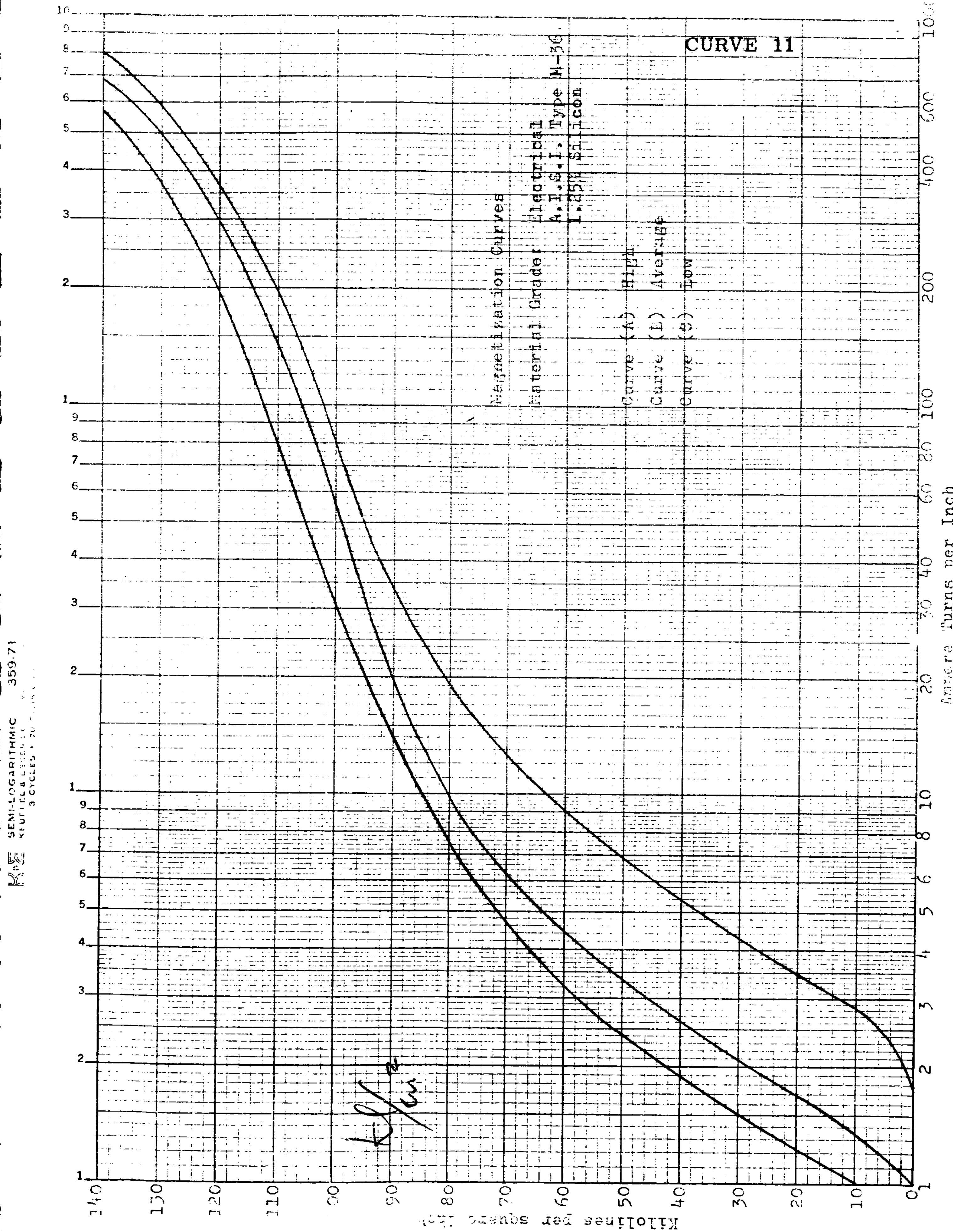
CURVE 9

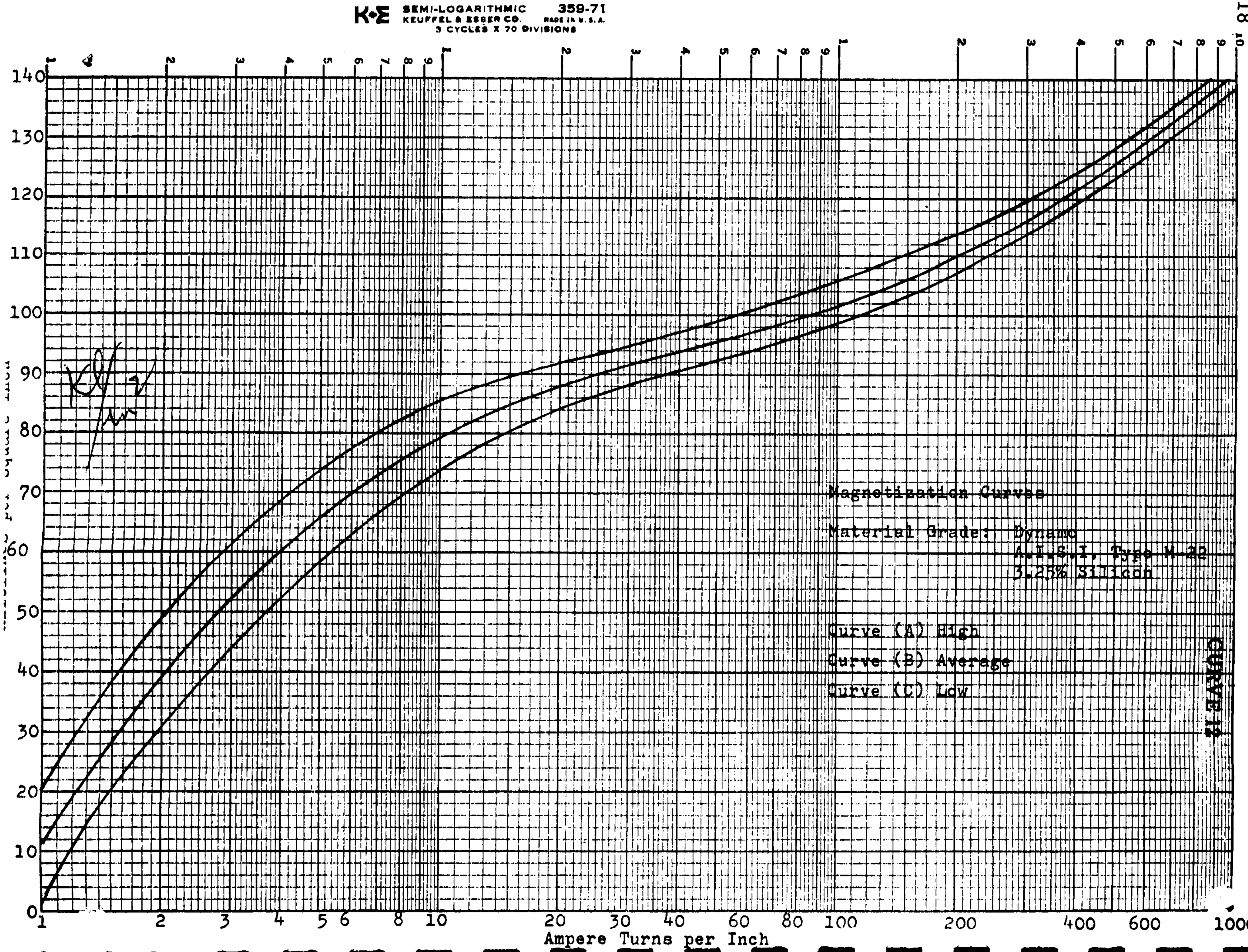


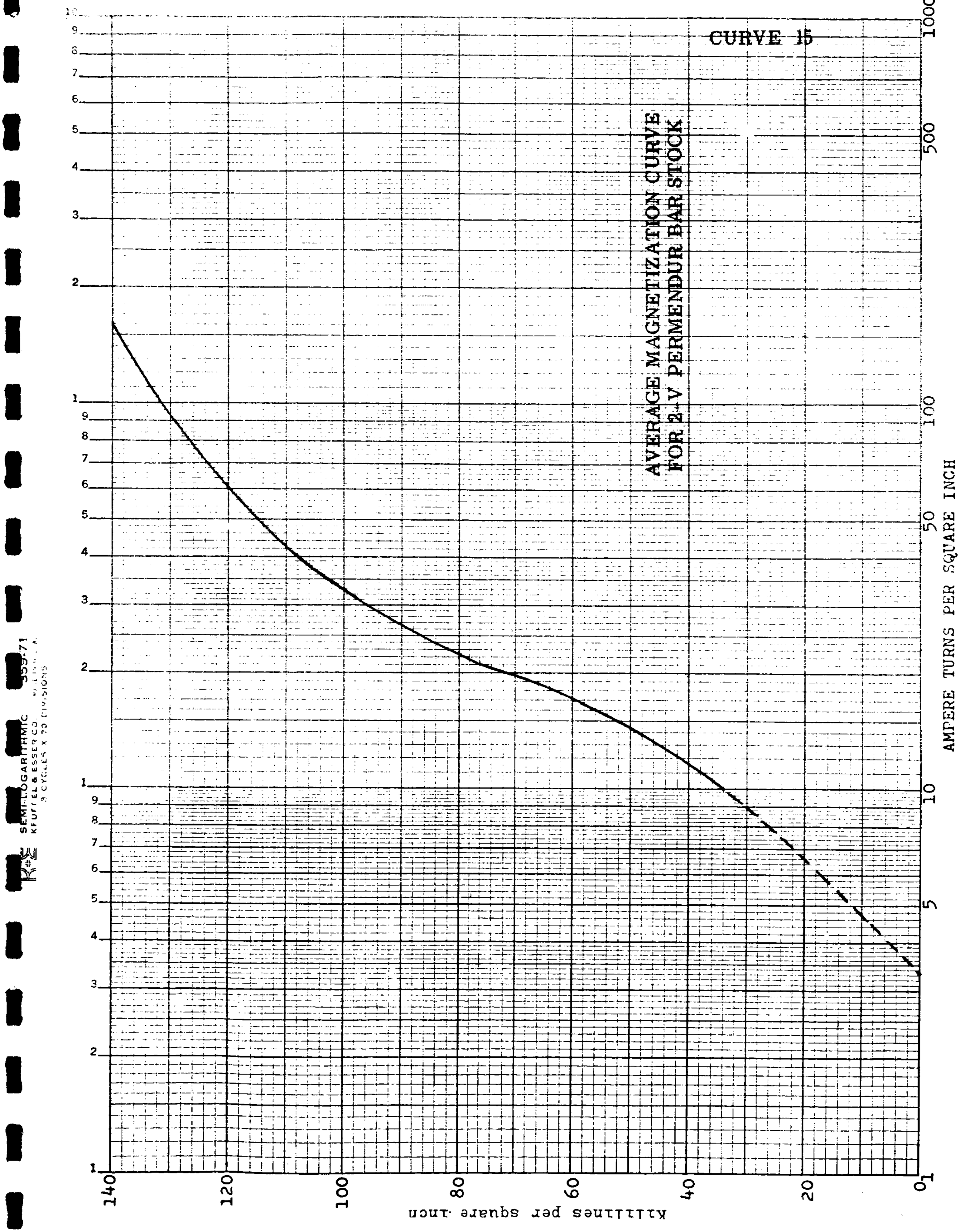
SALIENT POLE MACHINE CONSTANTS C_m $C_{q'}$

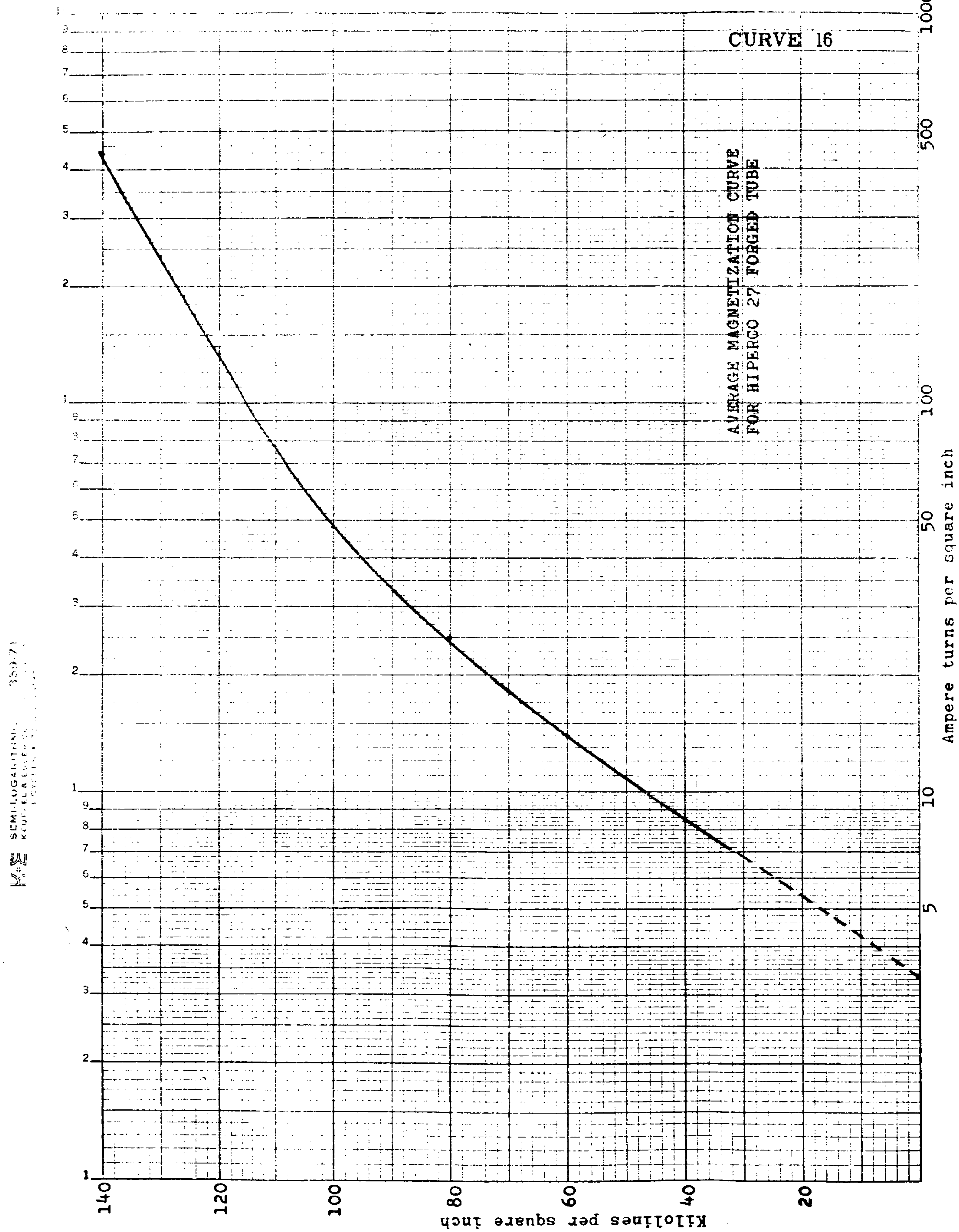

CURVE 10
Magnetization Curves
Material Grade: Commercially Pure Iron
Armco Ingot Iron
Ampere Turns per Inch
1
2
3
4
5
6
8
10
20
30
40
60
80
100
200
300
400
600
800
1000
0
10
20
40
60
80
100
120
Kl/in²

CURVE 11


Magnetization Curves
Material Grade: Electrical
A.I.S.I. Type M-36
1.25% Silicon
Curve High
Curve Average
Curve Low
Ampere Turns per Inch
Kilolines per square inch
SEMI-LOGARITHMIC
3 CYCLES
359-71

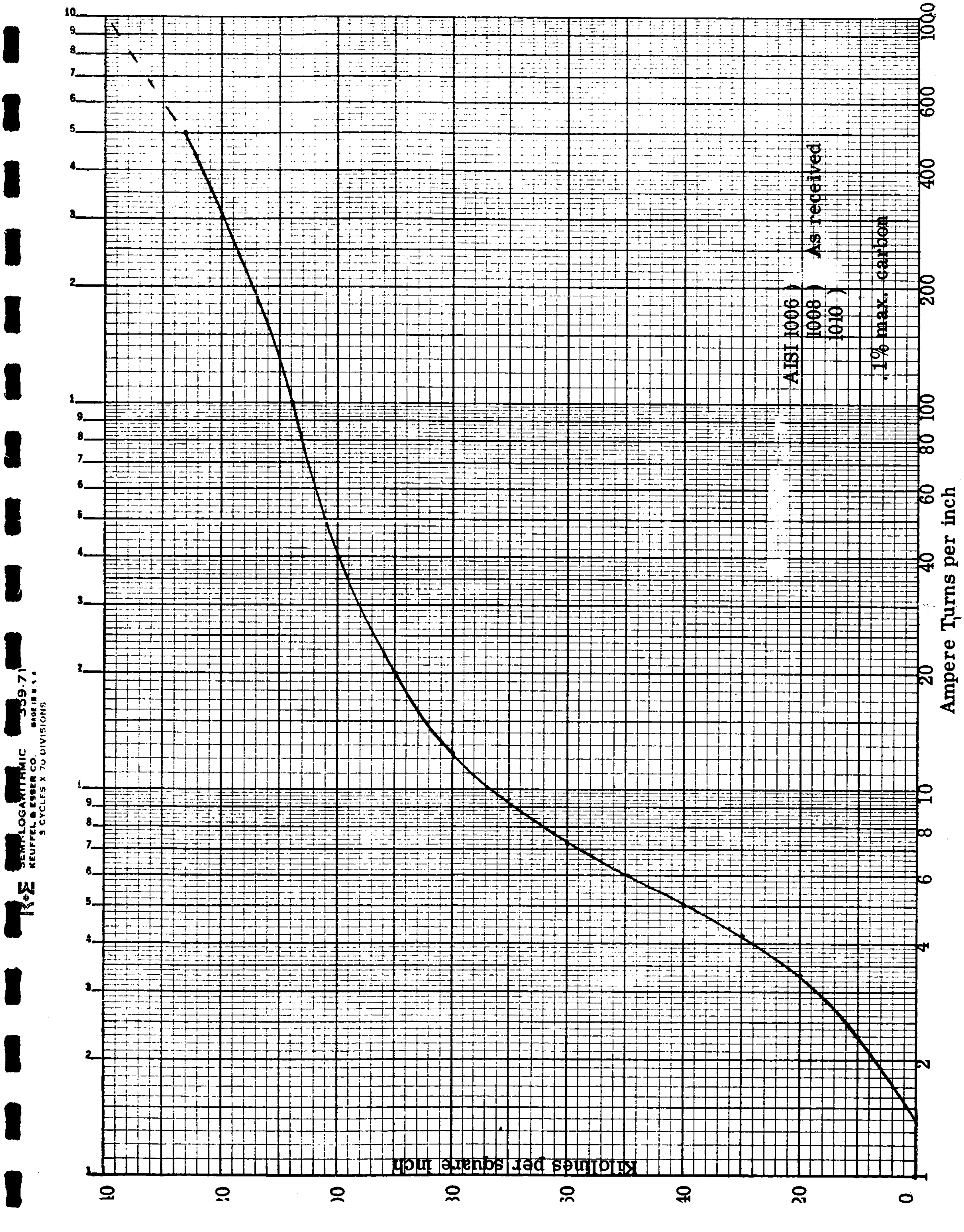

K+E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS
CURVE 12
Magnetization Curves
Material Grade: Dynamo
A.I.S.I, Type M-22
3.25% Silicon
Curve (A) High
Curve (B) Average
Curve (C) Low
Ampere Turns per Inch
1
2
3
4
5
6
8
10
20
30
40
60
80
100
200
400
600
1000
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140


CURVE 15
AVERAGE MAGNETIZATION CURVE
FOR 2-V PERMENDUR BAR STOCK
AMPERE TURNS PER SQUARE INCH
Kilolines per square inch
1000
500
100
50
10
5
1
140
120
100
80
60
40
20
0
SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO.
3 CYCLES X 70 DIVISIONS



CURVE 16
AVERAGE MAGNETIZATION CURVE
FOR HIPERCO 27 FORGED TUBE
Ampere turns per square inch
Kilolines per square inch
1000
500
100
50
10
5
140
120
100
80
60
40
20

CURVE 17



AISI 1006)
1008)
1010)
As received
.1% max. carbon
Ampere Turns per inch
Kilolines per square inch
SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO.
3 CYCLES X 70 DIVISIONS

Homopolar Inductor
~~SALIENT-POLE WOUND-POLE~~ SYMBOLS

Calculation Location	Symbol	Explanation
(78)	A	Ampere conductors per inch of stator periphery
(46)	a_c	Strand area (stator)
(89)	a_{cf}	Field conductor area
(85)	a_p	Pole area
(20)	B	Density
(128)	B_c	Core density
(125)	B_g	Gap density
(140)	B_p	Pole density no load
(163)	B_{pl}	Pole density full load
(127)	B_T	Tooth density
(89)	b_{bo}	Width of slot opening (damper)
(89)	b_{b1}	Width of rectangular slot (damper)
(76)	b_h	Pole head width
(22)	b_o	Width of slot opening (stator)
(76)	b_p	Pole body width
(22)	b_s	Stator slot dimension per Fig. 1
(15)	b_v	Width of duct
(22)	b_1	Stator slot dimensions per Fig. 1
(22)	b_2	Stator slot dimensions per Fig. 1
(22)	b_3	Stator slot dimensions per Fig. 1

Calculation Location	Symbol	Explanation
(74)	C_m	Demagnetizing factor
(73)	C_p	Average/maximum field form
(75)	C_q	Cross magnetization factor
(72)	C_w	Winding constant
(71)	C_1	Ratio max. to fund.
(32)	c	Parallel circuits
(12)	D	Stator outside diameter
(11)	d	Stator inside diameter
(35)	d_b	Diameter of bender pin
(11a)	d_r	Rotor outside diameter
(3)	E	Line volts
(145)	E_F	Field volts no load
(168)	E_{FFL}	Field volts full load
(4)	E_{PH}	Phase volts
(55)	EF_{top}	Eddy factor top
(56)	EF_{bot}	Eddy factor bottom
(130)	F_c	Stator core ampere turns
(131)	F_g	Air gap ampere turns
(165)	F_{FL}	Total ampere turns full load
(142)	F_{NL}	Total ampere turns no load
(141)	F_p	Pole ampere turns at no load
(164)	F_{PL}	Pole ampere turns at full load
(136)	F_{sc}	Short circuit ampere turns

Calculation Location	Symbol	Explanation
(129)	F_t	Stator tooth ampere turns
(147)	F&W	Friction and windage
(5a)	f	Frequency
(69)	g_e	Effective air gap
(59a)	g_{max}	Maximum air gap
(22)	h_o	
(22)	h_1	
(22)	h_2	
(22)	h_3	Stator slot dimension
(22)	h_s	
(22)	h_t	
(22)	h_w	
(22)		
(89)	h_{bo}	Height of slot opening
(89)	h_{b1}	Rectangular bar thickness
(24)	h_c	Depth below slot
(76)	h_p	Pole Height
(37)	h_{st}	Uninsulated strand height
(38)	h'_{st}	Distance between center line of strand
(8)	I_{PH}	Phase current
(166)	I_{FFL}	Field amperes at full load

Calculation Location	Symbol	Explanation
(143)	I_{FNL}	Field amperes at no load
(146)	I^2R_F	Field loss
(158)	I^2R_S	Stator copper loss
(9a)	K_c	Adjustment factor
(43)	K_d	Distribution factor
(18)	K_i	Stacking factor
(44)	K_p	Pitch factor
(67)	K_s	Carter coefficient
(42)	K_{SK}	Skew factor
(2)	KVA	Machine rating
(151)	K_1	Pole face loss factor
(19)	k	Watts per lb.
(48)	L_E	End extension one turn
(113)	L_F	Field self inductance
(13)	ℓ	Gross core length
(93)	ℓ_b	Damper bar length
(136)	ℓ_{e2}	Coil extension straight portion
(76)	ℓ_n	Pole head length
(76)	ℓ_p	Pole body length
(17)	ℓ_s	Solid core length
(49)	ℓ_t	1/2 mean turn
(100)	ℓ_{tr}	Mean length of field turns
(5)	m	Number of phases

Calculation Location	Symbol	Explanation
(34)	N_{st}	Strands per conductor
(92)	n_b	Number of damper bars
(45)	n_e	Effective conductors
(99)	n_p	Number of field turns
(30)	n_s	Conductor per slot
(14)	n_v	Number of ducts
(9)	P.F.	Power factor
(6)	p	Number of poles
(23)	Q	Number of slots
(53)	$R_{ph\ (cold)}$	Stator resistance at 20 °C
(54)	$R_{ph\ (hot)}$	Stator resistance at X °C
(107)	$R_{F\ (cold)}$	Field resistance at 20 °C
(108)	$R_{f\ (hot)}$	Field resistance at X °C
(137)	SCR	Short circuit ratio
(47)	s_s	Stator current density
(144)	s_f	Field current density
(133)	T_a	Armature time constant
(134)	T'_d	Transient time constant
(135)	T''_d	Subtransient time constant
(132)	T'_{do}	Open circuit time constant
(149)	W_c	Stator core loss
(172)	W_{DFL}	Damper loss at full load

Calculation Location	Symbol	Explanation
(157)	W_{DNL}	Damper loss at no load
(171)	W_{PFL}	Pole face losses at full load
(150)	W_{PNL}	Pole face loss at no load
(170)	W_{TFL}	Stator tooth loss at full load
(148)	W_{TNL}	Stator tooth loss at no load
(98a)	V_r	Peripheral speed of rotor
(79)	X	Reactance factor
(81)	X_{ad}	Reactance direct axis
(82)	X_{aq}	Reactance quadrature axis
(83)	X_d	Synchronous reactance direct axis
(119)	X_d'	Stator transient reactance
(120)	X_d''	Subtransient reactance direct axis
(115)	X_{Dd}	Leakage reactance direct axis
(117)	X_{Dq}	Leakage reactance quadrature axis
(118)	X_{du}'	Unsaturated transient reactance
(112)	X_f	Field leakage reactance
(80)	X_ℓ	Leakage
(84)	X_q	Synchronous reactance quadrature axis
(121)	X_q''	Subtransient reactance quadrature axis
(123)	X_o	Zero sequence reactance
(122)	X_2	Negative sequence reactance
(96)	$X_D\,^oC$	Expected damper bar ^oC

Calculation Location	Symbol	Explanation
(103)	X_F ^oC	Expected field temp. in ^oC
(50)	X_S ^oC	Expected temp. stator in ^oC
(95)	ρ_D	Resistivity of damper bar at 20^oC
(51)	ρ_S	Resistivity of stator cond at 20^oC
(104)	ρ_F	Resistivity of field conductor
(138)	ϕ_ℓ	Leakage flux at no load
(160)	$\phi_{\ell\ell}$	Leakage flux at full load
(126)	ϕ_P	Flux per pole
(139)	ϕ_{PT}	Total flux per pole at no load
(162)	ϕ_{PTL}	Total flux per pole at full load
(124)	ϕ_T	Total flux
(94)	τ_b	Damper bar pitch
(41)	τ_p	Pole pitch
(26)	τ_s	Slot pitch
(27)	$\tau_{s\ 1/3}$	Slot pitch 1/3 distance from narrowest point
(40)	τ_{sk}	Stator slot skew
(70)	λ_a	Air gap permeance
(63)	λ_E	End permeance
(86)	$\lambda_{e\ell}$	Pole end leakage permeance
(62)	λ_i	Stator conductor permeance
(88)	$\lambda_{s\ell}$	Pole side leakage permeance
(87)	$\lambda_{t\ell}$	Pole tip leakage permeance

(1)	--	<u>DESIGN NUMBER</u> - To be used for filing purposes
(2)	KVA	<u>GENERATOR KVA</u>
(3)	E	<u>LINE VOLTS</u>
(4)	E_{PH}	<u>PHASE VOLTS</u> - For 3 phase, Y [redacted] connected generator $E_{PH} = \frac{\text{(Line Volts)}}{\sqrt{3}} = \frac{(3)}{\sqrt{3}}$ Δ For 3 phase, [redacted] connected generator E_{PH} = (Line Volts) = (3)
(5)	m	<u>PHASES</u> - Number of
(5a)	f	<u>FREQUENCY</u> - In cycles per second
(6)	P	<u>POLES</u> - Number of
(7)	RPM	<u>SPEED</u> - In revolutions per minute
(8)	I_{PH}	<u>PHASE CURRENT</u> - In amperes at rated load
(9)	P.F.	<u>POWER FACTOR</u> - Given in per unit
(9a)	K_c	<u>ADJUSTMENT FACTOR</u> - When P. F. = 0. to .95 set K_c = 1.; when P. F. = .95 to 1. set K_c = 1.05
(10)	--	<u>LOAD POINTS</u> - The computer program is set up to have the 0.%, 100%, 150%, 200% load points as standard outputs. There is an additional space available on the output sheet for one optional load point. This optional

load point will be the designer's choice and can be selected anywhere in the range of 0 to 200% load. When an optional load calculation is required, insert the per unit load value on the input sheet. The optional load point will be calculated in addition to the standard points listed above. For example, insert .33 on the input sheet when the optional load calculation for 33% load is required in addition to the standard points.

If only the standard points are required, insert 0.0 on the input sheet and the optional load column will be blank.

(11)	d	<u>STATOR PUNCHING I.D.</u> - The inside diameter of the stator punching in inches.
(11a)	d_r	<u>ROTOR O.D.</u> - The outside diameter of the rotor in inches.
(12)	D	<u>PUNCHING O.D.</u> - The outside diameter of the stator punching in inches.
(13)	ℓ	<u>GROSS STATOR CORE LENGTH</u> - In inches.
(14)	n_v	<u>RADIAL DUCTS</u> - Number of.
(15)	b_v	<u>RADIAL DUCT WIDTH</u> - In inches.
(16)	K_i	<u>STACKING FACTOR</u> - This factor allows for the coating (core plating) on the punchings, the burrs due to slotting, and the deviations in flatness. Approximate values of K_i are given in Table IV.

THICKNESS OF LAMINATIONS (INCHES)	GAGE	K_i
.014	29	0.92
.018	26	0.93
.025	24	0.95
.028	23	0.97
.063	--	0.98
.125	--	0.99

TABLE IV

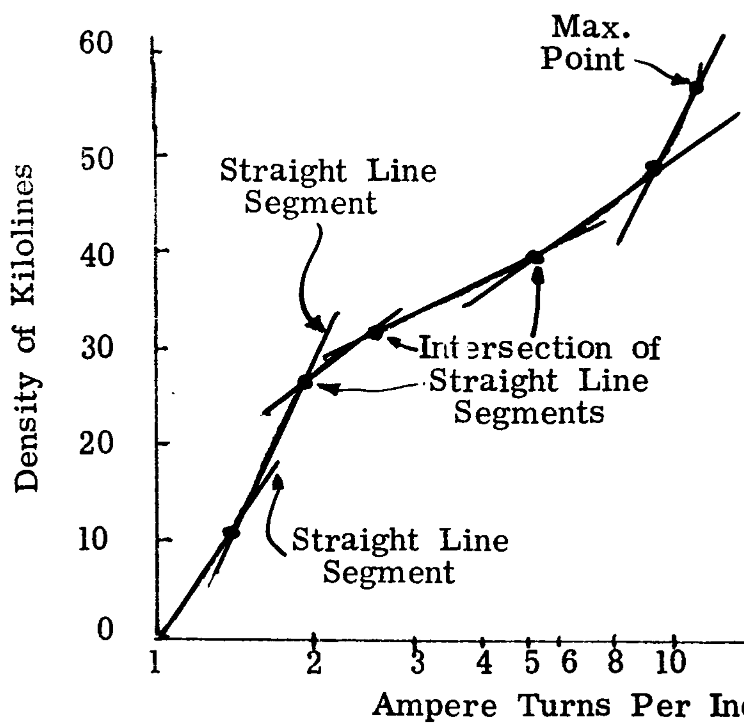
(17) ℓ_s SOLID CORE LENGTH - The solid length is the gross length times the stacking factor. If ventilating ducts are used, their length must be subtracted from the gross length also.

$$\ell_s = (K_i) \times (\ell - (n_v)(b_v)) = (16) \quad (13) - (14)\ (15)$$

(18) MATERIAL - This input is used in selecting the proper magnetization curves for stator, yoke, pole, and shaft; when different materials are used. Separate spaces are provided on the input sheet for each section mentioned above. Where curves are available on card decks, use the proper identifying code. Where card decks are not available submit data in the following manner:

The magnetization curve must be available on semi-log paper. Typical curves are shown in this manual on Curves 15 and 16. Draw straight line segments through the curve starting with zero density. Record the coordinates of the points where the straight line segments intersect. Submit these coordinates as input data for the magnetization curve. The maximum density point must be submitted first.

Refer to Figure below for complete sample



Sample Input Data

	Density	NI
1)	55	Max.
2)	0	0
3)	10	1.5
4)	27	1.9
5)	32	2.5
6)	40	4.9
7)	48	9.0
8)	55	12.0

(19) k **WATTS/LB** - Core loss per lb of stator lamination material. Must be given at the density specified in (20).

(20) B **DENSITY** - This value must correspond to the density used in Item (19) to pick the watts/lb. The density that is usually used is 77.4 kilolines/in^2.

(21) 1 <u>TYPE OF STATOR SLOT</u> - Refer to Figure 1, Page

2 for type of slot.

3 For (a) slot use 1. as an input

4 For (b) slot use 2. as an input

5 For (c) slot use 3. as an input

For (d) slot use 4. as an input

Type 5. is not a slot but instead a particular situation for an open slot where the winding has only one conductor per slot.

(22) b_0 <u>ALL SLOT DIMENSIONS</u> - Given in inches per Figure 1,

b_1 Page . Where the dimension does not apply

b_2 to the slot being used, insert 0. on input sheet.

b_3

$$b_s = \frac{b_1 + b_3}{2} = \frac{(22) + (22)}{2}$$

b_s

h_0

h_1

h_2

h_3

h_s

h_t

h_w

(23) Q <u>STATOR SLOTS</u> - Number of

(24) h_c <u>DEPTH BELOW SLOTS</u> - The depth of the stator core below the slots.

Due to mechanical strength reasons, h_c should never be less than 70% of h_s.

$$h_c = \frac{(D) - \left[(d) + 2(h_s)\right]}{2} = \frac{(12) - \left[(11) + 2(22)\right]}{2}$$

(25) | q | SLOTS PER POLE PER PHASE

$$q = \frac{(Q)}{(P)\ (m)} = \frac{(23)}{(6)\ (5)}$$

(26) | τ_s | STATOR SLOT PITCH

$$\tau_s = \frac{\pi(d)}{Q} = \frac{\pi(11)}{(23)}$$

(27) | $\tau_{s1/3}$ | STATOR SLOT PITCH - 1/3 distance up from narrowest section. For slot (a), (b), (c), and (e)

$$\tau_{s1/3} = \frac{\pi\left[(d) + .66(h_s)\right]}{(Q)} = \frac{\pi\left[(11) + .66\ (22)\right]}{(23)}$$

For slot (d)

$$\frac{\pi\left[(d) + 2(h_0) + 1.32(b_s)\right]}{(Q)} =$$

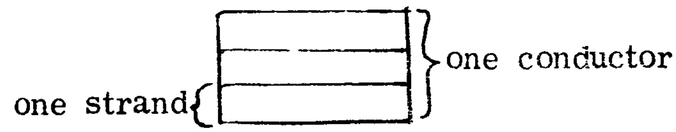
$$\frac{\pi\left[(11) + 2(22) + 1.32(22)\right]}{(23)}$$

(28) | -- | TYPE OF WINDING - Record whether the connection is "wye" of "delta". For "wye" conn use 1. for input. For "delta" use 0. for input.

(29) | -- | TYPE OF COIL - Record whether random wound or formed coils are used. For random wound coils use 0. for input. For formed coils use 1. for input.

(30)	n_S	<u>CONDUCTORS PER SLOT</u> - The actual number of conductors per slot. For random wound coils use a space factor of 75% to 80%. Where space factor is the percent of the total slot area that is available for insulated conductors after all other insulation areas have been subtracted out.
(31)	Y	<u>THROW</u> - Number of slots spanned. For example, with a coil side in slot 1 and the other coil side in slot 10, the throw is 9.
(31a)		<u>PER UNIT OF POLE PITCH SPANNED</u> - Ratio of the number of slots spanned to the number of slots in a pole pitch. This value must be between 1.0 and 0.5 to satisfy the limits of this program. $= \frac{(Y)}{(m)\ (q)} = \frac{(31)}{(5)\ (25)}$
(32)	C	<u>PARALLEL PATHS</u>, No. of - Number of parallel circuits per phase.
(33)	--	<u>STRAND DIA. OR WIDTH</u> - In inches. For round wire, use strand diameter. For rectangular wire, use strand width. This must be the largest of the two dimensions given for a ractangular wire.
(34)	N_{ST}	<u>NUMBER OF STRANDS PER CONDUCTOR IN DEPTH</u> - Applies to rectangular wire. In order to have a more flexible conductor and reduce eddy current loss, a stranded conductor is often used. For

example, when the space available for one conductor is .250 width x .250 depth, the actual conductor can be made up of 2 or 3 strands in depth as shown



For a more detailed explanation refer to section titled "Effective Resistance and Eddy Factor" in the Derivations in Appendix.

(34a) N'_{ST} NUMBER OF STRANDS PER CONDUCTOR - This number applies to the strands in depth and/or width and is used in calculating the conductor area. Item (34) is different in that it deals with strands in depth only and is used in calculating eddy factors.

(35) d_b DIAMETER OF BENDER PIN - in inches - This pin is used in forming coils. Use .25 inch for stator O.D. $<$ 8 inches use .50 inches for stator O.D. $>$ 8 inches.

(36) ℓ_{e2} COIL EXTENSION BEYOND CORE in Inches - Straight portion of coil that extends beyond stator core.

(37) h_{ST} HEIGHT OF UNINSULATED STRAND in Inches - This value is the vertical height of the strand and is used in eddy factor calculations. Set this value = 0 for round wire.

(38) h'_{ST} DISTANCE BETWEEN CENTERLINES OF STRANDS IN DEPTH in inches.

(39)	-	STATOR COIL STRAND THICKNESS in inches - For rectangular conductors only. For round wire insert 0. on input sheet. This must be the narrowest dimension of the two dimensions given for a rectangular wire.
(40)	τ_{SK}	SKEW - Stator slot skew in inches at stator I.D.
(41)	τ_P	POLE PITCH in inches. $$\tau_P = \frac{\pi(d)}{(P)} = \frac{\pi(11)}{(6)}$$
(42)	K_{SK}	SKEW FACTOR - The skew factor is the ratio of the voltage induced in the coils to the voltage that would be induced if there were no skew. When $\tau_{SK} = 0$, $K_{SK} = 1$ $$K_{SK} = \frac{\sin\left[\frac{\pi(\tau_{SK})}{2(\tau_P)}\right]}{\frac{\pi(\tau_{SK})}{2(\tau_P)}} = \frac{\sin\left[\frac{\pi(40)}{2(41)}\right]}{\frac{\pi(40)}{2(41)}}$$
(42a)		PHASE BELT ANGLE - Input. For phase belt angle = 60° insert 60 on input sheet. For phase belt angle = 120° insert 120 on input sheet.
(43)	K_d	DISTRIBUTION FACTOR - The distribution factor is the ratio of the voltage induced in the coils to the voltage that would be induced if the windings

were concentrated in a single slot. See Table 2 for compilation of distribution factors for the various harmonics.

For 60^o phase belt angle and q = integer when (42a) = 60 and (25) = integer.

$$K_d = \frac{\text{Sin } 30^o}{(q) \text{ Sin}\left[30/(q)\right]} = \frac{\text{Sin } 30^o}{(25) \text{ Sin}\left[30/(25)\right]}$$

For 60^o phase belt angle and (q) $\neq$ integer = N/B reduced to lowest terms.

When (43a) = 1 and (25) $\neq$ integer = N/B reduced to lowest terms

$$K_d = \frac{\text{Sin } 30^o}{(N) \text{ Sin}\left[30/(N)\right]} = \frac{\text{Sin } 30^o}{(43) \text{ Sin}\left[30/(43)\right]}$$

For 120^o phase belt angle and (q) = integer

When (43a) = 120 and (25) = integer

$$K_d = \frac{\text{Sin } 60^o}{2(q) \text{ Sin}\left[30/(q)\right]} = \frac{\text{Sin } 60^o}{2(25) \text{ Sin}\left[30/(25)\right]}$$

For 120^o phase belt angle and q $\neq$ integer

When (43a) = 120 and (25) $\neq$ integer = N/B reduced to lowest terms

$$K_d = \frac{\text{Sin } 60^o}{2(N) \text{ Sin}\left[30/(N)\right]} = \frac{\text{Sin } 60^o}{2(43) \text{ Sin}\left[30/(43)\right]}$$

(44) K_P **PITCH FACTOR** - The ratio of the voltage induced in the coil to the voltage that would be induced in a full pitched coil. See Table 1 for compilation of the pitch factors for the various harmonics.

$$K_P = \sin\left[\frac{(Y)}{(m)(q)} \times 90^o\right] = \sin\left[\frac{(31)}{(5)(25)} \times 90^o\right]$$

(45) n_e **TOTAL EFFECTIVE CONDUCTORS** - The actual number of effective series conductors in the stator winding taking into account the pitch and skew factors but not allowing for the distribution factor.

$$n_e = \frac{(Q)(n_S)(K_P)(K_{SK})}{(C)} = \frac{(23)(30)(44)(42)}{(32)}$$

(46) a_C **CONDUCTOR AREA OF STATOR WINDING** in (inches)^2 - The actual area of the conductor taking into account the corner radius on square and rectangular wire. See the following table for typical values of corner radii

If (39) = 0 then $a_c = .25\pi(\text{Dia})^2 = .25\pi(33)^2$

If (39) $\neq$ 0 then $a_C = (N'_{ST})\left[(\text{strand width})\ (\text{strand depth}) - (.858\ r_C^2)\right] = (34a)\left[(33)\ (39) - (.858\ r_C^2)\right]$

where $.858\ r_C^2$ is obtained from Table V below.

(39)	(33) .188	.189 (33) .75	(33) .751
.050	.000124	.000124	.000124
.072	.000210	.000124	.000124
.125	.000210	.00084	.000124
.165	.000840	.00084	.003350
.225	.001890	.00189	.003350
.438	--	.00335	.007540
.688	--	.00754	.01340
--	--	.03020	.03020

TABLE V

(47) S_S CURRENT DENSITY - Amperes per square inch of stator conductor

$$S_S = \frac{(I_{PH})}{(C)(a_c)} = \frac{(8)}{(32)(46)}$$

(48) L_E END EXTENSION LENGTH in inches - Can be an input or output.

For L_E to be output, insert 0. on input sheet.

For L_E to be input, calculate per following:

When (29) = 0. then:

$$L_E = .5 + \frac{K_T \pi (Y)\left[(d)+(h_s)\right]}{Q} = .5 + \frac{\begin{Bmatrix} 1.3 \text{ If } (6) = 2 \\ 1.5 \text{ If } (6) = 4 \\ 1.7 \text{ If } (6) \quad 4 \end{Bmatrix} \pi (31)\left[(11)+(22)\right]}{(23)}$$

When (29) = 1. then:

$$L_E = 2\ell_{e2} + \pi\left[\frac{h_1}{2} + \text{dia}\right] + y\left[\frac{\tau_s^2}{\sqrt{\tau_s^2 - b_s^2}}\right]$$

$$= 2 \quad (36) + \pi\left[\frac{(22)}{2} + (35)\right] + (31)\left[\frac{(26)^2}{\sqrt{(26)^2 - (22)^2}}\right]$$

(49) ℓ_t 1/2 MEAN TURN - The average length of one conductor in inches.

$$\ell_t = (\ell) + (L_E) = (13) + (44)$$

(50) X_S °C STATOR TEMP °C - Input temp at which F.L. losses will be calculated. No load losses and cold resistance will be calculated at 20°C.

(51) ρ_s **RESISTIVITY OF STATOR WINDING** - In micro ohm-inches @ 20^{o}C. If tables are available using units other than that given above, use Table VI for conversion to ohm-inches.

ρ	ohm-cm	ohm-in	ohm-cir mil/ft
1 ohm-cm =	1.000	0.3937	6.015×10^{6}
1 ohm-in =	2.540	1.000	1.528×10^{7}
1 ohm-cir mil/ft =	1.662×10^{-7}	6.545×10^{-8}	1.000

TABLE VI
Conversion Factors for Electrical Resistivity

(52) $\rho_{S\,(hot)}$ **RESISTIVITY OF STATOR WINDING** - Hot at $X_s\,^{o}C$ in micro ohm-inches

$$\rho_{S(hot)} = (\rho_S)\left[\frac{(X_s\,^{o}C) + 234.5}{254.5}\right] = (51)\left[\frac{(50) + 234.5}{254.5}\right]$$

(53) $R_{SPH\,(cold)}$ STATOR RESISTANCE/PHASE - Cold @ 20^{o}C in ohms

$$R_{SPH(cold)} = \frac{(\rho_s)(n_s)(Q)(\ell_t)}{(m)(a_c)(C)^2} \times 10^{-6} = \frac{(51)(30)(23)(49)}{(5)(46)(32)^2} \times 10^{-6}$$

(54) $R_{SPH\,(hot)}$ STATOR RESISTANCE/PHASE - Calculated @ X^{o}C in ohms

$$R_{SPH(hot)} = \frac{(\rho_{s\,hot})(n_s)(Q)(\ell_t)}{(m)(a_c)(C)^2} \times 10^{-6}\ \frac{(52)(30)(23)(49)}{(5)(46)(32)^2} \times 10^{-6}$$

(55) EF (top) EDDY FACTOR TOP - The eddy factor of the top coil. Calculate this value at the expected operating temperature of the machine. For round wire

$EF_{top} = 1$

$$EF_{top} = 1 + \left\{ .584 + \left[\frac{N_{st}^2 - 1}{16}\right]\left[\frac{h'_{st}\ell}{h_{st}\ell_t}\right]^2 \right\} 3.35 \times 10^{-3} \left[\frac{(h_{st})(n_s)(f)(a_c)}{(b_s)(\rho_S)_{hot}}\right]^2$$

$$= 1 + \left\{ .584 + \left[\frac{(34)^2 - 1}{16}\right]\left[\frac{(38)(13)}{(37)(49)}\right]^2 \right\} 3.35 \times 10^{-3} \left[\frac{(37)(30)(5a)(46)}{(22)(52)}\right]^2$$

(56) EF (bot) <u>EDDY FACTOR BOTTOM</u> - The eddy factor of the bottom coil at the expected operating temperature of the machine. For round wire $EF_{(bot)} = 1$

$$EF_{(bot)} = (EF_{(top)}) - 1.677 \left[\frac{(h_{st})(n_s)(f)(a_c)}{(b_s)(\rho_{S\,hot})}\right]^2 \times 10^{-3}$$

$$= (55) - 1.677 \left[\frac{(37)(30)(5a)(46)}{(22)(52)}\right] 10^{-3}$$

(57) b_{tm} <u>STATOR TOOTH WIDTH</u> 1/2 way down tooth in inches - For slots type (a), (b), (d) and (e), Figure I

$$b_{tm} = \frac{\pi\left[(d) + (h_s)\right]}{(Q)} - (b_s) = \frac{\pi\left[(11) + (22)\right]}{(23)} - (22)$$

(57a)	$b_{t\,1/3}$	STATOR TOOTH WIDTH 1/3 distance up from narrowest section For slots type (a), (b) and (e) $b_{t\,1/3} = (\tau_{s\,1/3}) - (b_s) = (27) - (22)$ For slot type (c) $b_{t\,1/3} = b_{tm} = (57)$ For slot type (d) $b_{t\,1/3} = (\tau_{1/3}) - \frac{2\sqrt{2}}{3}(b_s) = (27) - .94\,(22)$
(58)	b_t	TOOTH WIDTH AT STATOR I.D. in inches - For partially closed slot $b_t = \frac{\pi(d)}{(Q)} - b_0 = \frac{\pi(11)}{(23)} - (22)$ For open slot $b_t = \frac{\pi(d)}{(Q)} - b_s = \frac{\pi(11)}{(23)} - (22)$

(59)	g_{min}	MINIMUM AIR GAP in inches - For concentric pole face $g_{min} = g_{max}$. For non concentric pole face g_{min} = gap at the center of the pole.
(59a)	g_{max}	MAXIMUM AIR GAP in inches
(60)	C_X	REDUCTION FACTOR - Used in calculating conductor permeance and is dependent on the pitch and distribution factor. This factor can be obtained from Graph 1 with an assumed K_d of .955 or calculated as shown $$C_X = \frac{(K_X)}{(K_p)^2\ (K_d)^2} = \frac{(61)}{(44)^2\ (43)^2}$$
(61)	K_X	FACTOR TO ACCOUNT FOR DIFFERENCE in phase current in coil sides in same slot $$K_X = \frac{1}{4}\left[\frac{3(\gamma)}{(m)(q)} + 1\right] \quad \text{For 3 phase}$$ $$= \frac{1}{4}\left[\frac{3(31)}{(5)(25)} + 1\right]$$ $$K_X = \frac{(\gamma)}{(m)(q)} \quad \text{For 2 phase}$$ $$= \frac{(31)}{(5)(25)}$$ NOTE: See special case for (e) slot. Refer to calculation (62)
(62)	λ_i	CONDUCTOR PERMEANCE - The specific permeance for the portion of the stator current that is embedded in the iron. This permeance depends upon the configuration of the slot.

(a) For open slots

$$\lambda_i = (C_X)\,\frac{20}{(m)(q)}\left[\frac{(h_2)}{(b_s)} + \frac{(h_1)}{3(b_s)} + \frac{(b_t)^2}{16(\tau_s)(g)} + \frac{.35(b_t)}{(\tau_s)}\right]$$

$$\lambda_i = (60)\,\frac{20}{(5)(25)}\left[\frac{(22)}{(22)} + \frac{(22)}{3(22)} + \frac{(58)^2}{16(26)(59)} + \frac{.35(58)}{(26)}\right]$$

(b) For partially closed slots with constant slot width

$$\lambda_i = (C_X)\,\frac{20}{(m)(q)}\left[\frac{(h_o)}{(b_o)} + \frac{2(h_t)}{(b_o)+(b_s)} + \frac{(h_w)}{(b_s)} + \frac{(h_1)}{3(b_s)} + \frac{(b_t)^2}{16(\tau_s)(g)} + \frac{.35(b_t)}{(\tau_s)}\right]$$

$$\lambda_i = (60)\,\frac{20}{(5)(25)}\left[\frac{(22)}{(22)} + \frac{2(22)}{(22)+(22)} + \frac{(22)}{(22)} + \frac{(22)}{3(22)} + \frac{(58)^2}{16(26)(59)} + \frac{.35(58)}{(26)}\right]$$

(d) For round slots

$$\lambda_i = (C_X)\,\frac{20}{(m)(q)}\left[.62 + \frac{(h_o)}{(b_o)}\right]$$

$$\lambda_i = (60)\,\frac{20}{(5)(25)}\left[.62 + \frac{(22)}{(22)}\right]$$

(e) For open slots with a winding of one conductor per slot

$$\lambda_i = (C_X)\,\frac{20}{(m)(q)}\left[\frac{(h_2)}{(b_s)} + \frac{(h_1)}{3(b_s)} + .6 + \frac{(g)}{2(\tau_s)} + \frac{(\tau_s)}{4(g)}\right]$$

$$\lambda_i = (60)\,\frac{20}{(5)(25)}\left[\frac{(22)}{(22)} + \frac{(22)}{3(22)} + .6 + \frac{(59)}{2(26)} + \frac{(26)}{4(59)}\right]$$

$$\left((C_X) = \frac{1}{(K_p^2)(K_d^2)}\right)$$

$$(K_X) = 1$$

(63) K_E **LEAKAGE REACTIVE FACTOR** for end turn

$$K_E = \frac{\text{Calculated value } (L_E)}{\text{Value } (L_E) \text{ from Graph 1}} \quad \text{(For machines where (11)>8")}$$

where L_E = (48) and abscisa of Graph 1 = $(\gamma)(\tau_s)$ = (31)(26)

$$K_E = \sqrt{\frac{\text{Calculated value of } (L_E)}{\text{Value } (L_E) \text{ from Graph 1}}} \quad \text{(For machines where (11)<8")}$$

(64) λ_E **END WINDING PERMEANCE** - The specific permeance for the end extension portion of the stator winding

$$\lambda_E = \frac{6.28(K_E)}{(\ell)(K_d)^2}\left[\frac{\phi_E L_E}{2n}\right] = \frac{6.28(63)}{(13)(43)^2}\left[\frac{Q_E L_E}{2n}\right]$$

The term $\left[\frac{\phi_E L_E}{2n}\right]$ is obtained from Graph 1.

The symbols used in this (term) do not apply to those of this design manual. Reference information for the symbol origin is included on Graph 1.

(65) -- **WEIGHT OF COPPER** - The weight of stator copper in lbs.

$$\text{\#'s copper} = .321(n_s)(Q)(a_c)(\ell_t) = .321(30)(23)(46)(49)$$

(66) -- **WEIGHT OF STATOR IRON** - in lbs.

$$\text{\#'s iron} = .566\left\{(b_{tm})(Q)(\ell_s)(h_s) + \pi\left[(D) - (h_c)\right](h_c)(\ell_s)\right\}$$

$$.566\left\{(57)(23)(17)(22) + \pi\left[(12) - (24)\right](24)(17)\right\}$$

(67) K_s **CARTER COEFFICIENT**

$$K_s = \frac{(\tau_s)\left[5(g) + (b_s)\right]}{(\tau_s)\left[5(g) + (b_s)\right] - (b_s)^2} \quad \text{(For open slots)}$$

$$K_s = \frac{(26)\left[5(59) + (22)\right]}{(26)\left[5(59) + (22)\right] - (22)^2}$$

$$K_s = \frac{\tau_s\left[4.44(g) + .75(b_o)\right]}{\tau_s\left[4.44(g) + .75(b_o)\right] - (b_o)^2} \quad \text{(For partially closed slots)}$$

$$K_s = \frac{(26)\left[4.44(59) + .75(22)\right]}{(26)\left[4.44(59) + .75(22)\right] - (22)^2}$$

(68) -- AIR GAP AREA - The area of the gap surface at the stator bore

$$\text{Gap Area} = \pi(d)(\ell) = \pi(11)(13)$$

(69) g_e EFFECTIVE AIR GAP

$$g_e = (K_s)(g) = (67)(59)$$

(70) λ_a AIR GAP PERMEANCE - The specific permeance of the air gap

$$\lambda_a = \frac{6.38(d)}{(P)(g_e)} = \frac{6.38(11)}{(6)(69)}$$

(71) C_1 THE RATIO OF MAXIMUM FUNDAMENTAL of the field form to the actual maximum of the field form - This term can be an input or output. For C_1 to be output insert 0. on input sheet. For C_1 to be input, determine C_1 as follows:

For pole heads with only one radius, C_1 is obtained from curve #4. The abscisa is "pole embrace" (α) = (77). The graphical flux plotting method of determining C_1 is explained in the section titled "Derivations" in the Appendix.

(72)	C_W	<u>WINDING CONSTANT</u> - The ratio of the RMS line voltage for a full pitched winding to that which would be introduced in all the conductors in series if the density were uniform and equal to the maximum value. This value can be an input or output. For C_W to be an output, insert 0. on input sheet. For C_W to be an input, calculate as follows: $$C_W = \frac{(E)(C_1)(K_d)}{\sqrt{2}\,(E_{PH})(m)} = \frac{(3)(71)(43)}{\sqrt{2}\,(4)(5)}$$ Assuming $K_d = .955$, then $C_W = .225\ C_1$ for three phase delta machines and $C_W = .390\ C_1$ for three phase star machines.
(73)	C_P	<u>POLE CONSTANT</u> - The ratio of the average to the maximum value of the field form. This ratio can be an input or output. For C_P to be an output, insert 0. on input sheet. For C_P to be an input, determine as follows: For pole heads with more than one radius C_P is calculated from the same field form that was used to determine C_1, and this method is described in the section titled "Derivations" in the Appendix. For pole heads with only one radius C_P is obtained from curve #4. Note the correction factor at the top of the curve.
(74)	C_M	<u>DEMAGNETIZING FACTOR</u> - direct axis - This factor can be an input or output. For C_M to be an output, insert 0. on input sheet. For C_M to be an input, determine as follows:

$$C_M = \frac{(\alpha)\pi + \sin[(\alpha)\pi]}{4\sin[(\alpha)\ \pi/2]} = \frac{(77)\pi + \sin[(77)\pi]}{4\sin[(77)\ \pi/2]}$$

C_M can also be obtained from curve 9.

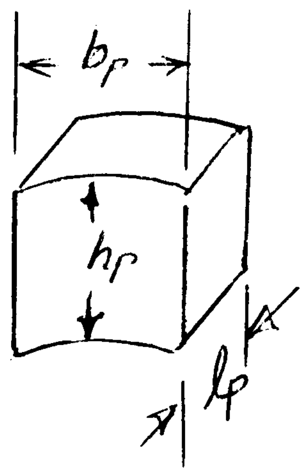
(75) C_q CROSS MAGNETIZING FACTOR - quadrature axis - This factor can be an input or output. For C_q to be an output, insert 0. on input sheet. For C_q to be an input, determine as follows:

$$\left.\begin{aligned} C_q &= \frac{1/2\cos[(\alpha)\ \pi/2] + (\alpha)\pi - \sin[(\alpha)\pi]}{4\sin[(\alpha)\ \pi/2]} \\ &= \frac{1/2\cos[(77)\ \pi/2] + (77)\pi - \sin[(77)\pi]}{4\sin[(77)\ \pi/2]} \end{aligned}\right\} \text{VALID FOR CONCENTRIC POLES.}$$

C_q can also be obtained from curve 9.

(76) -- POLE DIMENSIONS LOCATIONS per Figure 2

Where:



ℓ_p length of pole (one end only)

b_p width of pole

h_p height of pole at center

all dimensions in inches

(77) α POLE EMBRACE

$$\alpha = \frac{b_p}{\tau_p} = \frac{(76)}{(41)}$$

(78) | A | AMPERE CONDUCTORS per inch - The effective ampere conductors per inch of stator periphery. This factor indicates the "specific loading" of the machine. Its value will increase with the rating and size of the machine and also will increase with the number of poles. It will decrease with increases in voltage or frequency. A is generally higher in single phase machines than in polyphase ones.

$$A = \frac{(I_{PH})(n_S)(K_P)}{(C)(\tau_S)} = \frac{(8)(30)(44)}{(32)(26)}$$

(79) | X | REACTANCE FACTOR - The reactance factor is the quantity by which the specific permeance must be multiplied to give percent reactance. It is the percent reactance for unit specific permeance, or the percent of normal voltage induced by a fundamental flux per pole per inch numerically equal to the fundamental armature ampere turns at rated current. Specific permeance is defined as the average flux per pole per inch of core length produced by unit ampere turns per pole.

$$X = \frac{100(A)(K_d)}{\sqrt{2}\,(C_1)(B_g) \times 10^3} = \frac{100\,(78)(43)}{\sqrt{2}\,(71)(125) \times 10^3}$$

(80) | X_ℓ | LEAKAGE REACTANCE - The leakage reactance of the stator for steady state conditions. When (5) = 3, calculate as follows:

$$X_\ell = 2X\left[(\lambda_i) + (\lambda_E)\right] = 2(79)\left[(62) + (64)\right]$$

In the case of two phase machines a component due to belt leakage must be included in the stator leakage reactance. This component is due to the harmonics caused by the concentration of the MMF into a small number of phase belts per pole and is negligible for three phase machines. When (5) = 2, calculate as follows:

$$\lambda_B = \frac{0.1(d)}{(P)(g_e)}\left[\frac{\sin\left[\frac{3(y)}{(m)(q)}\right]90^o}{(K_p)}\right] = \frac{0.1(11)}{(6)(69)}\left[\frac{\sin\left[\frac{3(31)}{(5)(25)}\right]90^o}{(44)}\right]$$

$X_\ell = 2\chi\left[(\lambda_i) + (\lambda_E) + (\lambda_B)\right]$ where $\lambda_B = 0$ for 3 phase machines.

$X_\ell = 2(79)\left[(62) +)64) + (80)\right]$

(81) X_{ad} REACTANCE - direct axis - This is the fictitious reactance due to armature reaction in the direct axis.

$$X_{ad} = (X)(\lambda_a)(C_1)(C_M) = (79)(70)(71)(74)$$

(82) X_{aq} REACTANCE - quadrature axis - This is the fictitious reactance due to armature reaction in the direct axis.

$$X_{aq} = (X)(C_q)(\lambda_a) = (79)(75)(70)$$

(83) X_d SYNCHRONOUS REACTANCE - direct axis - The steady state short circuit reactance in the direct axis.

$$X_d = (X_\ell) + (X_{ad}) = (80) + (81)$$

(84) X_q SYNCHRONOUS REACTANCE - quadrature axis - The steady state short circuit reactance in the quadrature axis.

$$X_q = (X_\ell) + (X_{aq}) = (80) + (82)$$

(85) a_p POLE AREA - The effective cross sectional area of the pole.

$$a_p = (b_p)(\ell_p)(K_i) = (76)(76)(16)$$

(98a)	V_r	PERIPHERAL SPEED - The velocity of the rotor surface in fee t per minute $$V_r = \frac{(d_r)(RPM)}{12} = \frac{(11a)(7)}{12}$$
(99)	N_P	NUMBER OF FIELD TURNS
(100)	tr	MEAN LENGTH OF FIELD TURN
(101)	--	FIELD CONDUCTOR DIMENSIONS
(103)	$X_f{}^oC$	FIELD TEMP IN ^oC - Input temp at which full load field loss is to be calculated.
(104)	ρ_f	RESISTIVITY of field conductor at 20^oC in micro ohm-inches. Refer to table given in Item (51) for conversion factors.
(105)	ρ_f (hot)	RESISTIVITY of field conductor at $X_f{}^oC$ $$\rho_{f\ (hot)} = \rho_f \left[\frac{(X_f{}^oD) \quad 234.5}{254.5}\right] = (104)\left[\frac{(103) \quad 234.5}{254.5}\right]$$
(106)	a_{cf}	CONDUCTOR AREA OF FIELD WDG - Calculate same as stator conductor area (46) except substitute (102) for (39) (101) for (33)

(107) R_f (cold) **COLD FIELD RESISTANCE** @ $20^{O}C$

$$R_{f\,(cold)} = (\rho_f) \frac{(N_f)(\ell_{tf})}{(a_{cf})} = (104) \frac{(99)\,(100)}{(106)}$$

(108) R_f (hot) **HOT FIELD RESISTANCE** - Calculated at $X_f{}^{O}C$ (103)

$$R_f\ (hot) = (\rho_{f\ hot}) \frac{(N_f)\ (\ell_{tf})}{(a_{cf})} = (105) \frac{(99)(100)}{(106)}$$

(108a) -- **WEIGHT OF FIELD COPPER** in lbs

#'s of copper $= .321\ (N_f)\ (\ell_{tf})(a_{cf})$

$= .321(99)\ (100)(106)$

(108b) -- **WEIGHT OF ROTOR IRON** - Because of the large number of different pole shapes, one standard formula cannot be used for calculating rotor iron weight. Therefore, the computer will not calculate rotor iron weight.

(113) **FIELD SELF INDUCTANCE**

$$L_f = (N_f)^2 \left[C_p \lambda_a \frac{\pi}{2} \ell_p + P_f \right] x\ 10^{-8}$$

(118) X'_{du} **UNSATURATED TRANSIENT REACTANCE**

$$X'_{du} = (X_{\ell}) + (X_f) = (80) + (112)$$

X_d' — <u>SATURATED TRANSIENT REACTANCE</u>

$$X_d' = .88(X_{du}') = .88(118)$$

X_d'' — <u>SUBTRANSIENT REACTANCE</u> in direct axis

When no damper bars exist, i.e. when (92) = 0

$$X_d'' = (X_d') = (119)$$

X_q'' — <u>SUBTRANSIENT REACTANCE</u> in quadrature axis

When no damper bars exist, i.e. when (92) = 0

$$X_q'' = X_q = (84)$$

X_2 — <u>NEGATIVE SEQUENCE REACTANCE</u> - The reactance due to the field which rotates at synchronous speed in a direction opposite to that of the rotor.

$$X_2 = .5\left[X_d'' + X_q''\right] = .5\left[(120) + (121)\right]$$

X_0 — <u>ZERO SEQUENCE REACTANCE</u> - The reactance drop across any one phase (star connected) for unit current in each of the phases. The machine must be star connected for otherwise no zero sequence current can flow and the term then has no significance.

(132)	T'_{do}	<u>OPEN CIRCUIT TIME CONSTANT</u> - The time constant of the field winding with the stator open circuited and with negligible external resistance and inductance in the field circuit. Field resistance at room temperature ($20^{o}C$) is used in this calculation. $T'_{do} = \frac{L_F}{R_F} = \frac{(113)}{(107)}$
(133)	T_a	<u>ARMATURE TIME CONSTANT</u> - Time constant of the D.C. component. In this calculation stator resistance at room temperature ($20^{o}C$) is used. $T_a = \frac{X_2}{200\pi(f)(r_a)} = \frac{(122)}{200\pi(5a)(133)}$ where $r_a = \frac{(m)(I_{PH})^2(R_{SPH\,cold})}{\text{Rated KVA}} = \frac{(5)(8)^2(53)}{(2)}$
(134)	T'_d	<u>TRANSIENT TIME CONSTANT</u> - The time constant of the transient reactance component of the alternating wave. $T'_d = \frac{(X'_d)}{(X_d)}(T'_{do}) = \frac{(119)}{(83)}(132)$
(135)	T''_d	<u>SUBTRANSIENT TIME CONSTANT</u> - The time constant of the subtransient component of the alternating wave. This value has been determined empirically from tests on large machines. Use following values. T''_d = .035 second at 60 cycle T''_d = .005 second at 400 cycle
(136)	F_{SC}	<u>SHORT CIRCUIT AMPERE TURNS</u> - The field ampere turns required to circulate rated stator current when the stator is short circuited. $F_{SC} = (X_d)(F_g) = (83)(131)$

ϕ_T — TOTAL FLUX IN KILO LINES

$$\phi_T = \frac{6(E)10^6}{(C_W)(n_e)(RPM)} = \frac{6(3)10^6}{(72)(45)(17)}$$

B_g — GAP DENSITY in Kilo Lines/in^2 - The maximum flux density in the air gap

$$B_g = \frac{(\phi_T)}{\pi(d)(\ell)} = \frac{(124)}{\pi(11)(13)}$$

ϕ_P — FLUX PER POLE in Kilo Lines

$$\phi_P = \frac{(\phi_T)(C_P)}{(P)} = \frac{(124)(73)}{(6)}$$

B_t — TOOTH DENSITY in Kilo Lines/in^2 - The flux density in the stator tooth at 1/3 of the distance from the minimum section.

$$B_t = \frac{\phi_T}{(Q)(\ell_s)(b_{t\,1/3})} = \frac{(124)}{(23)(17)(57a)}$$

B_c — CORE DENSITY in Kilo Lines/in^2 - The flux density in the stator core

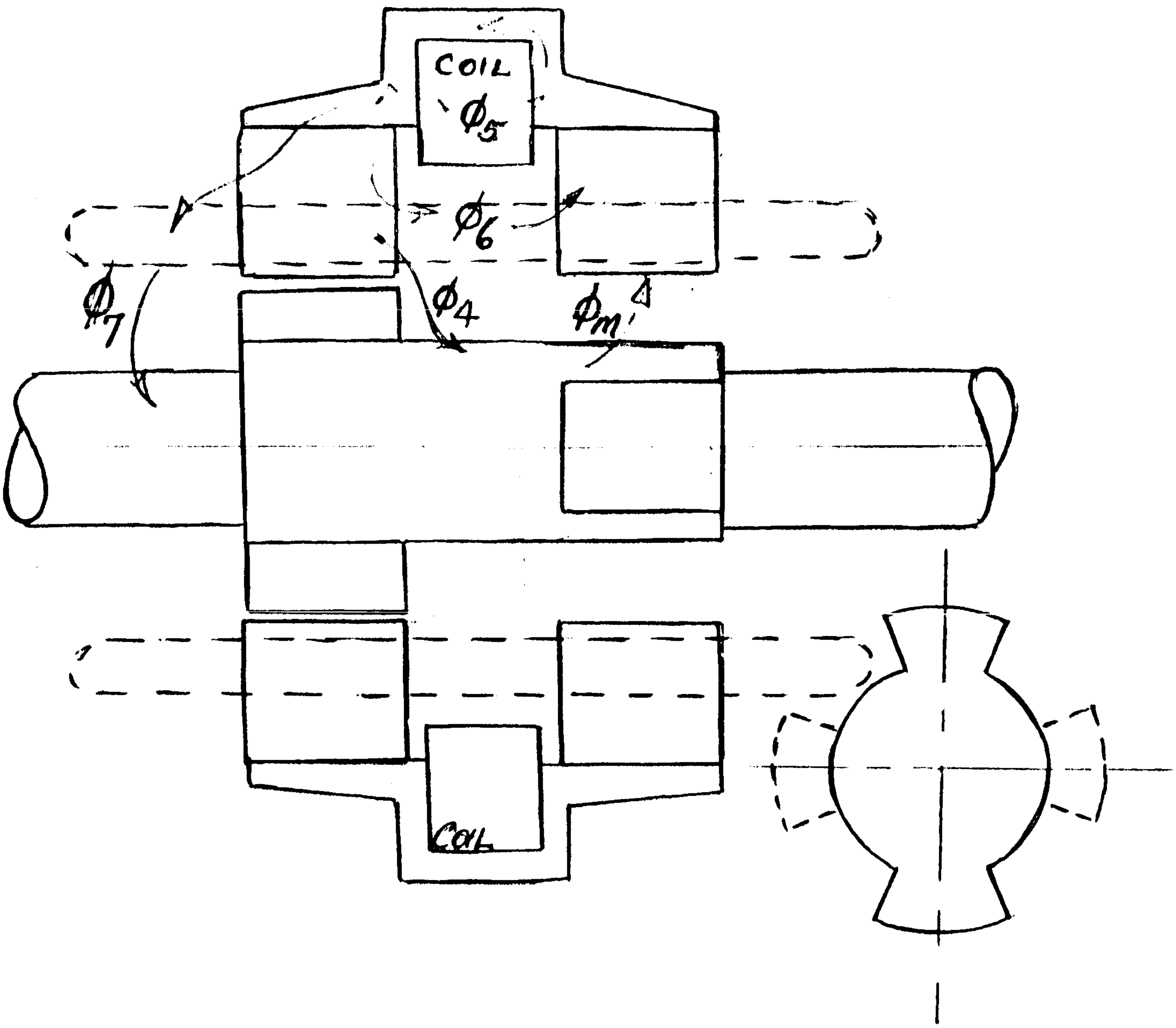
$$B_c = \frac{(\phi_P)}{A_c}$$

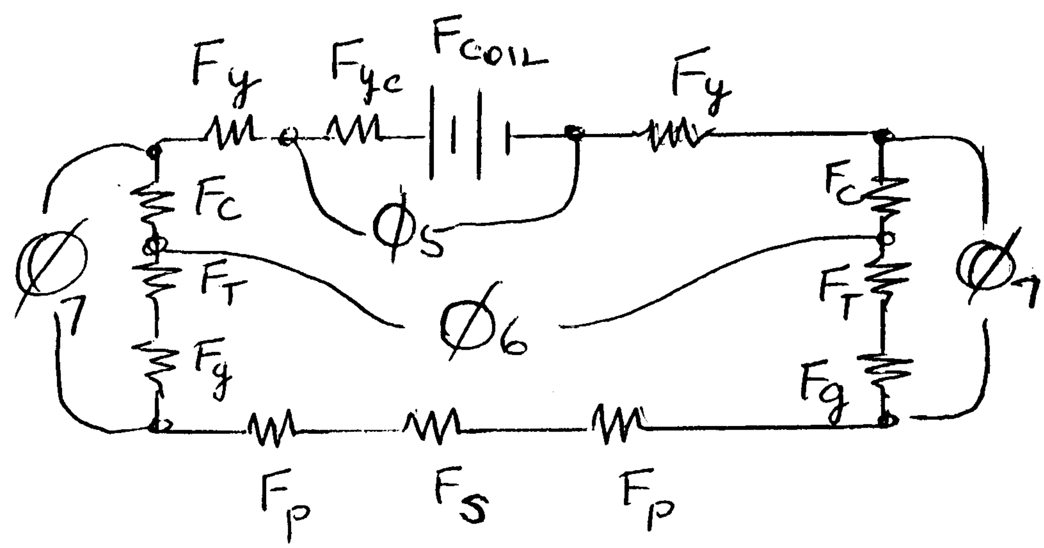
A_c — EFFECTIVE AREA OF THE CORE

$$A_c = \frac{(D - 2\,dbs)\,\pi\,d}{p}\,\ell_{s'}$$

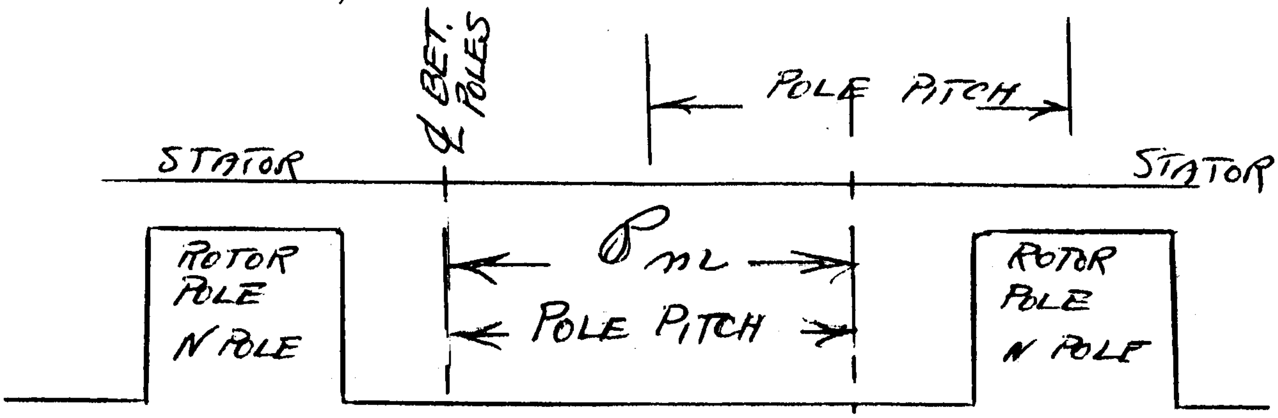
F_g — AIR GAP AMPERE TURNS - The field ampere turns per pole required to force flux across the air gap when operating at no load with rated voltage.

$$F_g = \frac{(B_g)(g_e)}{3.19} = \frac{(125)(69)}{3.19}$$


COIL
ϕ_5
ϕ_6
ϕ_7
ϕ_4
ϕ_m
COIL

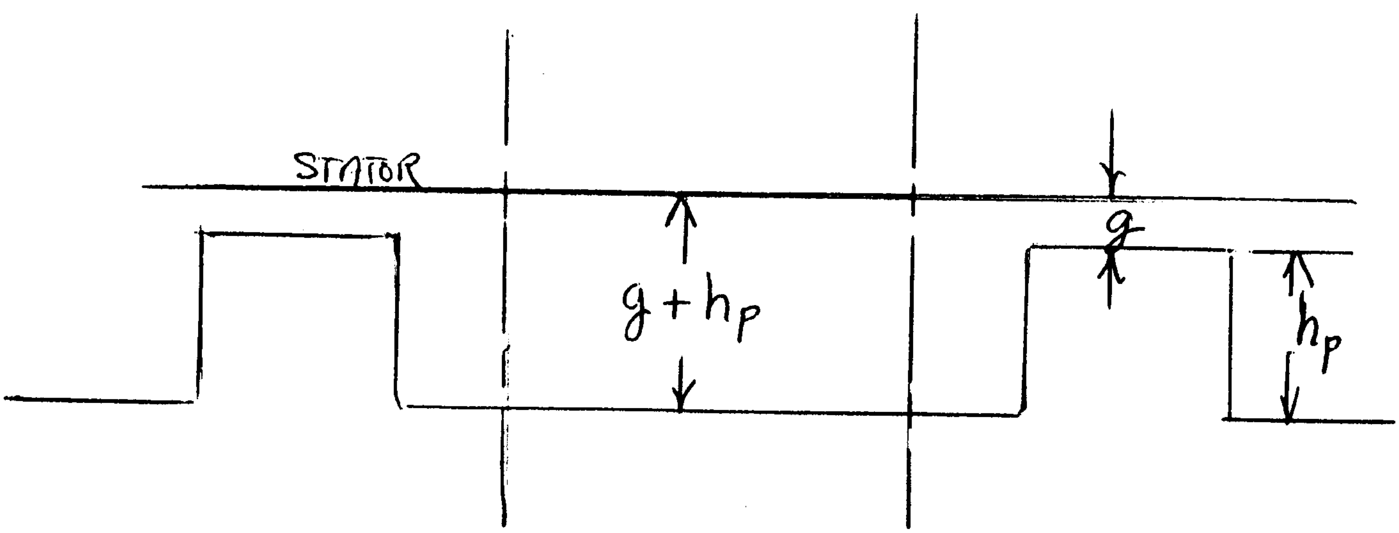

F_y
F_{yc}
F_{COIL}
F_y
F_C
F_C
ϕ_5
ϕ_7
F_T
ϕ_6
F_T
ϕ_7
F_g
F_g
F_p
F_S
F_p

P_m = Leakage permeance path from rotor to stator between pole lobes, or rotor teeth.



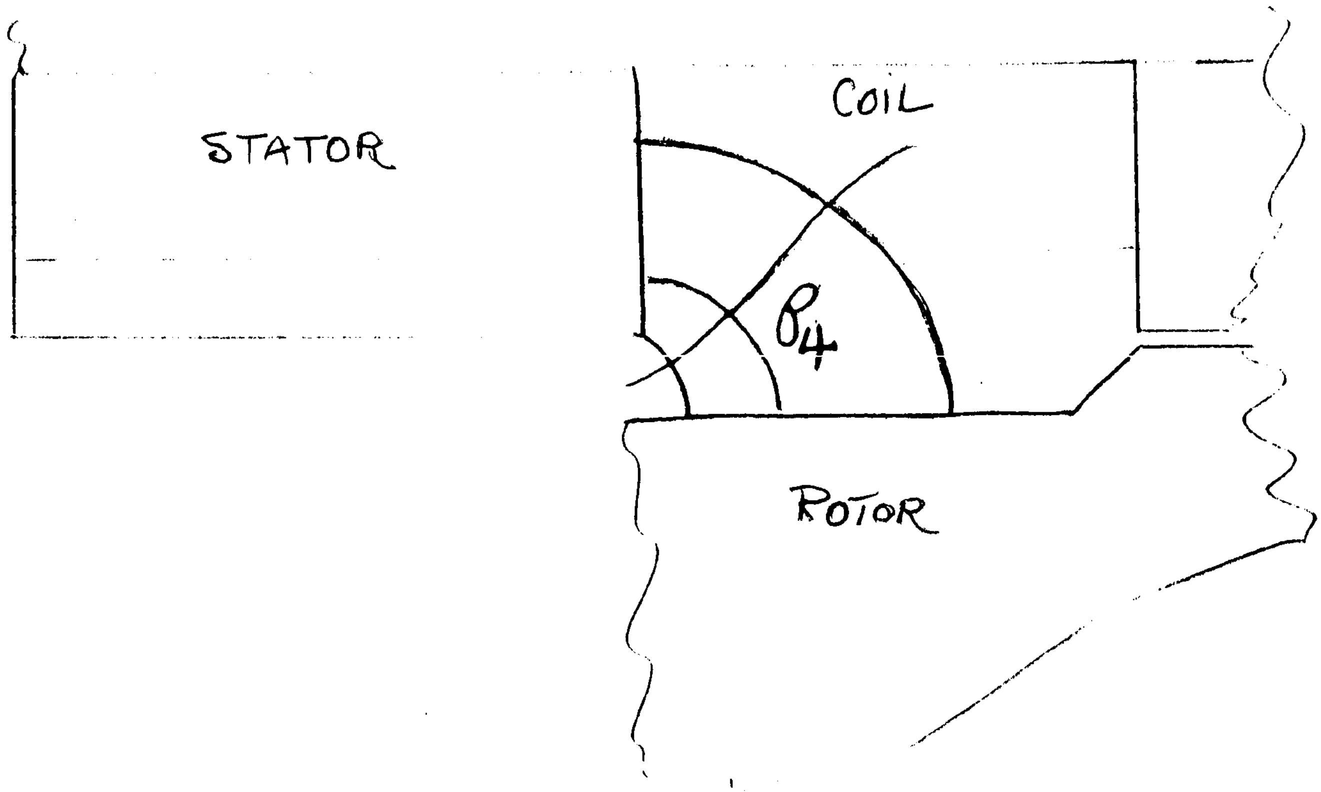
$$P_m = \frac{3.19\ (\pi d_r)\ \ell}{P\ (h_p + g)}$$

h_p = EFFECTIVE POLE HEIGHT WHEN THE ROTOR IS TAPERED AND ACTUAL HEIGHT WHEN THE ROTOR IS STRAIGHT



FLUX LEVEL BETWEEN LOBES AT N.L. IS $\dfrac{g_e}{g + h_p}(B_g)$

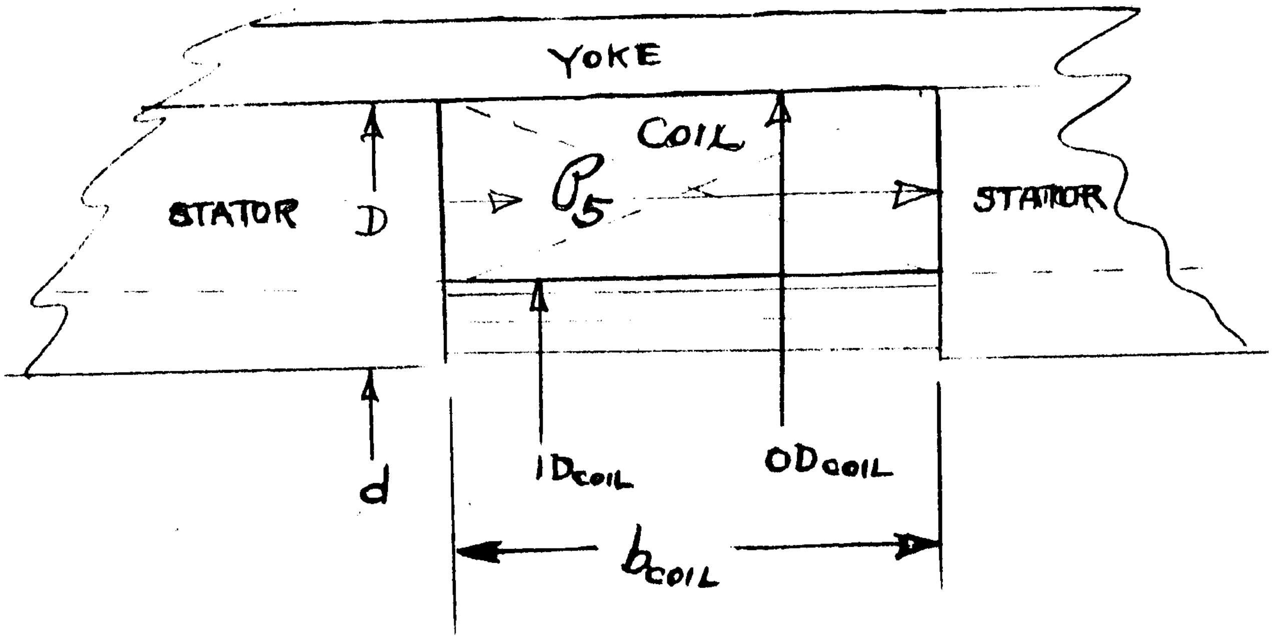
P_4 The permeance of the leakage path from the inner (or inboard) edge of the stator stack, to the center shaft portion of the rotor.



$$P_4 = \frac{P}{2}\left\{\frac{\pi(d+h_s)\frac{b_{coil}}{2}(3.19)\frac{1}{P}}{h_p+h_s}\right\}$$

$$P_4 = \frac{10\ (d+h_s)\,b_{coil}}{h_p+h_s}$$

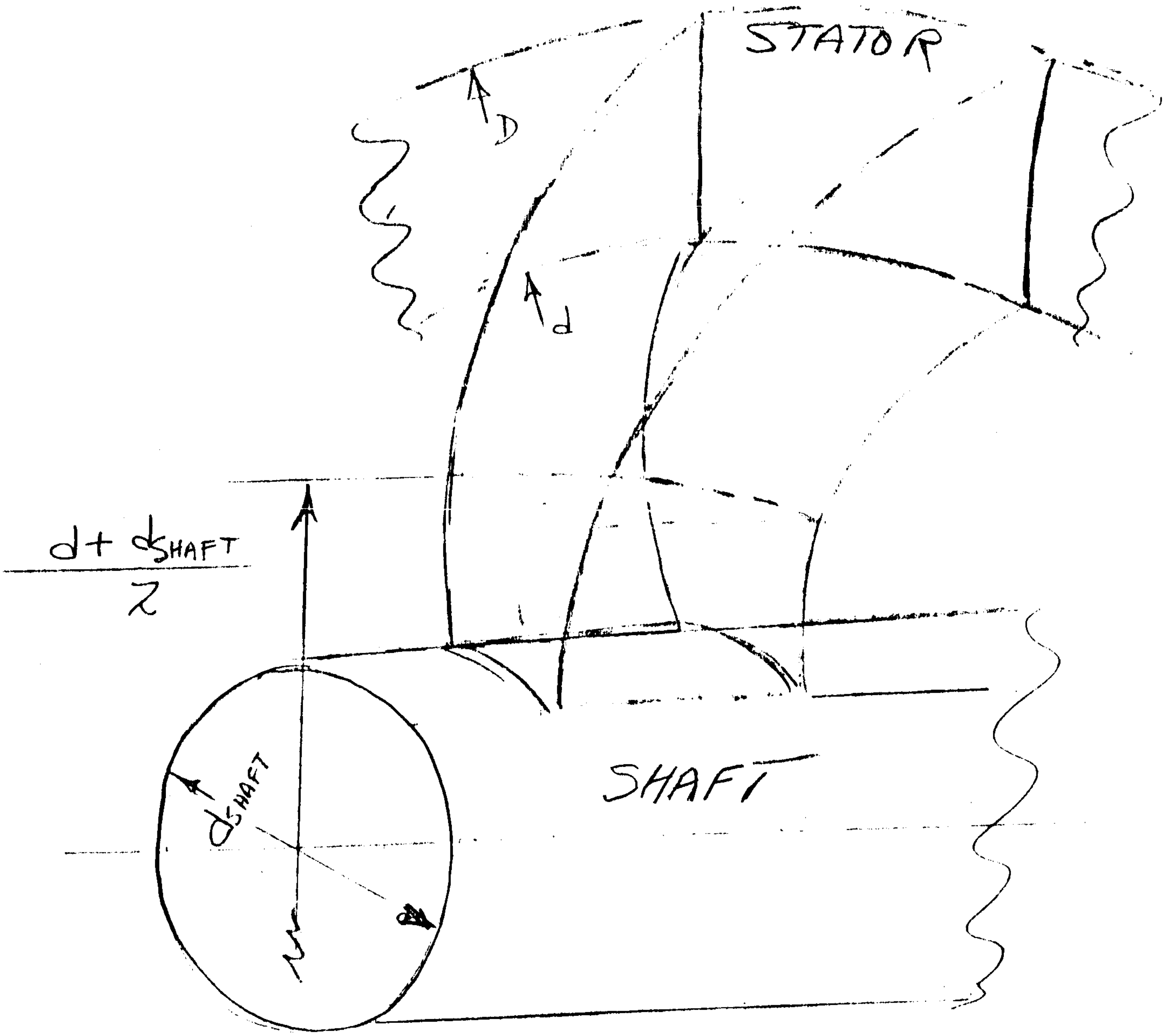
P_5 = Leakage permeance across the field coil.



$$\mathcal{P}_5 = \frac{3.19 \left[\frac{OD_{COIL} - ID_{COIL}}{2}\right]\left[\frac{OD_{COIL} + ID_{COIL}}{2}\right]\pi\left(\frac{2}{3}\right)}{b_{COIL}}$$

P_6 = Leakage permeance from stator to stator.

$$\mathcal{P}_6 = \frac{3.19 \left[\frac{ID_{COIL} - d}{2}\right]\left[\frac{ID_{COIL} + d}{2}\right]\pi}{b_{COIL}}$$



P_7 = Leakage permeance from stators to shaft and rotor.

$$P_7 = \pi \frac{\left[\frac{d + d_{SHAFT}}{2}\right]\left[\frac{D-d}{2}\right]\frac{3.19}{2}}{\frac{1}{2}\left[D - d_{SHAFT}\right]}$$

ϕ_m = Flux leaking across the rotor area where there is no pole, no load

ϕ_m' = Approximation of ϕ_m

ϕ_{ML} = ϕ_m leaking under load conditions

ϕ_4 = Flux leaking from the inboard side of a stator stack to the rotor (shaft center section) at no load

ϕ_4' = Approximation of ϕ_4

ϕ_{4L} = ϕ_4 leaking under load conditions

ϕ_5 = Flux leaking across the field coil itself, no load

ϕ_5' = Approximation of ϕ_5

ϕ_{5L} = ϕ_5 leaking under load conditions

ϕ_6 = Flux leaking between stators, below the field coil and at no load

ϕ_6' = Approximation of ϕ_6

ϕ_{6L} = ϕ_6 leaking under load conditions

ϕ_7 = Flux leaking from stator into rotor end shaft extension.

ϕ_7' = Approximation of ϕ_7

ϕ_{7L} = ϕ_7 leaking under load conditions

ϕ_{SH} = Flux in shaft at no load

ϕ_{SH}' = Approximation of ϕ_{SH}

ϕ_{SHL} = ϕ_{SH} under load

ϕ_y = Flux in the yoke outside the stator stack

ϕ_{yc} = Flux in the yoke around the field coil when the housing is jogged out to accommodate the field coil.

ϕ_y = ϕ_{yc} in a straight housing.

CALCULATING THE PERFORMANCE OF THE HOMOPOLAR INDUCTOR

The procedure recommended for hand calculations is as follows:

1.0 Calculate ϕ_T, B_T, ϕ_P and make all of the stator calculations listed on the left hand side of the stator design sheet, page 2.

Use C_1 and C_p values obtained from curve 4, page 11.

1.1 Calculate P_m, P_4, P_5, P_6, P_7.

2.0 <u>No-load Calculations</u>

2.1 Calculate $AT_g = \frac{Bg\ ge}{3.19}$

2.2 Calculate $\phi_m = P_m\ AT_g$

2.3 Calculate $AT_g + AT_m = AT_g + \frac{P\phi_m\ ge}{3.19\ Ag}$

2.4 Calculate $\phi_4 = P_4 \left[AT_g - AT_m\right]$

$$O_5 = P_5 \left[2AT_g - 2AT_m\right]$$

$$\phi_6 = P_6 \left[2AT_g - 2AT_m\right]$$

$$\phi_7 = P_7 \left[AT_g - AT_m\right]$$

2.5 $$B_{TNL} = \frac{\phi_T + \phi_m P}{A_T}$$

2.6 $$B_c = \frac{\phi_P + \phi_m}{A_{core}}$$

2.7 $$\phi_{shaft} = \frac{P}{2}\,\phi_P + P\,\phi_m + \phi_4 + \phi_7$$

2.8 $$B_{shaft} = \frac{\phi_{SH}}{A_{SH}}$$

2.9 $$\phi_y = \phi_{SH} + \phi_6 + \phi_5$$

2.10 $$B_y = \frac{\phi_y}{A_y}$$

2.11 $$F_y = \ell_y \left[\frac{NI}{in} \text{ at } B_y\right]$$

2.12 The total ampere turns drop in the flux circuit is =

$$F_{TNL} = 2\left[F_g + F_{\phi m} + F_T + F_C + F_P\right] + F_{SH} + F_y$$

3.0 <u>Full-load Calculations</u>

3.1 Calculate $e_d = \text{Cos}\,\alpha + x_d \sin\psi$

3.2 Calculate $F'_{TL} = F_T\ (1 + \text{Cos}\,\phi)$

3.3 $$F_{dm} = \frac{.45\, N_e\, I_{ph}\, C_m\, K_d}{P}$$

3.4 $$\phi'_{mL} = P_m\left[F_{dm} + F_g\, e_d\right]$$

3.5 $$F'_{gL} = F_g e_d + \frac{P \phi_m ge}{3.19 Ag}$$

3.6 $$\phi'_{PL} = \phi_P \left[e_d - .93 \chi ad \sin \psi \right]$$

3.7 $$B'_{PL} = \frac{\phi'_{PL} + \phi_m}{A_{pole}}$$

3.8 $$F'_{PL} = h_p \left[NI \text{ at } B'_{PL} \right]$$

3.9 $$\phi'_{4L} = P_4 \left[F'_{gL} + F'_{TL} + F'_{PL} \right]$$

3.10 $$\phi'_{5L} = P_5 \left[2 F'_{gL} + 2 F'_{TL} + 2 F'_{PL} \right]$$

3.11 $$\phi'_{6L} = P_6 \left[2 F'_{gL} + 2 F'_{TL} + 2 F'_{PL} \right]$$

3.12 $$\phi'_{7L} = P_7 \left[F'_{gL} + F'_{TL} + F'_{PL} \right]$$

3.13 $$\phi'_{SHL} = \phi'_{PL} \frac{P}{2} + P \phi_m + \phi'_{4L} + \phi'_{7L}$$

3.14 $$B'_{SHL} = \frac{\phi'_{SHL}}{A_{SH}}$$

3.15 $$F'_{SHL} = \ell_{SH} \left[NI \text{ at } B'_{SHL} \right]$$

3.16 $$\phi_{mL} = P_m \left[F_{dm} + F'_{gL} \right]$$

3.17 $$\phi_{PL} = \phi'_{PL} + \phi_{mL}$$

3.18 $$B_{PL} = \frac{\phi_{PL}}{A_p}$$

3.19 $$F_{PL} = h_p \left[NI \text{ at } B_{PL} \right]$$

3.20 $$F_{gL} = F_g e_d + \frac{P\,\phi_m\,ge}{Ag\ 3.19}$$

3.21 $$B_{TL} = B_T + \frac{4\ \phi_{mL}}{A_T}$$

3.22 $$F_{TL} = h_T \left[NI \text{ at } B_{TL} \right] (1 + \cos\theta)$$

3.23 $$\phi_{7L} = P_7 \left[F_{TL} + F_{gL} + F_{PL} \right]$$

3.24 $$\phi_{4L} = P_4 \left[F_{TL} + F_{gL} + F_{PL} \right]$$

3.25 $$\phi_{shaft\ L} = \frac{\phi_{PL}\,(P)}{2} - \frac{P}{2}\ \phi_{mL} - \phi_{7L} - \phi_{4L}$$

3.26 $$B_{SHL} = \frac{\phi_{SHL}}{A_{SH}}$$

3.27 $$F_{SHL} = \ell_{SH} \left[NI \text{ at } B_{SHL} \right]$$

3.28 $$\phi_{5L} = P_5 \left[2F_{TL} + 2F_{gL} + 2F_{FL} + F_{SHL} \right]$$

3.29 $$\phi_{6L} = P_6 \left[2F_{TL} + 2F_{gL} + 2F_{PL} + F_{SHL} \right]$$

3.30 $$\phi_{core\ L} = \phi_{PL} + \frac{\phi_{5L} + \phi_{6L}}{P}$$

ϕ_4 and ϕ_7 are leakage fluxes between pole positions and do not add to the flux density in the core.

3.31 $$B_{core\ L} = \frac{\phi_{cL}}{A_{cL}}$$

3.32 $$F_{cL} = dbs \left[NI/in \text{ at } B_{cL} \right]$$

3.33 $$\phi_{yL} = \phi_{SHL} + \phi_{6L} + \phi_{5L}$$

3.34 $$B_{yL} = \frac{\phi_{yL}}{A_y}$$

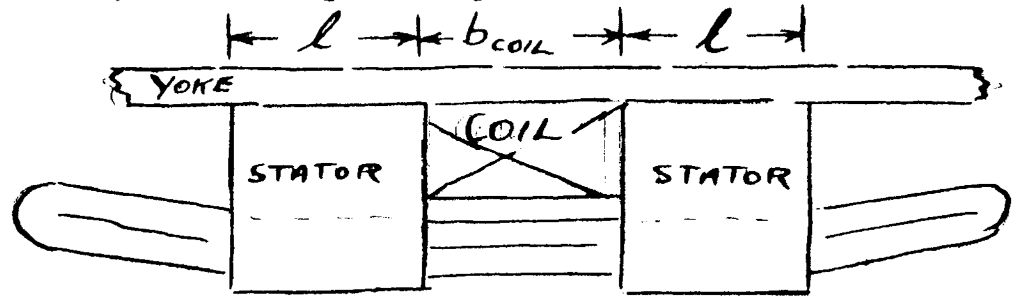
3.35 $$F_{yL} = \ell_y \left[NI/in \text{ at } B_{yL} \right]$$

3.36 $$F_{total_L} = 2F_{gL} + 2F_{TL} + 2F_{cL} + 2F_{PL} + F_{SHL} + F_{yL}$$

YOKE FLUX AND LENGTH

There are three common types of housing or yoke construction and each must be calculated differently.

1. The first type of housing is straight and of uniform thickness

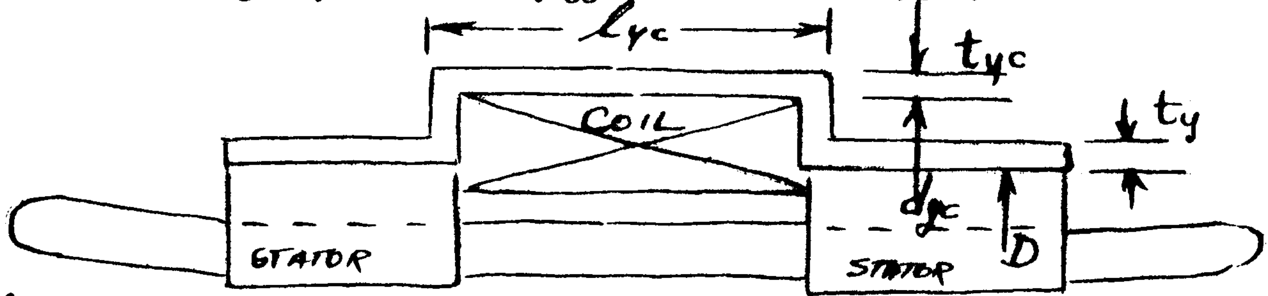


The coil is located between the stator stacks and is between the output winding and the housing or yoke.

$$\ell_y = b_{coil} + 2/3\,\ell$$

assuming that the effective length of the yoke, for the flux density calculated is 1/3 of the stack length.

2. In the second type of housing design, the excitation coil is so located that the housing or yoke must be jogged out to accomodate it.



$$\ell_{yCT} = \ell_{yc} + 2t_{yc} + (d_{yc} - D)$$

The flux in ℓ_{yc} is $)\phi_{yL}$

The flux in the housing or yoke, that is directly outside the stator stack is:

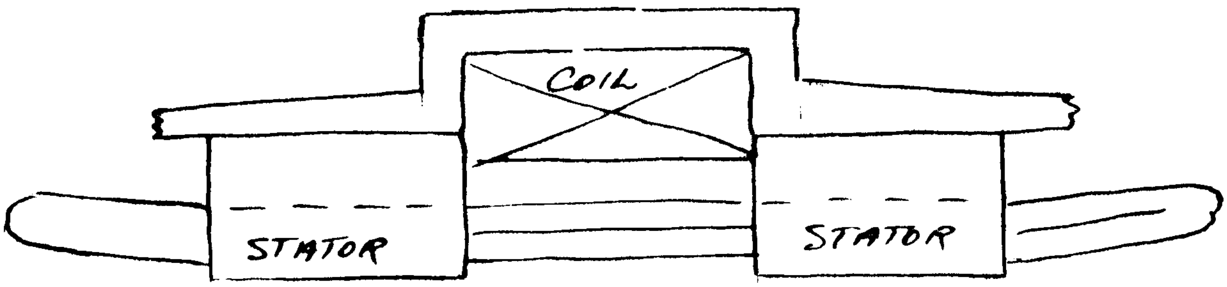
$$\ell_y = 2/3\,\ell$$

$\phi_{y2L} = \phi_{yL} - \phi_{5L}$ since we have calculated a flux value for case 1, the straight, uniform thickness housing.

3. In the third configuration, the housing is tapered over the stator and the yoke density is approximately uniform over most of the stator stack length. The yoke length in this case can be taken as $3/4\,\ell$ over each stack or

$$\ell_y = 3/2\,\ell \quad \& \quad \ell_{yCT} = \ell_{yc} + 2t_{yc} + [d_{yc} - D]$$

AS IN CASE 2.



The flux in the yoke directly outside the stack is

$$\phi_{y3L} = \phi_{yL} - \phi_{5L}$$

(143)	I_{FNL}	FIELD CURRENT - at no load $I_{FNL} = (F_{NL})/(N_F) = (142)/(99)$
(144)	S_F	CURRENT DENSITY - at no load. Amperes per square inch of field conductor. $S_F = (I_{FNL})/(a_{cf}) = (143)/(106)$
(145)	E_F	FIELD VOLTS - at no load. This calculation is made with cold field resistance at 20°C for no load condition. $E_F = (I_{FNL})(R_{f\ cold}) = (143)(107)$
(146)	I^2R	ROTOR I^2R - at no load. The copper loss in the field winding is calculated with cold field resistance at 20°C for no load condition. $I^2R = (I_{FNL})^2 (R_{f\ cold}) = (143)^2 (107)$

(147)	F&W	<u>FRICTION & WINDAGE LOSS</u> - The best values are obtained by using existing data. For ratioing purposes, the loss can be assumed to vary approximately as the 5/2 power of the rotor diameter and as the 3/2 power of the RPM. When no existing data is available, the following calculation can be used for an approximate answer. Insert 0. when computer is to calculate F&W. Insert actual F&W when available. Use same value for all load conditions. $F\&W = 2.52 \times 10^{-6} (d_r)^{2.5}\ 2\ell_n\ (RPM)^{1.5}$ $= 2.52 \times 10^{-6} (11a)^{2.5}\ 2(76)\ (7)^{1.5}$
(148)	W_{TNL}	<u>STATOR TEETH LOSS</u> - at no load. The no load loss (W_{TNL}) consists of eddy current and hysteresis losses in the iron. For a given frequency the no load tooth loss will vary as the square of the flux density. The losses in the two stators are approximately the same as if all the voltage were generated in a single stator.

$$W_{TNL} = .453(b_{t\,1/3})(Q)(\ell_s)(h_s)(K_Q)$$

$$= .453(57a)(23)(17)(22)(148)$$

$$\text{Where } K_Q = (k)\left[\frac{(B_t)}{(B)}\right]^2 = (19)\left[\frac{(127)}{(20)}\right]^2$$

(149) W_c STATOR CORE LOSS - The stator core losses are due to eddy currents and hysteresis and do not change under load conditions. For a given frequency the core loss will vary as the square of the flux density (B_c).

$$W_c = 1.42\left[(D) - (h_c)\right](h_c)(\ell_s)(K_Q)$$

$$= 1.42\left[(12) - (24)\right](24)(17)(149)$$

$$\text{Where } K_Q = (k)\left[\frac{(B_c)}{(B)}\right]^2 = (19)\left[\frac{(128)}{(20)}\right]^2$$

(150) W_{NPL} POLE FACE LOSS - at no load. The pole surface losses are due to slot ripple caused by the stator slots. They depend upon the width of the stator slot opening, the air gap, and the stator slot ripple frequency. The no load pole face loss (W_{PNL}) can be obtained from Graph 2. Graph 2 is plotted on the bases of open slots. In order to apply this curve to partially open slots, substitute b_o for b_s. For a better understanding of Graph 2, use the following sample.

K_1 as given on Graph 2 is derived empirically and depends on lamination material and thickness. Those values given on Graph 2 have been used with success. K_1 is an input and must be specified. See item (151) for values of K_1.

(150) (Cont.)

K_2 is shown as being plotted as a function of $(B_G)^{2.5}$ Also note that upper scale is to be used. Another note in the lower right hand corner of graph indicates that for a solid line (_____), the factor is read from the left scale, and for a broken or dashed line (____ - ____ -), the right scale should be read. For example, find K_2 when B_G = 30 kilo lines. First locate 30 on upper scale. Read down to the intersection of solid line plot of $K_2 = f(B_G)^{2.5}$. At this intersection read the left scale for K_2. $K_2 = .28$. Also refer to item (152) for K_2 calculations.

K_3 is shown as a solid line plot as a function of $(F_{SLT})^{1.65}$ The note on this plot indicates that the upper scale X 10 should be used. Note F_{SLT} = slot frequency. For an example, find K_3 when F_{SLT} = 1000. Use upper scale X 10 to locate 1000. Read down to intersection of solid line plot of $K_3 = f(F_{SLT})^{1.65}$. At this intersection read the left scale for K_3. $K_3 = 1.35$. Also refer to item (153) for K_3 calculations.

For K_4 use same procedure as outlined above except use lower scale. Do not confuse the dashed line in this plot with the note to use the right scale. The note does not apply in this case. Read left scale. Also refer to item (154) for K_4 calculations.

For K_5 use bottom scale and substitute b_o for b_s when using partially closed slot. Read left scale when using solid plot. Use right scale when using dashed plot. Also refer to item (155) for K_5 calculations.

(150) (Cont.)

For K_6 use the scale attached for C_1 and read K_6 from left scale. Also refer to item (156) for K_6 calculations.

The above factors (K_2), (K_3), (K_4), (K_5), (K_6) can also be calculated as shown in (152), (153), (154), (155), (156), respectively.

$$W_{PNL} = \pi(d)(\ell)(K_1)(K_2)(K_3)(K_4)(K_5)(K_6)$$
$$= \pi(11)(13)(151)(152)(153)(154)(155)(156)$$

(151) K_1

K_1 is derived empirically and depends on lamination material and thickness. The values used successfully for K_1 are shown on Graph 2. They are

K_1 = 1.17 for .028 lam thickness, low carbon steel
= 1.75 for .063 lam thickness, low carbon steel
= 3.5 for .125 lam thickness, low carbon steel
= 7.0 for solid

K_1 is an input and must be specified on input sheet.

(152) K_2

K_2 can be obtained from Graph 2 (see item 150) for explanation of Graph 2) or it can be calculated as follows:

$$K_2 = f(B_G) = 6.1 \times 10^{-5} (B_G)^{2.5}$$
$$= 6.1 \times 10^{-5} (125)^{2.5}$$

(153) K_3

K_3 can be obtained from Graph 2 (see item 150) for explanation of Graph 2) or it can be calculated as follows:

$$K_3 = f(F_{SLT}) = 1.5147 \times 10^{-5} (F_{SLT})^{1.65}$$
$$= 1.5147 \times 10^{-5} (153)^{1.65}$$

$$\text{Where } F_{SLT} = \frac{(RPM)}{60} (Q)$$
$$= \frac{(7)}{60} (23)$$

(154)	K_4	K_4 can be obtained from Graph 2 (see item (150) for explanation of Graph 2) or it can be calculated as follows: For $\tau_s \leqq .9$ $K_4 = f(\tau_s) = .81(\tau_s)^{1.285}$ $= .81(26)^{1.285}$ For $.9 \leqq \tau_s \leqq 2.0$ $K_4 = f(\tau_s) = .79(\tau_s)^{1.145}$ $= .79(26)^{1.145}$ For $\tau_s > 2.0$ $K_4 = f(\tau_s) = .92(\tau_s)^{.79}$ $= .92(26)^{.79}$
(155)	K_5	K_5 can be obtained from Graph 2 (see item (150) for explanation of Graph 2) or it can be calculated as follows: For $(b_s)/(g) \leqq 1.7$ $K_5 = f(b_s/g) = .3\left[(b_s)/(g)\right]^{2.31}$ $= .3\left[(22)/(59)\right]^{2.31}$ NOTE: For partially open slots substitute b_o for b_s in equations shown. For $1.7 < (b_s)/(g) \leqq 3$ $K_5 = f(b_s)/(g) = .35\left[(b_s)/(g)\right]^{2}$ $= .35\left[(22)/(59)\right]^{2}$

For $3 < (b_s)/(g) \leq 5$

$$K_5 = f(b_s)/(g) = .625\left[(b_s)/(g)\right]^{1.4}$$

$$= .625\left[(22)/(59)\right]^{1.4}$$

For $(b_s)/(g) > 5$

$$K_5 = f\left[(b_s)\right]/(g) = 1.38\left[(b_s)/(g)\right]^{.965}$$

$$= 1.38\left[(22)/(59)\right]^{.965}$$

(156) | K_6 | K_6 can be obtained from Graph 2 (see item (150) for explanation of Graph 2) or it can be calculated as follows:

$$K_6 = f(C_1) = 10^{\left[.9323(C_1) - 1.60596\right]}$$

$$= 10^{\left[.9323(71) - 1.60596\right]}$$

(158) | I^2R | STATOR I^2R - at no load. This item = 0. Refer to item (173) for 100% load stator I^2R.

(158a) | -- | EDDY LOSS - at no load. This item = 0. Refer to item (173a) for 100% load eddy loss.

(159) | -- | TOTAL LOSSES - at no load. Sum of all losses

Total losses = (Rotor I^2R) + (F & W) + (Stator Teeth Loss) + (Stator Core Loss) + (Pole Face Loss) + (Damper Loss)

= (146) + (147) + (148) + (149) + (150) + (157)

(166)	I_{FFL}	<u>FIELD CURRENT</u> at 100% load $I_{FFL} = (F_{FL})/(N_F) = (165)/(99)$
(167)	--	<u>CURRENT DENSITY</u> at 100% load Current density $= (I_{FFL})/(a_{cf}) = (166)/(106)$
(168)	E_{FFL}	<u>FIELD VOLTS</u> at 100% load - This calculation is made with hot field resistance at expected temperature at 100% load. Field Volts $= (I_{FFL})\ (R_{f\ hot}) = (166)(108)$
(169)	I^2R_F	<u>FIELD I^2R</u> at 100% load - The copper loss in the field winding is calculated with hot field resistance at expected temperature for 100% load condition. Field $I^2R = (I_{FFL})^2(R_{F\ hot}) = (166)^2(108)$
(170)	W_{TFL}	<u>STATOR TEETH LOSS</u> at 100% load - The stator tooth loss under load increases over that of no load because of the parasitic fluxes caused by the ripple due to flux distortion. $W_{TFL} = \left\{ 2\left[.27(X_d) \quad \frac{(\%\ Load)}{100}\right]^{1.8} + 1 \right\} (W_{TNL})$ NOTE (X_d) is in per unit $= \left\{ 2\left[.27(83) \quad 1 + 1 \right\} (148)$

POLE FACE LOSS at 100% load

$$W_{PFL} = \left\{ \left[\frac{(K_{sc})(I_{PH}) \frac{(\%\ \text{Load})}{100} (n_s)}{(C)(F_g)} \right]^2 + 1 \right\} (W_{PNL})$$

$$= \left\{ \left[\frac{(171)(8)\ 1\ (30)}{(32)(131)} \right]^2 + 1 \right\} (150)$$

(K_{sc}) is obtained from Graph 3

(173) | I^2R | STATOR I^2R at 100% load - The copper loss based on the D.C. resistance of the winding. Calculate at the maximum expected operating temperature.

$$I^2R = (m)(I_{PH})^2 (R_{SPH\,hot}) \frac{(\%\ \text{Load})}{100}$$

$$= (5)(8)^2 (54)\ 1.$$

(173a) | -- | EDDY LOSS - Stator I^2R loss due to skin effect

$$\text{Eddy Loss} = \left[\frac{(EF_{top}) + (EF_{bot})}{2} - 1 \right] (\text{Stator } I^2R)$$

$$= \left[\frac{(55) - (56)}{2} - 1 \right] (173)$$

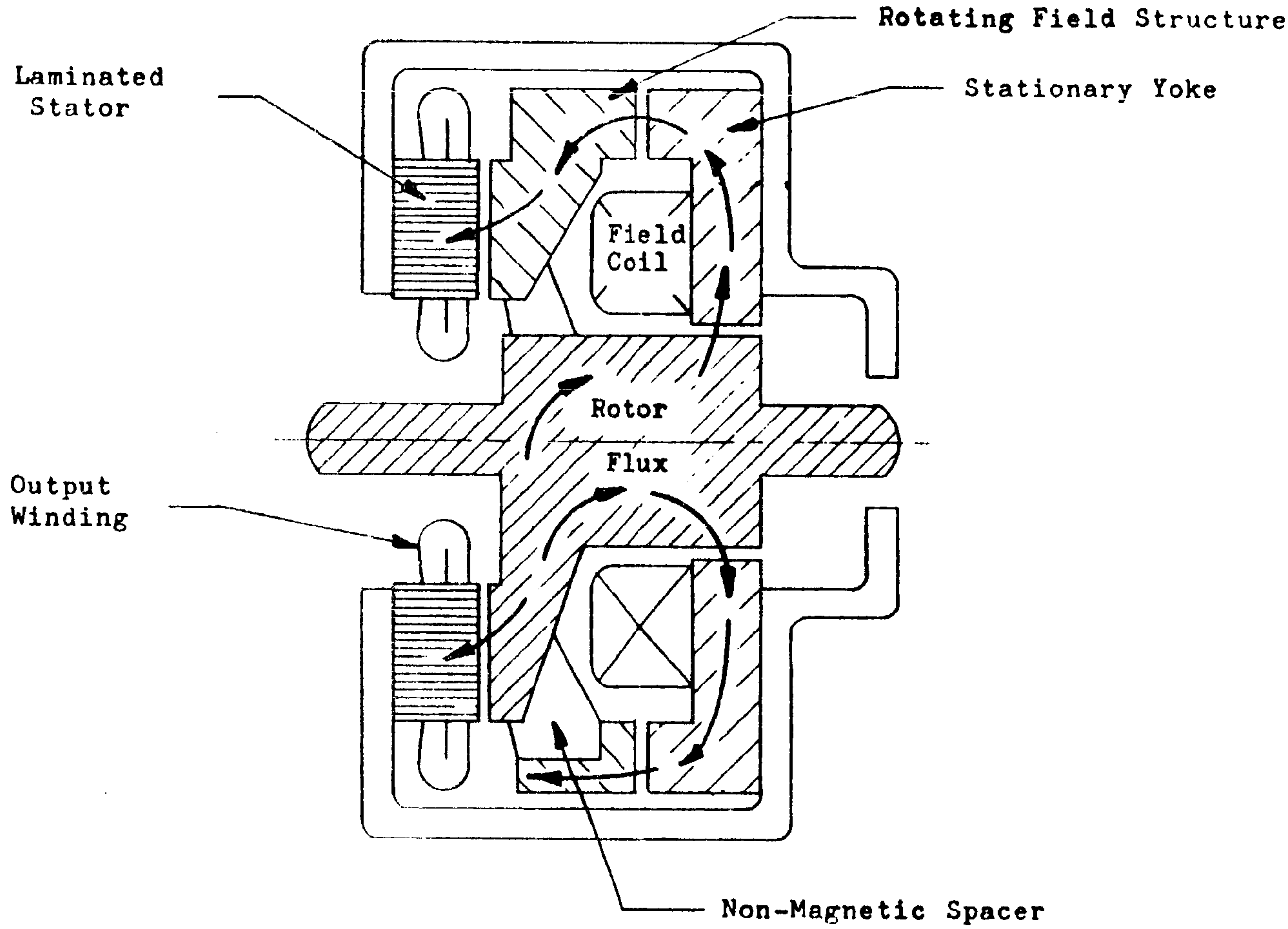
(174) | -- | TOTAL LOSSES at 100% load - sum of all losses at 100% load

Total Losses = (Field I^2R) + (F & W) + (Stator Teeth Loss)
+ (Stator Core Loss) + (Pole Face Loss)
(Stator I^2R) (Eddy Loss)

= (169) + (147) + (170) + (149) + (171) + (172) + (173) + (173a)

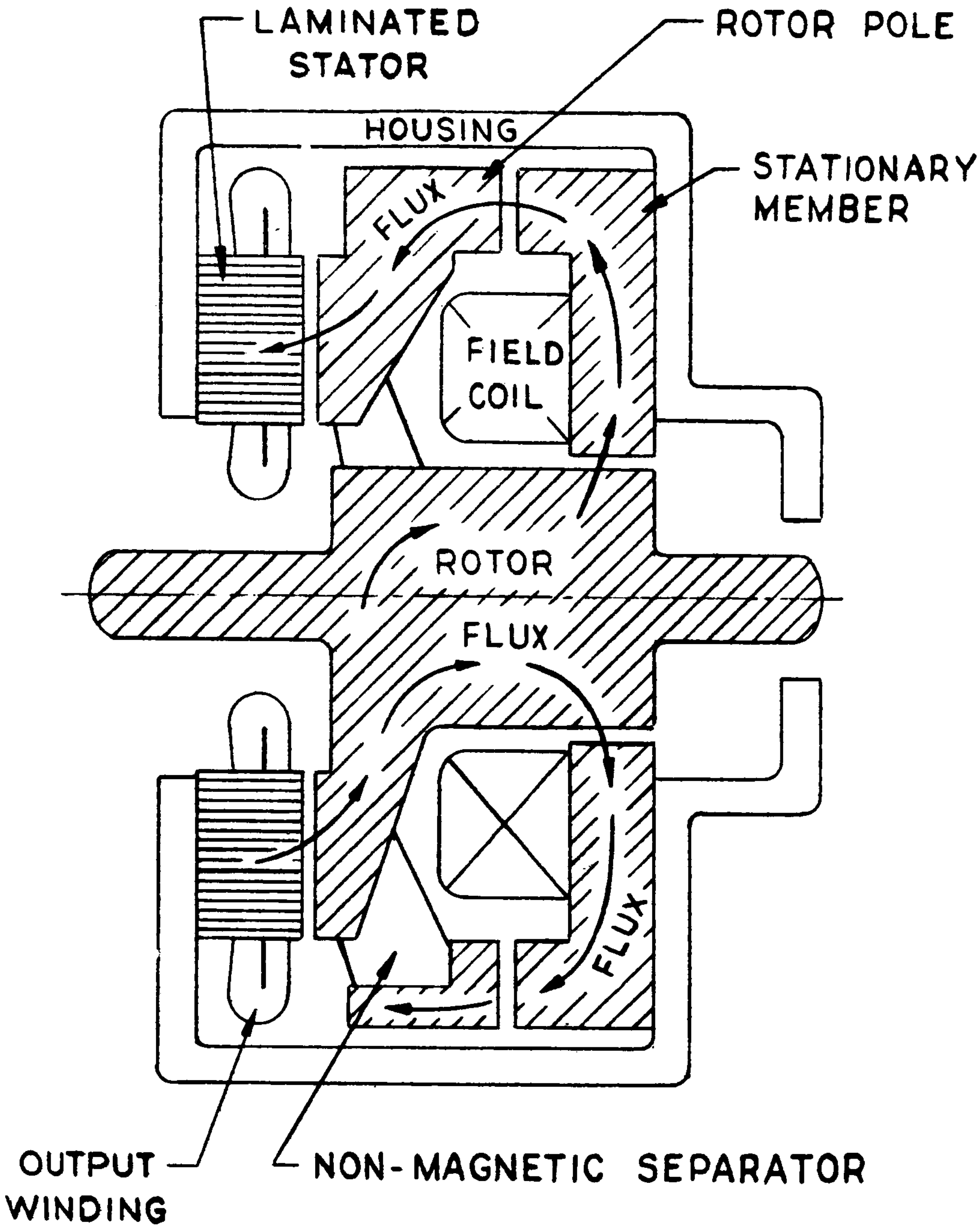
(175)	--	RATING IN WATTS at 100% load
		$$\text{Rating} = 3(E_{PH})(I_{PH}) \quad (P.F.)\ \frac{(\%\ \text{Load})}{100}$$
		$$= 3(4)(8) \quad (9)(1.)$$
(176)	--	RATING & Σ LOSSES = (175) + (174)
(177)	--	% LOSSES $= \left[\Sigma\ \text{Losses} / \text{Rating} + \Sigma\ \text{Losses}\right] 100$
		$$= \left[(174)/(177)\right] 100$$
(178)	--	% EFFICIENCY = 100% - % Losses
		= 100% - (177)
		Item (160) through (178) are 100% load calculations. These items can be recalculated for any load condition by simply inserting the values that correspond to the % load being calculated. The factor $\frac{(\%\ \text{Load})}{100}$ takes care of (I_{PH}) as it changes with load.
		Note that values for F & W (147) and W_C (Stator Core Loss) (149) do not change with load,

DISK-TYPE, OR AXIAL AIR-GAP
LUNDELL TYPE AC GENERATORS



H

AXIAL AIR-GAP, LUNDELL TYPE, A.C. GENERATOR



DISK TYPE LUNDELL

(No Model.)

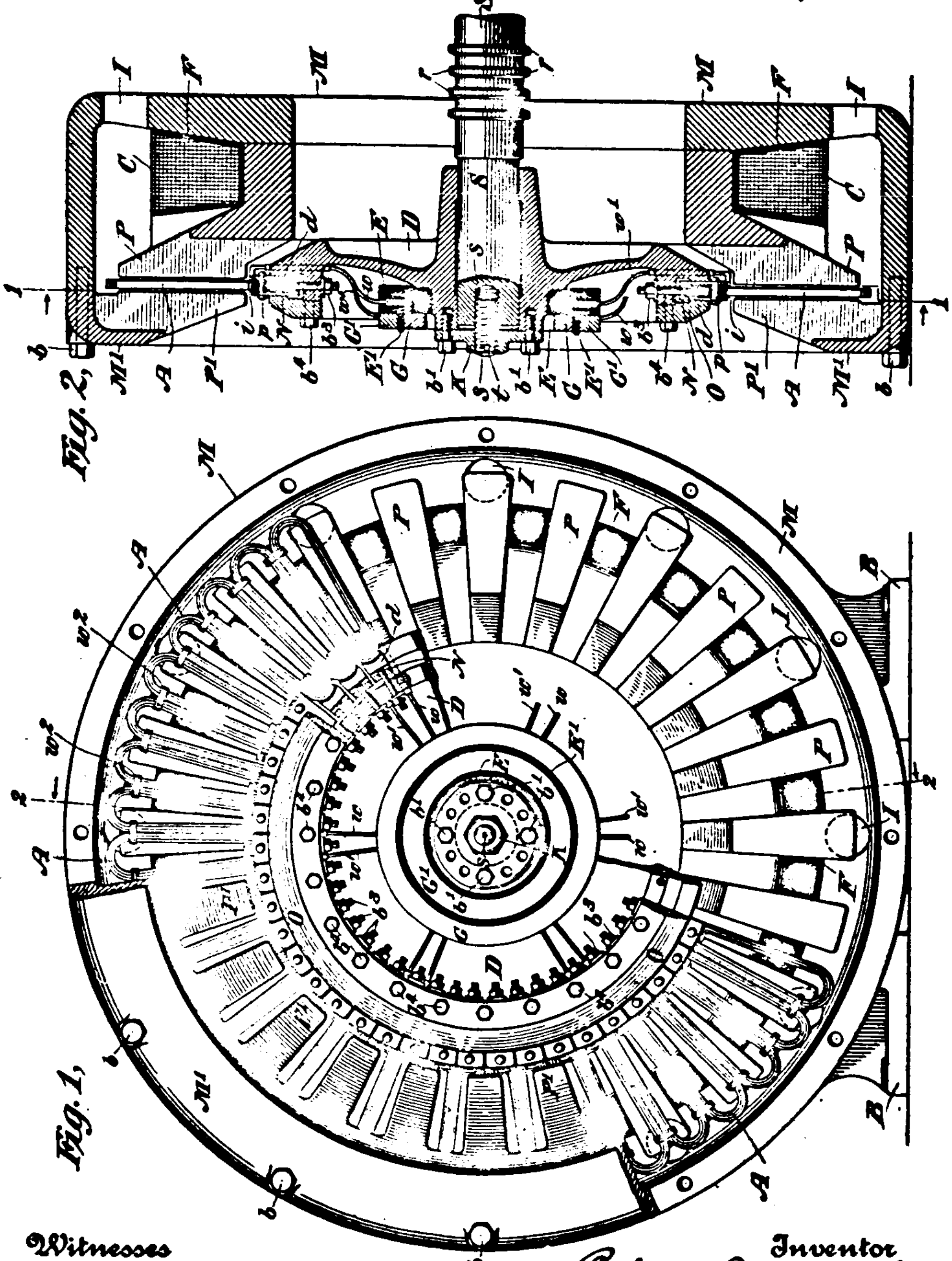
2 Sheets—Sheet 1.

R. LUNDELL.

DYNAMO ELECTRIC MACHINE.

No. 490,809.

Patented Jan. 31, 1893.



Witnesses

C. E. Ashley

C. W. Chamberlain

Inventor

Robert Lundell

By his Attorney

Charles J. Kintner

The original Lundell generator patented by Robert Lundell in 1893 was an axial air-gap generator with the output windings rotating.

The newer brushless, axial air-gap generator has the field structure rotating and the output winding is stationary. The brushes are eliminated through the use of auxiliary air-gaps.

The weight of this machine is approximately the same as that of a radial gap Lundell generator of the same rating, speed and frequency. It can be built with two stators and one field coil for maximum output at a given diameter.

The output of the disk-type or axial-gap Lundell generator is a function of the third power of the stator diameter, $(D)^3$.

If a single-stator axial-gap generator and a radial-gap generator are built with the same KVA, frequency, RPM, air-gap flux density, and stator ampere loading (or the same reactances) the rotor of the disk-type generator will be a minimum of two (2) times the diameter of the radial-gap generator. See derivations in Second Quarterly Report.

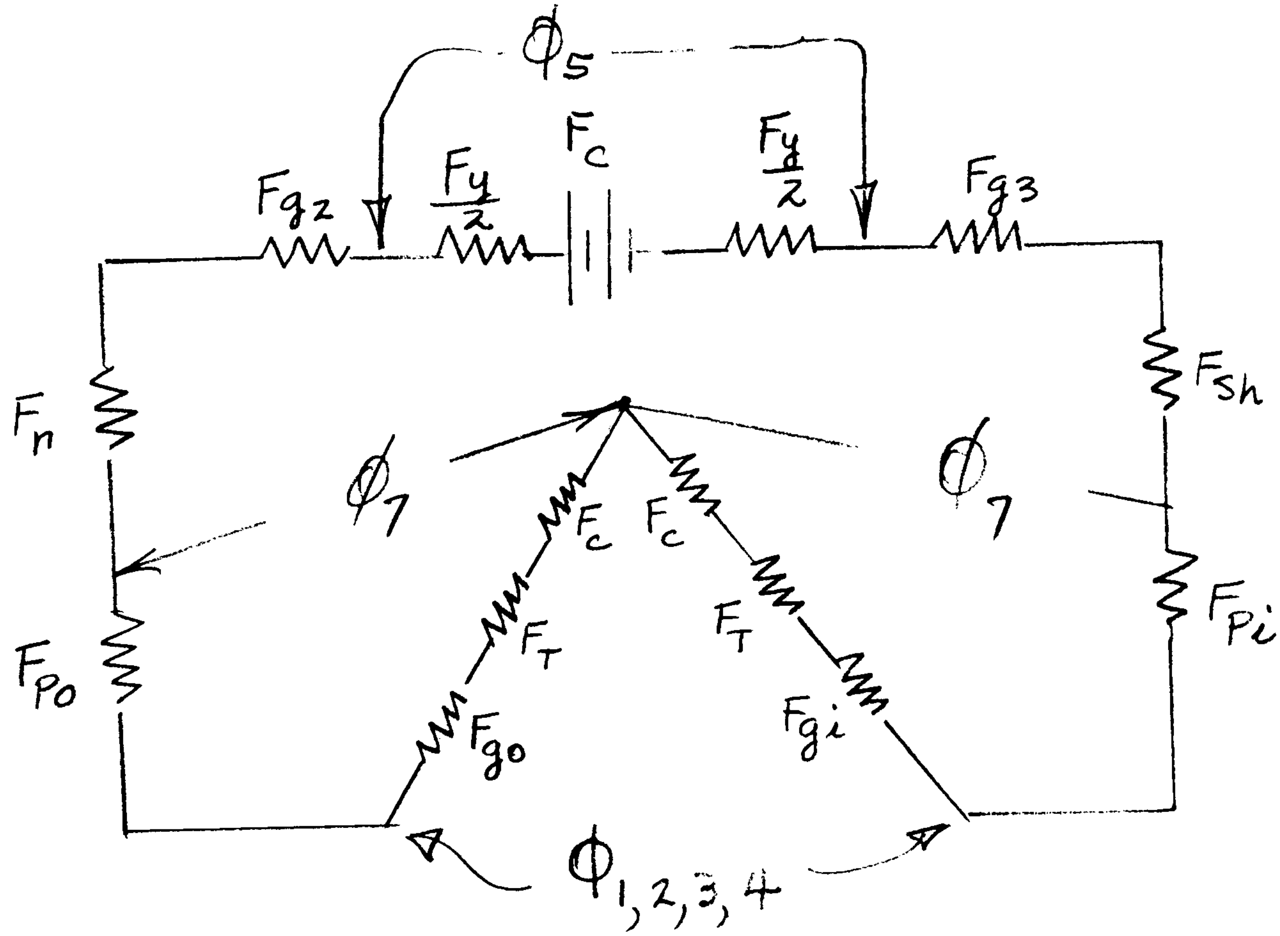
At the same rating and conditions of load, the single-stator axial air-gap machine operates at four (4) times the stress level of the radial-gap machine.

The outputs of all of the Lundell generators are functions of the third power of their rotor diameters $(d)^3$, but for equal maximum rotor stresses, the radial gap generator will have $(2)^3$ or 8 times the output of the disk-type machine.

Though the machine weights are comparable, the disk-type machine has more WR^2 and more gyroscopic moment than the radial gap Lundell machines.

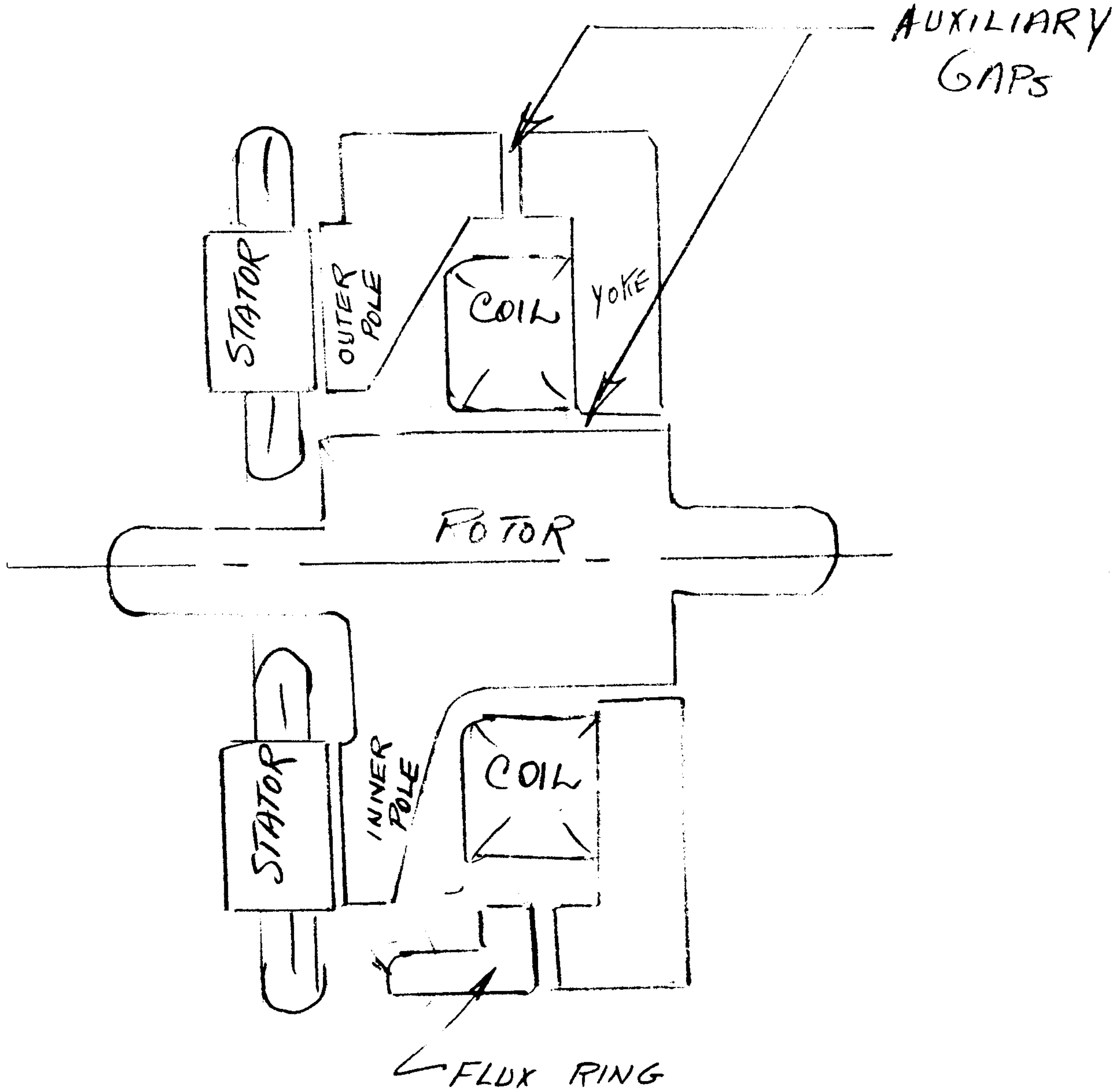
The attractive force, due to air-gap flux, between the rotor and stator of the single-stator machine is great and the single-stator configuration cannot be used with fluid bearings. The more balanced two-stator design must be used with fluid bearings.

In some cases, the axial air-gap machines are advantageous because of their physical configuration and the design procedure is included in this study for completeness.

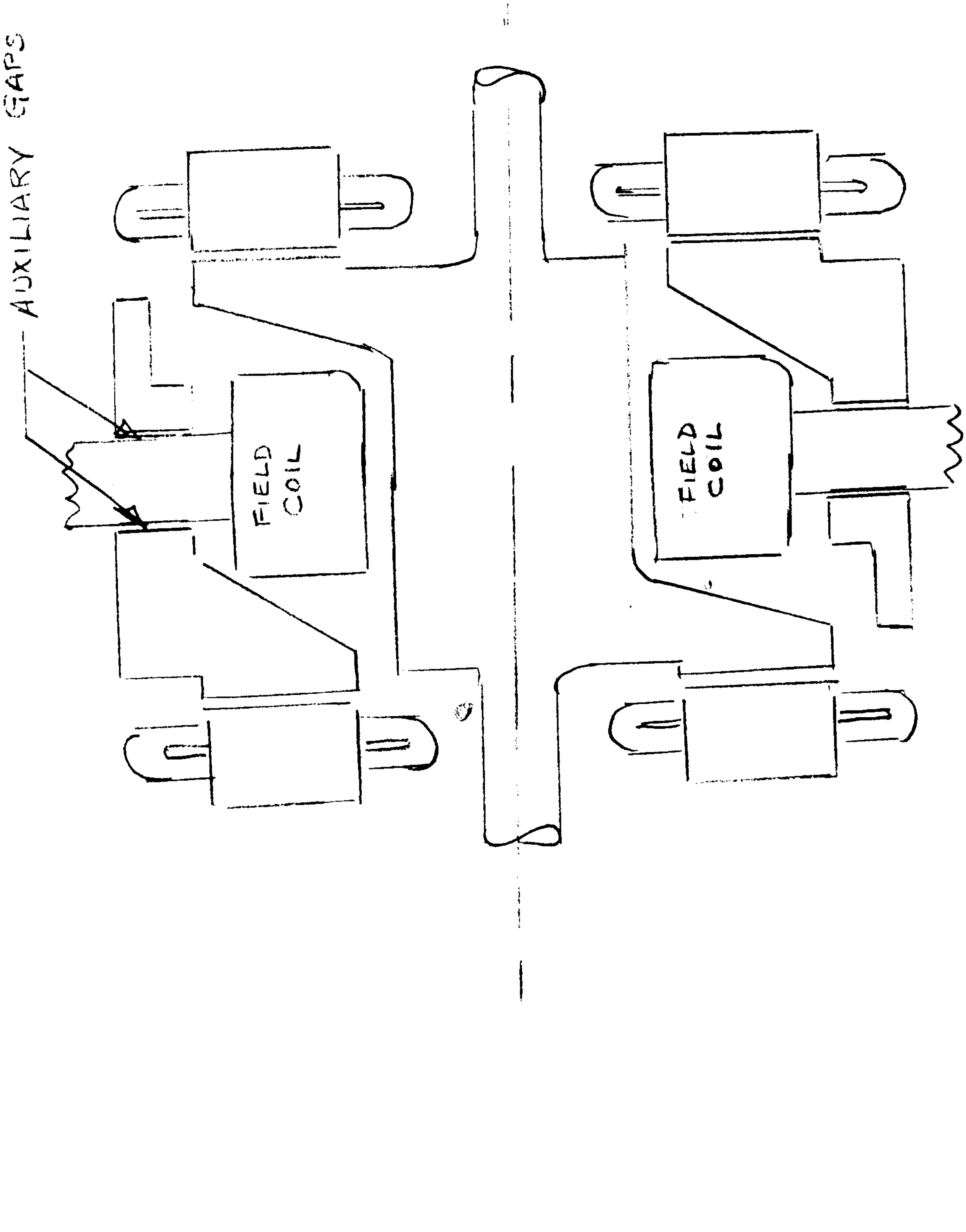


COMPLETE FLUX CIRCUIT OF A SINGLE-STATOR, AXIAL-GAP, LUNDELL, A-C GENERATOR

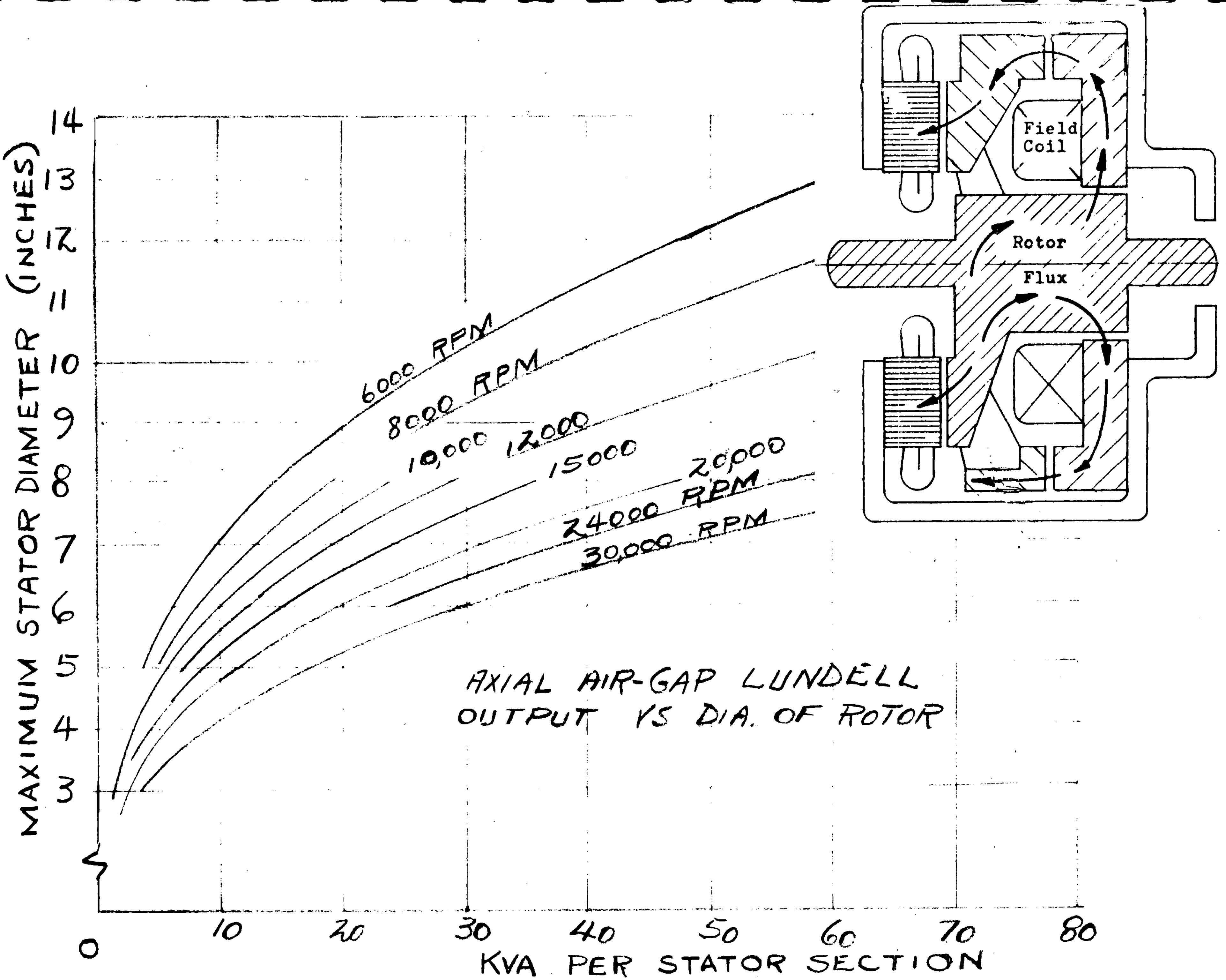
LEAKAGE FLUXES ARE INCLUDED



ELECTROMAGNETIC PARTS OF A SINGLE STATOR, AXIAL GAP, LUNDELL, A-C GENERATOR



ELECTROMAGNETIC PARTS OF
A TWO-STATOR, AXIAL GAP, LUNDELL
A-C GENERATOR



Field Coil
Rotor
Flux
6000 RPM
8000 RPM
10,000
12,000
15000
20000 RPM
24000 RPM
30,000 RPM
AXIAL AIR-GAP LUNDELL
OUTPUT VS DIA. OF ROTOR
MAXIMUM STATOR DIAMETER (INCHES)
KVA PER STATOR SECTION
14
13
12
11
10
9
8
7
6
5
4
3
0
10
20
30
40
50
60
70
80

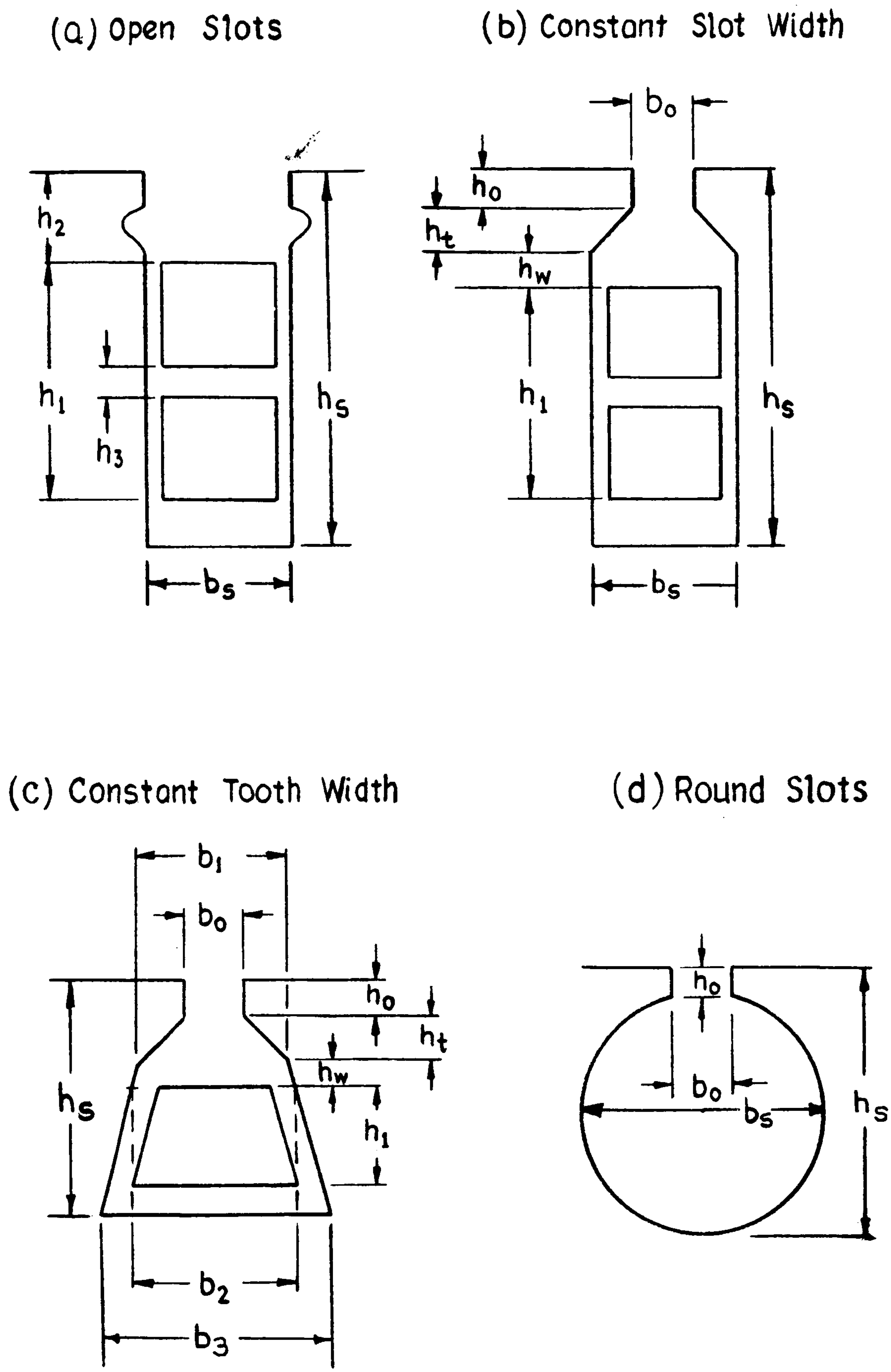

(a) Open Slots
(b) Constant Slot Width
(c) Constant Tooth Width
(d) Round Slots
h_2
h_1
h_3
h_s
b_s
b_o
h_o
h_t
h_w
b_1
b_2
b_3

FIG 1

TABLE - 1

CHORD FACTORS K_p FOR HARMONICS AT DIFFERENT PITCHES

q														y/mq																									
1	3/3																															2/3							
2	6/6																5/6															4/6							
3	9/9											8/9										7/9										6/9							
4	12/12								11/12								10/12								9/12							8/12							7/12
5	15/15						14/15							13/15						12/15							11/15					10/15				9/15			
6	18/18					17/18						16/18					15/18					14/18						13/18				12/18							
7	21/21				20/21					19/21					18/21				17/21					16/21					15/21			14/21							
8	24/24			23/24					22/24				21/24				20/24				19/24				18/24					17/24		16/24			15/24				14/24
9	27/27		26/27					25/27				24/27				23/27		22/27				21/27				20/27					19/27	18/27		17/27				16/27	
10	30/30	29/30					28/30				27/30			26/30			25/30			24/30			23/30				22/30					21/30	20/30	19/30			18/30		
y/mq	1.0	.967	.964	959	952	945	934	926	917	905	906	889	875	867	857	852	833	815	810	800	792	779	767	762	750	741	734	722	712	709	705	700	667	634	630	625	600	592	584
1	1.0	999	998	998	997	996	995	993	991	989	988	985	981	978	975	973	966	958	956	951	947	940	934	931	924	918	914	906	901	897	894	891	866	839	835	831	809	800	793
3	1.0	988	985	981	975	966	951	940	924	901	891	865	831	809	782	766	707	643	623	588	556	500	454	434	383	342	309	259	222	195	174	156	0.0	156	174	195	309	342	383
5	1.0	966	958	947	931	906	866	835	793	733	707	643	556	500	434	396	259	116	075	0.0	065	174	259	295	383	447	500	574	623	660	690	707	866	966	974	982	1.0	998	993
7	1.0	943	918	897	866	819	743	686	609	500	454	342	177	105	0.0	058	259	449	500	588	659	766	839	866	924	958	978	996	1.0	998	993	988	866	629	588	559	309	225	139
9	1.0	897	866	831	782	707	588	500	383	223	156	0.0	191	304	434	500	707	866	901	951	981	1.0	998	975	924	866	804	707	623	556	500	454	0.0	358	407	462	500	755	875
11	1.0	834	802	750	680	574	407	287	131	075	156	342	556	660	782	795	966	998	989	951	897	766	629	583	383	231	105	087	222	321	396	454	866	999	994	981	809	721	618
13	1.0	777	727	659	563	423	208	058	131	365	454	643	831	914	975	993	966	802	733	588	442	174	052	148	383	550	669	818	901	947	973	988	866	371	294	195	309	459	598
15	1.0	707	643	556	434	259	0.0	174	383	624	707	866	981	1.0	975	940	707	342	223	0.0	195	500	707	782	924	981	1.0	966	907	831	766	707	0.0	696	760	831	1.0	982	930
17	1.0	629	550	442	295	087	208	346	609	826	891	985	981	914	782	686	259	231	365	588	752	940	999	997	924	802	669	423	222	065	058	156	866	940	898	831	309	100	113
19	1.0	545	449	321	149	087	407	597	793	956	986	985	831	669	434	287	259	727	826	951	998	940	777	680	363	116	105	423	623	752	835	891	866	072	047	194	809	952	989
21	1.0	454	342	195	0.0	259	588	766	924	1.0	988	866	556	309	0.0	174	707	985	1.0	951	831	500	156	0.0	383	643	804	966	1.0	981	940	891	0.0	881	935	981	809	628	402
23	1.0	358	231	064	149	423	743	894	991	956	891	643	195	105	434	597	966	918	826	588	321	174	545	680	924	998	978	819	623	442	287	156	866	792	696	555	309	566	778
25	1.0	259	116	064	295	574	866	973	991	826	707	342	195	500	782	894	966	550	365	0.0	321	766	966	997	924	727	500	087	222	442	597	707	866	061	383	555	1.0	951	809
27	1.0	170	0.0	195	434	707	951	1.0	924	624	454	0.0	556	809	975	1.0	707	0.0	223	558	831	1.0	897	782	383	0.0	305	707	901	981	1.0	988	0.0	983	1.0	981	309	025	356
29	1.0	052	116	321	563	819	995	973	793	365	156	342	831	978	975	894	259	550	733	951	948	766	358	148	383	727	914	996	901	752	597	454	866	569	412	194	809	965	995
31	1.0	052	231	442	680	906	995	894	109	075	156	643	981	978	782	597	259	918	989	951	752	174	358	583	924	998	914	574	222	065	287	454	866	517	673	831	809	525	162
33	1.0	156	342	556	782	966	951	766	383	223	454	866	981	809	434	174	707	985	901	588	195	500	891	975	924	643	309	259	623	831	940	988	0.0	992	946	831	309	666	910
35	1.0	259	449	659	866	996	866	597	131	500	707	985	831	500	0.0	287	966	727	500	0.0	442	940	966	866	383	116	500	906	1.0	947	835	707	866	294	079	194	1.0	905	637
37	1.0	358	550	750	931	996	743	396	131	793	891	985	556	105	434	686	966	231	075	588	897	940	545	295	383	802	978	906	623	321	058	156	866	855	884	981	309	150	577
39	1.0	454	643	831	975	966	588	174	383	901	988	866	195	309	782	940	707	342	623	951	981	500	156	434	924	985	809	259	222	556	766	891	0.0	909	780	555	805	990	939
41	1.0	545	727	891	997	906	407	058	609	989	988	643	195	669	975	933	259	802	956	951	659	174	777	931	924	550	105	574	901	998	973	891	866	010	264	555	809	414	088
43	1.0	629	802	947	997	819	208	287	793	989	891	342	556	914	975	795	259	998	956	588	065	766	999	931	383	231	669	996	901	660	346	156	866	916	990	981	309	754	984
45	1.0	707	866	987	925	707	0.0	500	924	901	707	0.0	831	1.0	782	500	707	866	623	0.0	556	1.0	707	434	383	866	1.0	701	222	145	500	707	0.0	739	523	194	1.0	844	262
47	1.0	777	918	998	931	574	208	686	991	733	454	342	981	914	434	058	966	449	075	588	947	766	052	295	924	958	669	087	623	897	993	988	866	312	575	831	305	273	762
49	1.0	839	958	998	856	423	407	835	991	500	156	643	981	669	0.0	396	966	116	500	951	947	174	629	866	224	597	105	819	1.0	847	690	454	866	994	979	831	809	1.0	823
51	1.0	891	985	981	782	254	588	940	924	223	156	866	831	309	434	766	707	643	901	951	556	500	988	975	383	342	809	966	623	195	174	454	0.0	501	203	194	805	297	333
53	1.0	934	998	947	680	087	743	993	793	075	454	985	556	105	782	973	259	958	989	588	065	940	839	583	383	918	978	423	222	660	894	988	866	585	818	981	309	721	997
55	1.0	966	998	897	563	087	866	993	605	365	707	985	195	500	975	973	259	958	733	0.0	659	940	259	148	924	918	500	423	901	998	894	707	866	979	853	555	1.0	771	187
57	1.0	988	985	831	434	255	951	940	383	624	891	866	195	809	975	766	707	643	223	588	981	500	454	782	924	342	309	966	901	556	174	156	0.0	215	141	555	309	407	899
59	1.0	999	958	756	295	423	995	835	131	826	988	643	556	978	782	396	966	116	365	951	897	174	934	997	383	597	914	819	222	321	690	891	866	803	965	981	809	994	777
61	1.0	999	978	659	149	574	995	686	131	956	988	342	831	978	434	058	966	449	826	951	442	766	934	680	383	958	914	087	623	947	993	891	866	872	625	194	809	175	557
63	1.0	988	866	556	0.0	707	051	500	383	1.0	891	0.0	981	809	0.0	500	707	[illegible]	1.0	589	195	1.0	454	0.0	924	866	309	707	1.0	831	500	156	0.0	091	468	831	309	894	947

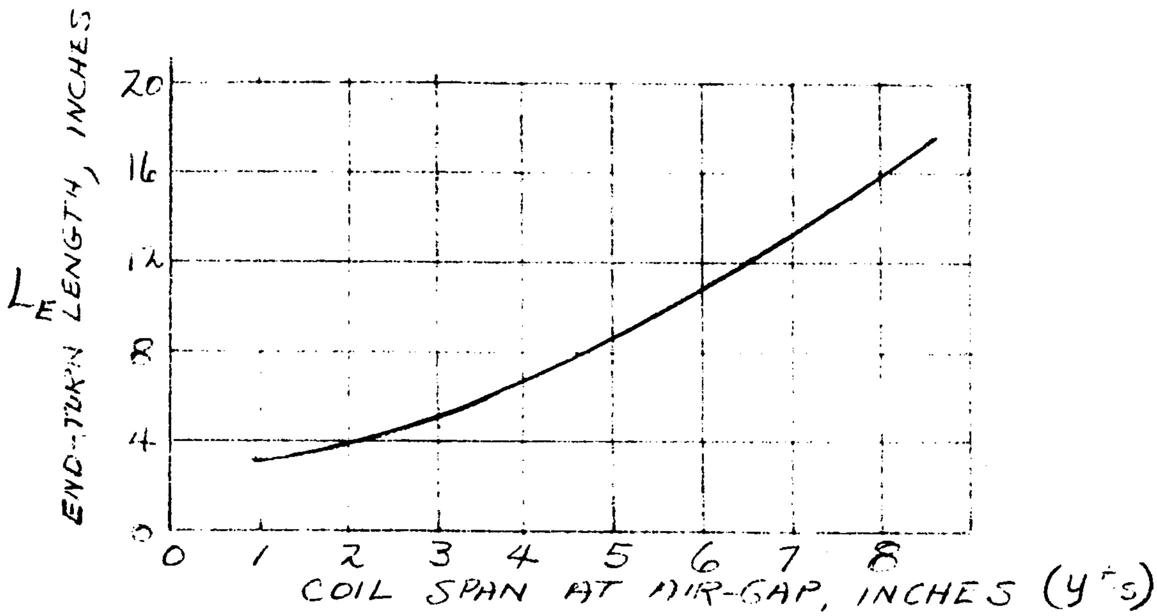
VALUES OF K_{dn} FOR INTEGRAL-SLOT, 3φ WINDINGS - TABLE 2

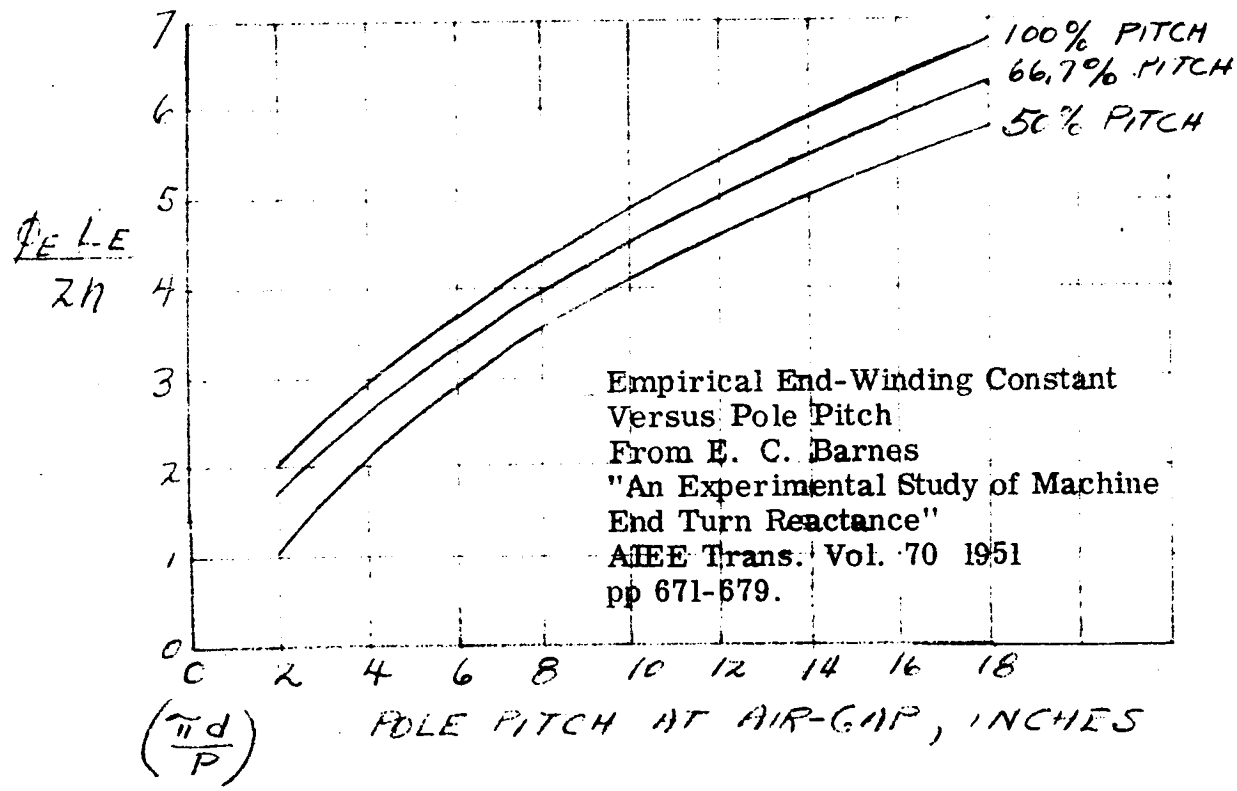
n	K_{dn} - HARMONIC DISTRIBUTION FACTORS									
q=	2	3	4	5	6	7	8	9	10	∞
1	.966	.960	.958	.957	.957	.957	.956	.955	.955	.955
3	.707	.667	.654	.646	.644	.642	.641	.640	.639	.636
5	.259	.217	.205	.200	.197	.195	.194	.194	.193	.191
7	-.259	-.177	-.158	-.149	-.145	-.143	-.141	-.140	-.140	-.136
9	-.707	-.333	-.270	-.247	-.236	-.229	-.225	-.222	-.220	-.212
11	-.966	-.177	-.126	-.110	-.102	-.097	-.095	-.093	-.092	-.087
13	-.966	.217	.126	.102	.092	.086	.083	.081	.079	.073
15	-.707	.667	.270	.200	.172	.158	.150	.145	.141	.127
17	-.259	.960	.158	.102	.084	.075	.070	.066	.064	.056
19	.259	.960	-.205	-.110	-.084	-.072	-.066	-.062	-.060	-.059
21	.707	.667	-.654	-.247	-.172	-.143	-.127	-.118	-.112	-.091
23	.966	.217	-.958	-.149	-.092	-.072	-.063	-.057	-.054	-.041
25	.966	-.177	-.958	.200	.102	.075	.063	.056	.052	.038
27	.707	-.333	-.654	.646	.236	.158	.127	.111	.101	.071
29	.259	-.177	-.205	.957	.145	.086	.066	.056	.050	.033
31	-.259	.217	.158	.957	-.197	-.097	- 070	-.057	-.050	-.031
33	-.709	.667	.270	.646	-.644	-.229	-.150	-.118	-.101	-.058
35	-.966	.960	.126	.200	-.957	-.143	-.083	-.062	-.052	-.027
37	-.966	.960	-.126	-.149	-.957	.195	.095	.066	.654	.026
39	-.707	.667	-.270	-.247	-.644	.642	.225	.145	.112	.049
41	-.259	.217	-.158	-.110	-.197	.957	.141	.081	-.060	.023
43	.259	-.177	.205	.102	.145	.957	-.194	-.093	-.064	-.022
45	.707	-.333	.654	.200	.236	.642	-.641	-.222	-.141	-.042
47	.966	-.177	.958	.102	.102	.195	-.956	-.140	-.079	-.020
49	.966	.217	.958	-.110	-.092	-.143	-.956	.194	.092	.019
51	.707	.667	.654	-.247	-.172	-.229	-.641	.640	.220	.038
53	.259	.960	.205	-.149	-.084	-.097	-.194	.955	.140	.018
55	-.259	.960	-.158	.200	.084	.086	.141	.955	-.193	-.017
57	-.707	.667	-.270	.646	.172	.158	.225	.640	-.639	-.033
59	-.966	.217	-.126	.957	.092	.075	.095	.194	-.955	-.016
61	-.966	-.177	.126	.957	-.102	-.072	-.083	-.140	-.955	.016
63	-.707	-.333	.270	.646	-.236	-.143	-.150	-.222	-.639	.030
65	-.259	-.177	.158	.200	-.145	-.072	-.070	-.093	-.193	.015

ROUND COPPER WIRE TABLE 3

SIZE AWG	BARE DIAMETER	AREA □"	Ω/1000' @25°C	SINGLE FORMVAR	HEAVY FORMVAR	SINGLE GLASS FORMVAR	BARE WT. #/1000'	SINGLE GLASS SILICONE	DOUBLE GLASS SILICONE
36	.0050	.0000196	424	.0056	.0060		.0757		
35	.0056	.0000246	338	.0062	.0066		.0949		
34	.0063	.0000312	266	.0070	.0074		.1201		
33	.0071	.0000396	210	.0079	.0084		.1526		
32	.0080	.0000503	165	.0088	.0094	.0121	.1937		
31	.0089	.0000622	134	.0097	.0104	.0130	.2398		
30	.0100	.0000785	106	.0108	.0116	.0142	.3025	.0132	.0152
29	.0113	.000100	83.1	.0122	.0130	.0156	.3866	.0145	.0165
28	.0126	.000125	66.4	.0135	.0144	.0169	.4806	.0158	.0178
27	.0142	.000158	52.6	.0152	.0161	.0186	.6101	.0174	.0194
26	.0159	.000199	41.7	.0169	.0179	.0203	.7650	.0191	.0211
25	.0179	.000252	33.0	.0190	.0200	.0224	.970	.0211	.0231
24	.0201	.000317	26.2	.0213	.0223	.0263	1.223	.0251	.0276
23	.0226	.000401	20.7	.0238	.0249	.0289	1.546	.0276	.0301
22	.0254	.000507	16.4	.0266	.0277	.0317	1.937	.0303	.0328
21	.0285	.000638	13.0	.0299	.0310	.0349	2.459	.0335	.0360
20	.0320	.000804	10.3	.0334	.0346	.0384	3.099	.0370	.0395
19	.0360	.00102	8.14	.0374	.0386	.0424	3.900	.0409	.0434
18	.0403	.00126	6.59	.0418	.0431	.0468	4.914	.0453	.0478
17	.0453	.00159	5.22	.0469	.0482	.0519	6.213	.0503	.0528
16	.0508	.00204	4.07	.0524	.0538	.0575	7.812	.0558	.0583
15	.0571	.00255	3.26	.0588	.0602	.0639	9.87	.0621	.0646
14	.0641	.00322	2.58	.0659	.0673	.0710	12.44	.0691	.0716
13	.072	.00407	2.04	.0738	.0753	.0789	15.69	.0770	.0795
12	.0808	.00515	1.61	.0827	.0842	.0877	19.76	.0858	.0883
11	.0907	.00650	1.28	.0927	.0942	.0977	24.90	.0957	.0982
10	.102	.00817	1.02	.1039	.1055	.1089	31.43	.1069	.1094
9	.114	.0102	.814	.1165	.1181	.1225	39.62	.1204	.1254
8	.129	.0131	.634	.1306	.1323	.1366	49.98	.1345	.1395
7	.144	.0163	.510	.1465	.1482	.1525	63.03	.1503	.1553
6	.162	.0206	.403	.1643	.1661	.1703	79.44	.1680	.1730
5	.182	.0260	.319	.1842	.1861	.1902	100.2	.1879	.1929
4	.204	.0327	.254				126.3	.2103	.2153
3	.229	.0412	.202				159.3		
2	.258	.0523	.159				200.9		
0	.325	.0830	.100						
2/0	.365	.105	.0791						
4/0	.460	.166	.0500						

CURVE 1





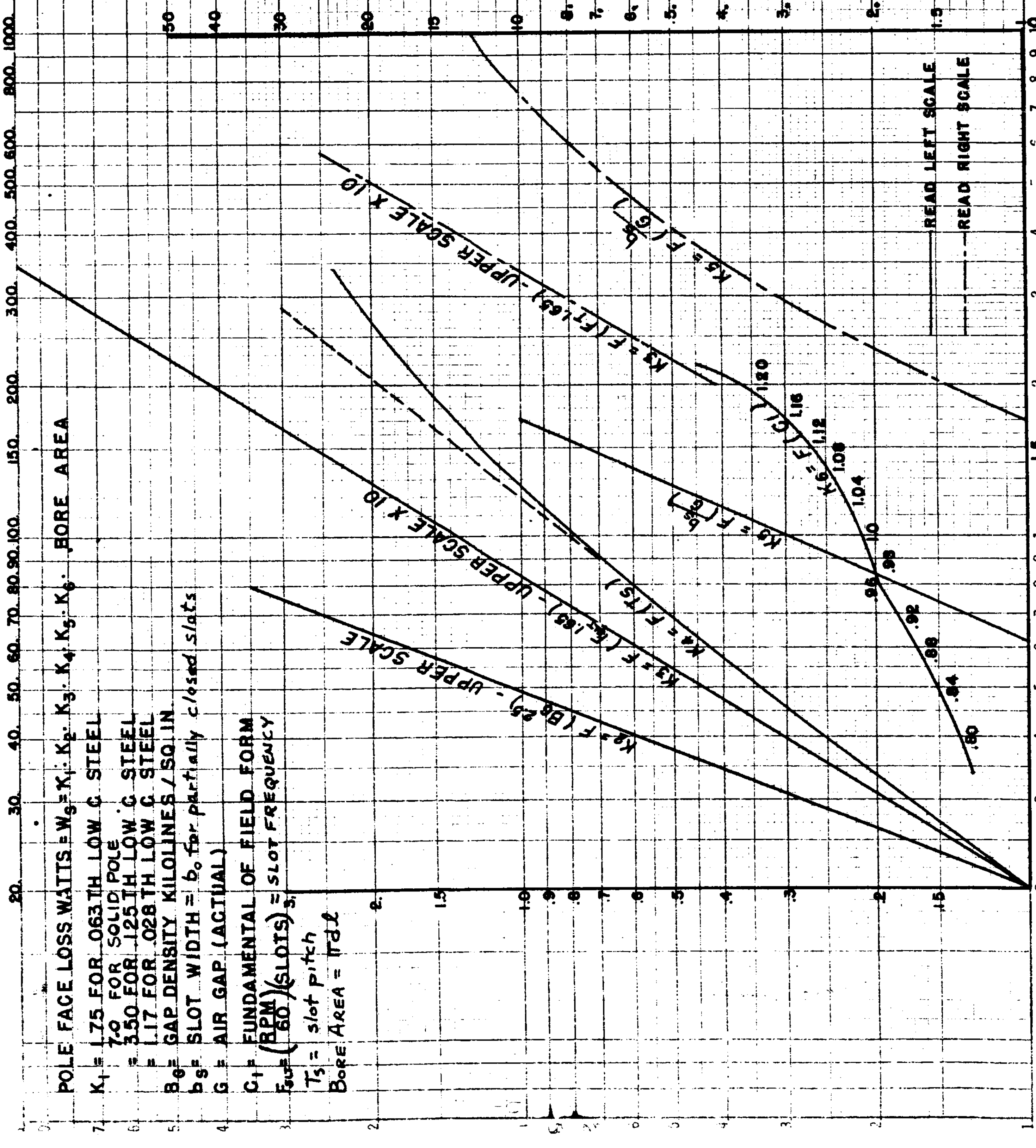
Empirical End-Winding Constant
Versus Pole Pitch
From E. C. Barnes
"An Experimental Study of Machine
End Turn Reactance"
AIEE Trans. Vol. 70 1951
pp 671-679.

CURVE 2

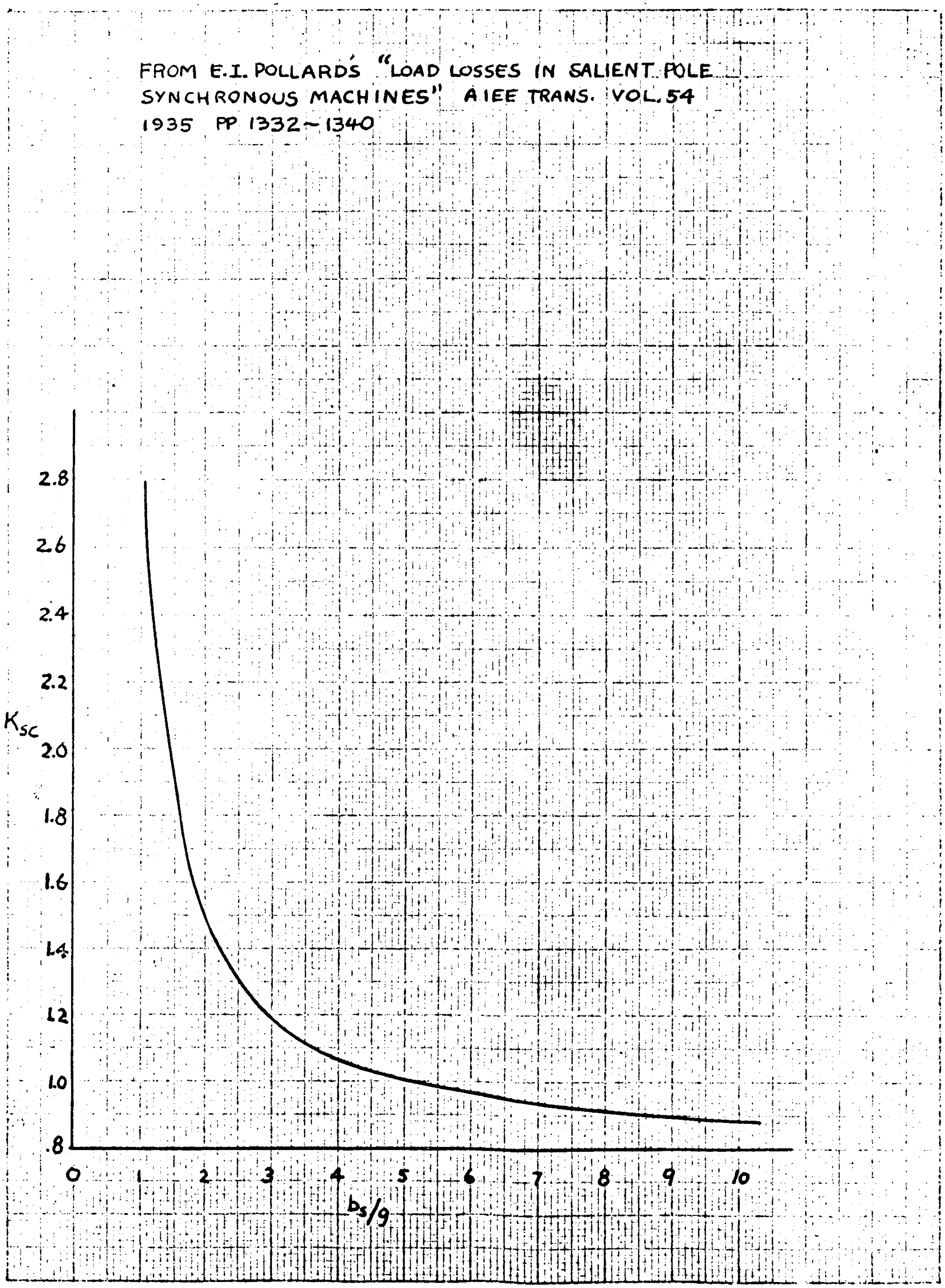
From Kennard and Spooner "Surface Iron Losses with Respect to Laminated Materials", Trans. AIEE, Vol. 43, 1924, pp 262-281.

DRAWN BY J.A.T.

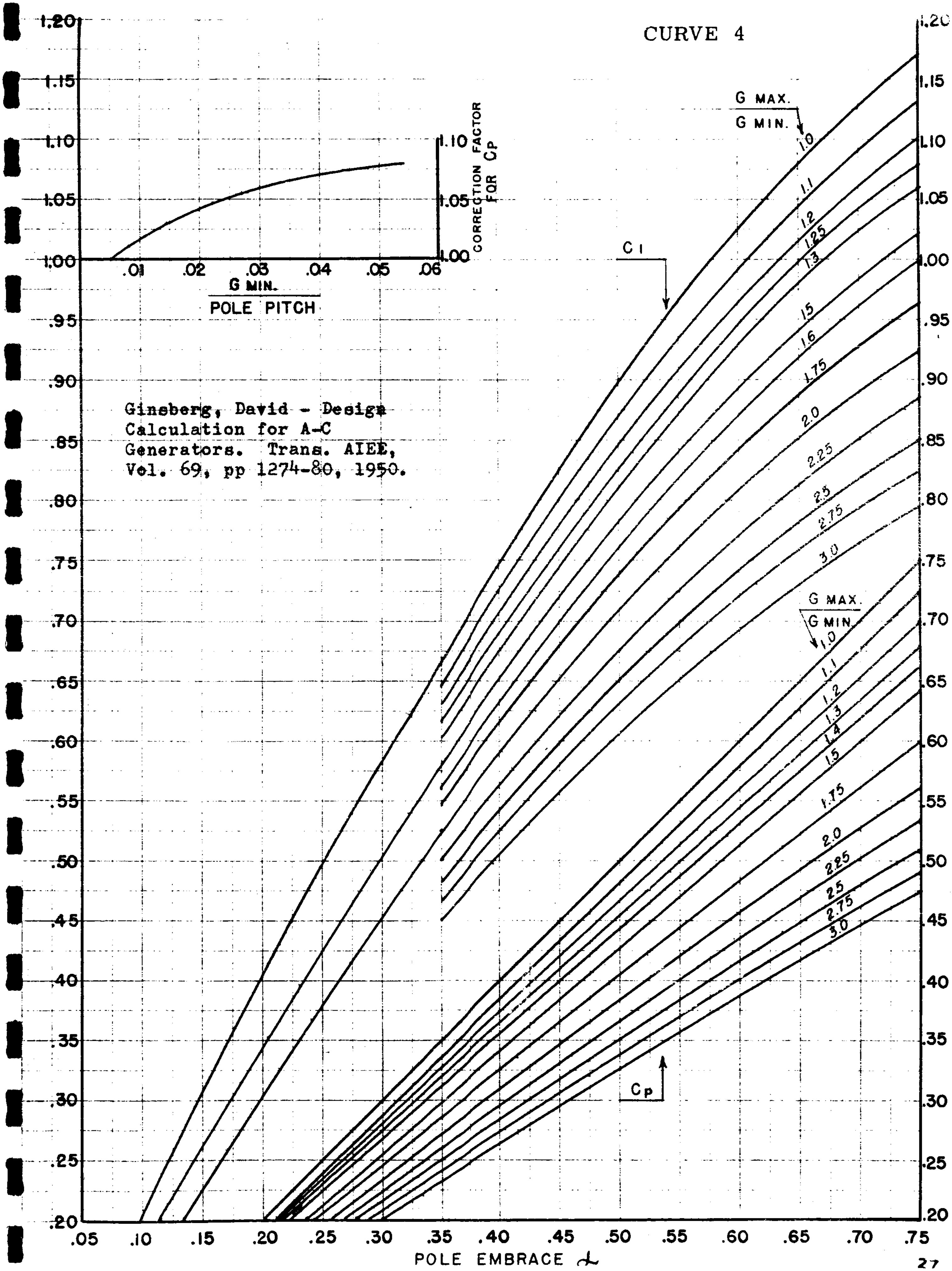
REFER TO ITEM (150) IN SALIENT POLE DESIGN MANUAL FOR SAMPLE USE OF THIS CURVE



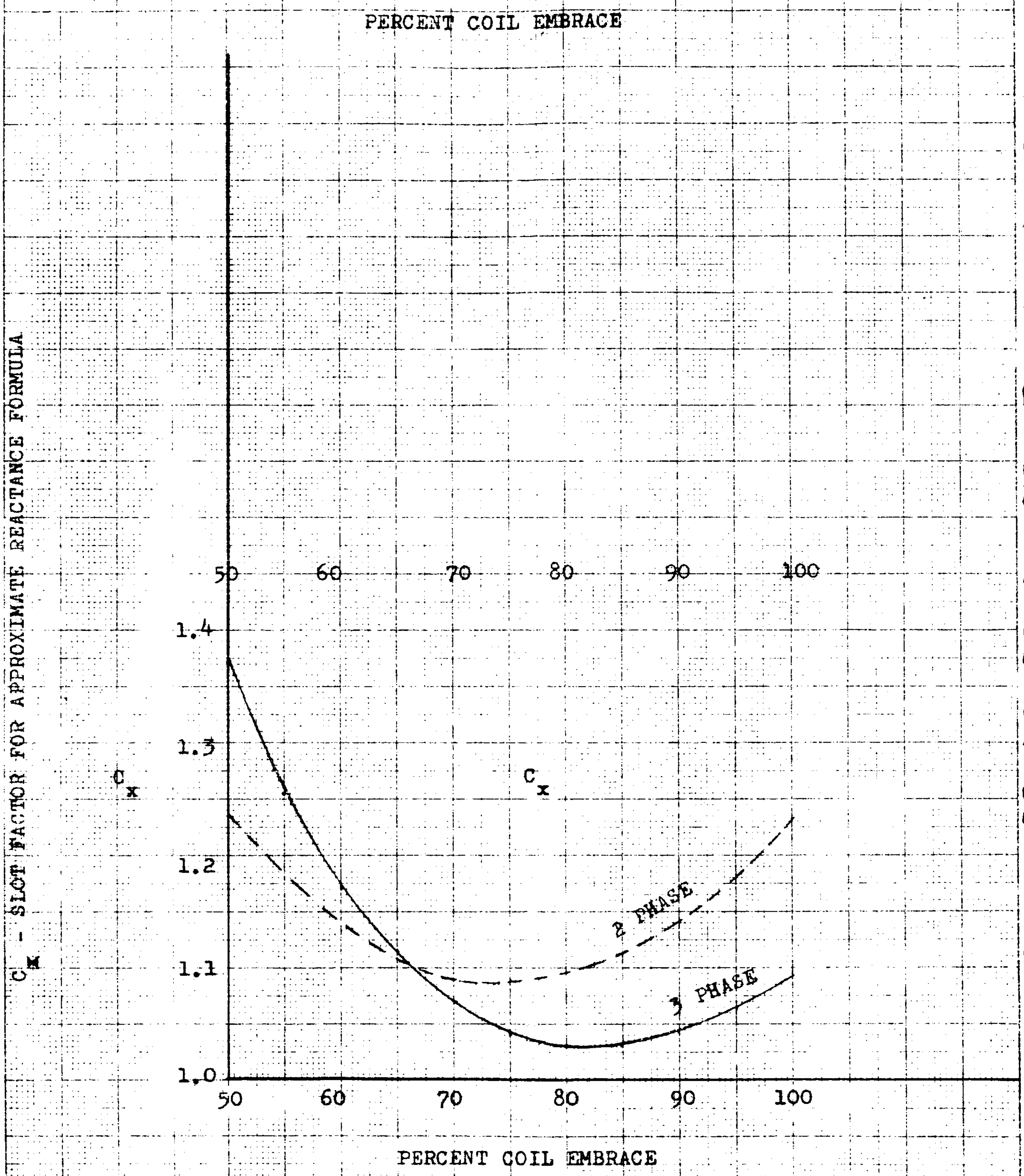
CURVE-3



FROM E.I. POLLARD'S "LOAD LOSSES IN SALIENT POLE SYNCHRONOUS MACHINES" AIEE TRANS. VOL. 54 1935 PP 1332~1340
K_{sc}
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
0
1
2
3
4
5
6
7
8
9
10
b_s/g


CURVE 4
Ginsberg, David - Design Calculation for A-C Generators. Trans. AIEE, Vol. 69, pp 1274-80, 1950.
CORRECTION FACTOR FOR Cp
G MIN. / POLE PITCH
G MAX. / G MIN.
C1
Cp
POLE EMBRACE

CURVE 5



10 X 10 TO THE CM 359 14

$$D.L. = \frac{1.246\, p\, n_b\, l_b\, \rho}{10^6\, a_b}\left[\tau_s B_g K_p K_g\right]^2 \left[K_{f1}\left(\frac{K_{w1}}{2\lambda_s + \frac{\lambda_g}{K_{\phi 1}}}\right)^2 + K_{f2}\left(\frac{K_{w2}}{2\lambda_s + \frac{\lambda_g}{K_{\phi 2}}}\right)^2\right]$$

= LOSS IN KW

$\lambda_s = \frac{h_r}{b_r} + \lambda_t + \lambda_c$

a_b = BAR AREA IN SQ IN.

n_b = BARS/POLE

$\lambda_g = \frac{\tau_b}{K_g g} = \frac{\tau_b}{g'}$

p = NO. POLES

l_b = LENGTH BAR IN.

K_g = CARTER'S COEFFICIENT (TOTAL)

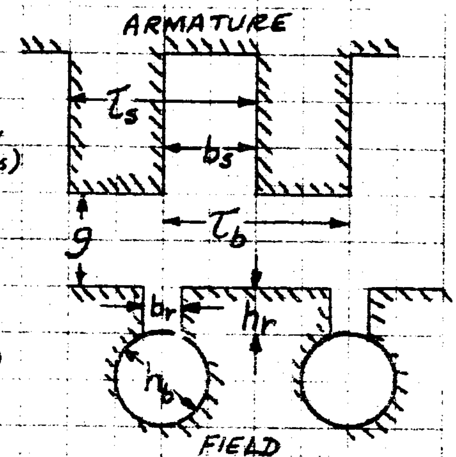
$K_p = fn(b_s/g)$, CURVE (a) (b_s = b_o for partially closed slots)

K_{f1} AND $K_{f2} = fn(f_s/\rho)$ CURVE (b)

ρ = DAMPER BAR RESISTIVITY (MICROHMS PER CU.IN.)

K_{W1} AND $K_{W2} = fn(b_s/\tau_s)$, CURVE (C_1) AND (C_2)

$K_{\phi 1}$ AND $K_{\phi 2} = fn(\tau_b/\tau_s)$, CURVE ($d_1$) AND ($d_2$)

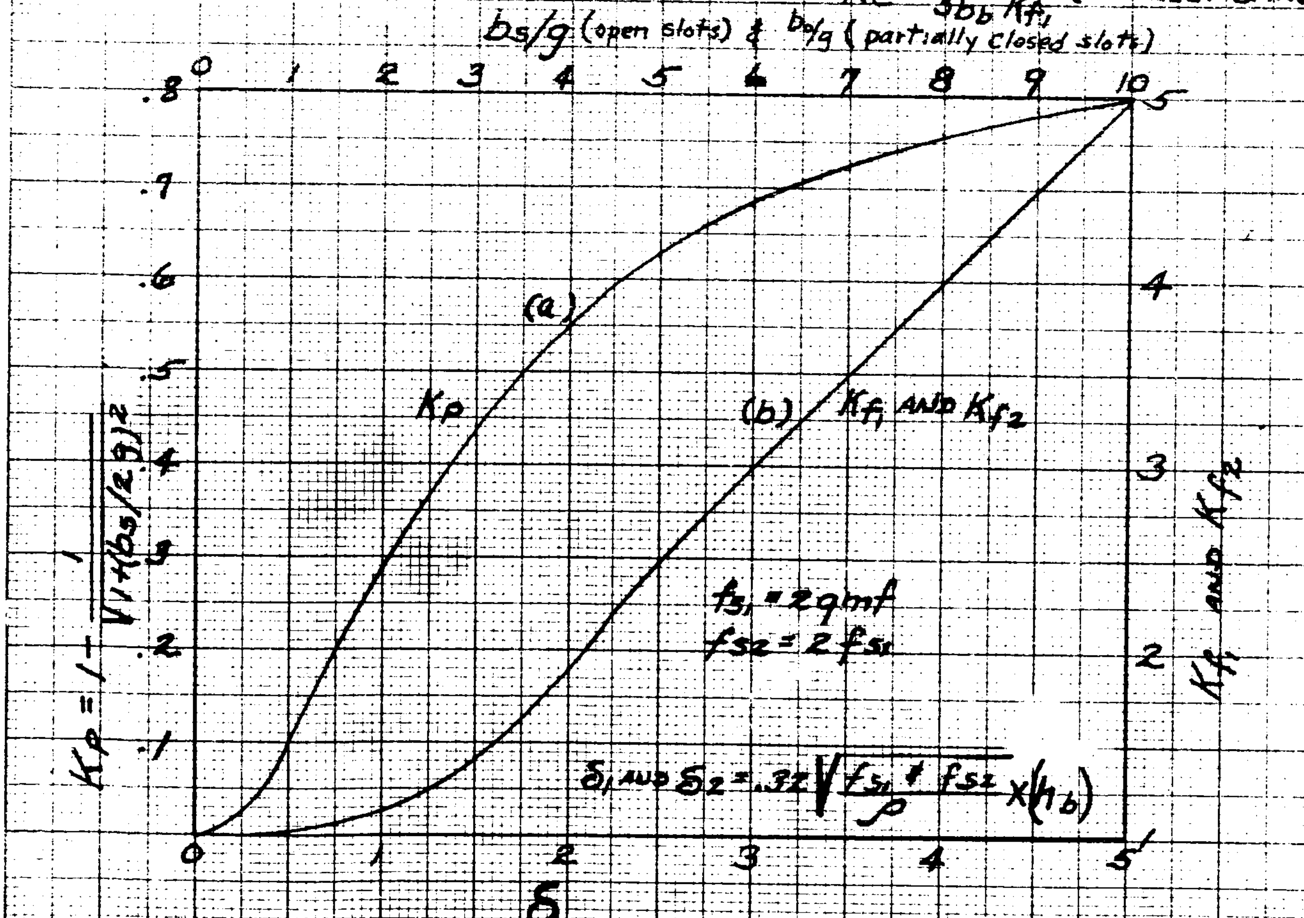


$\lambda_t = fn(b_r/gK_g)$ CURVE (e)

B_g IS IN KILOLINES PER SQ INCH

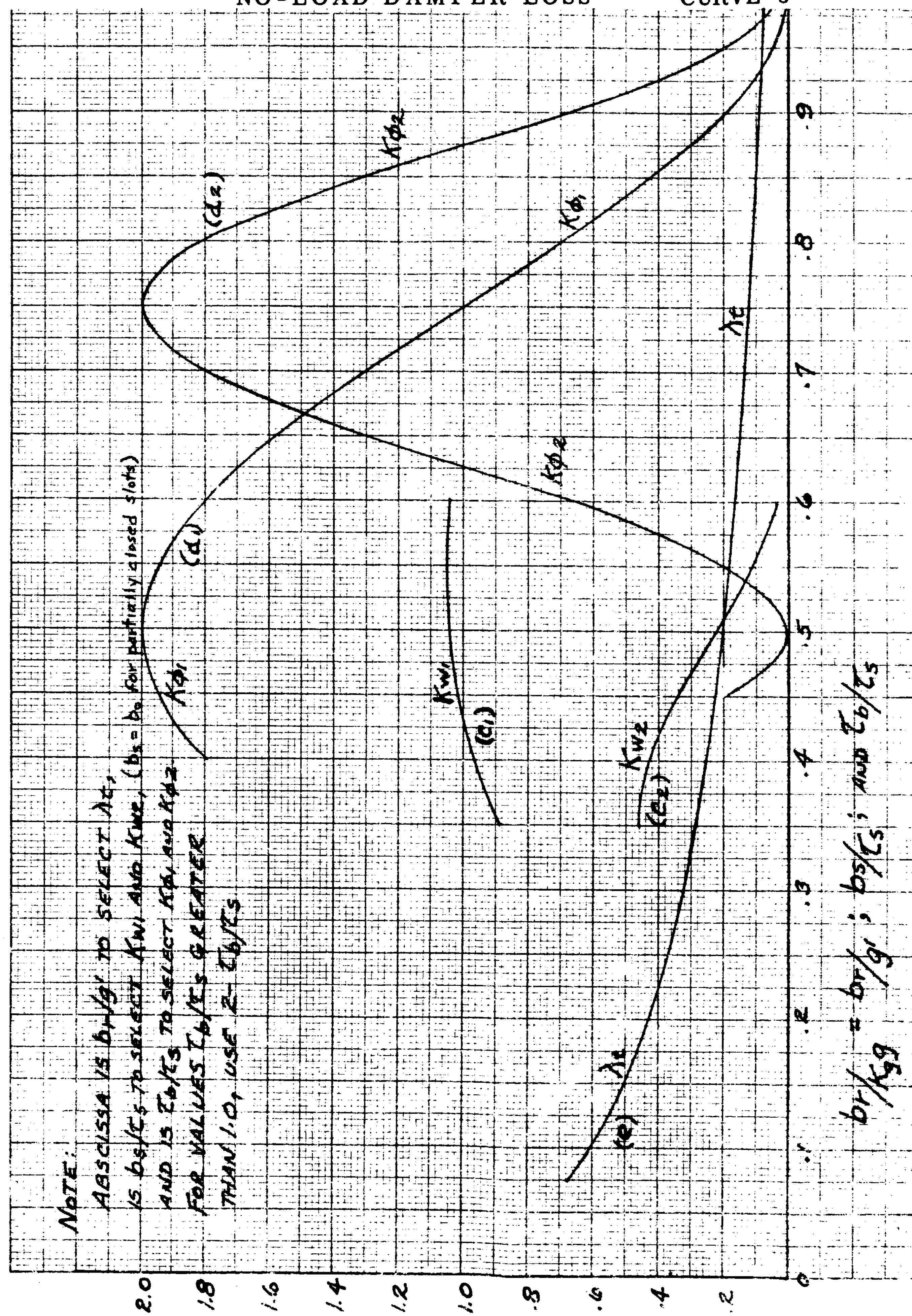
$\lambda_c = \frac{.75}{K_{f1}}$ (FOR ROUND OR SQ. BARS)

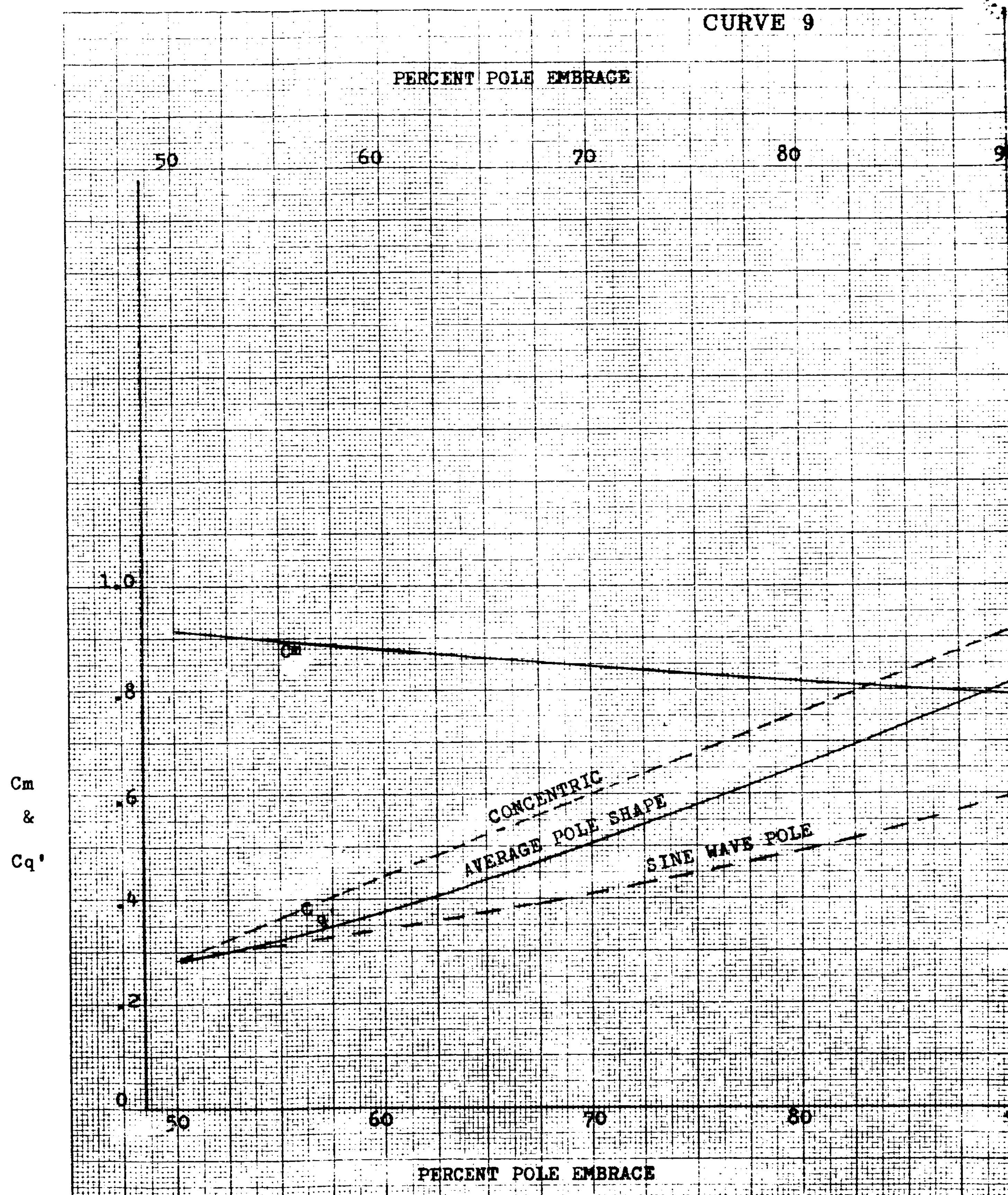
$\lambda_c = \frac{h_b}{3 b_b K_{f1}}$ (FOR RECT. BARS)



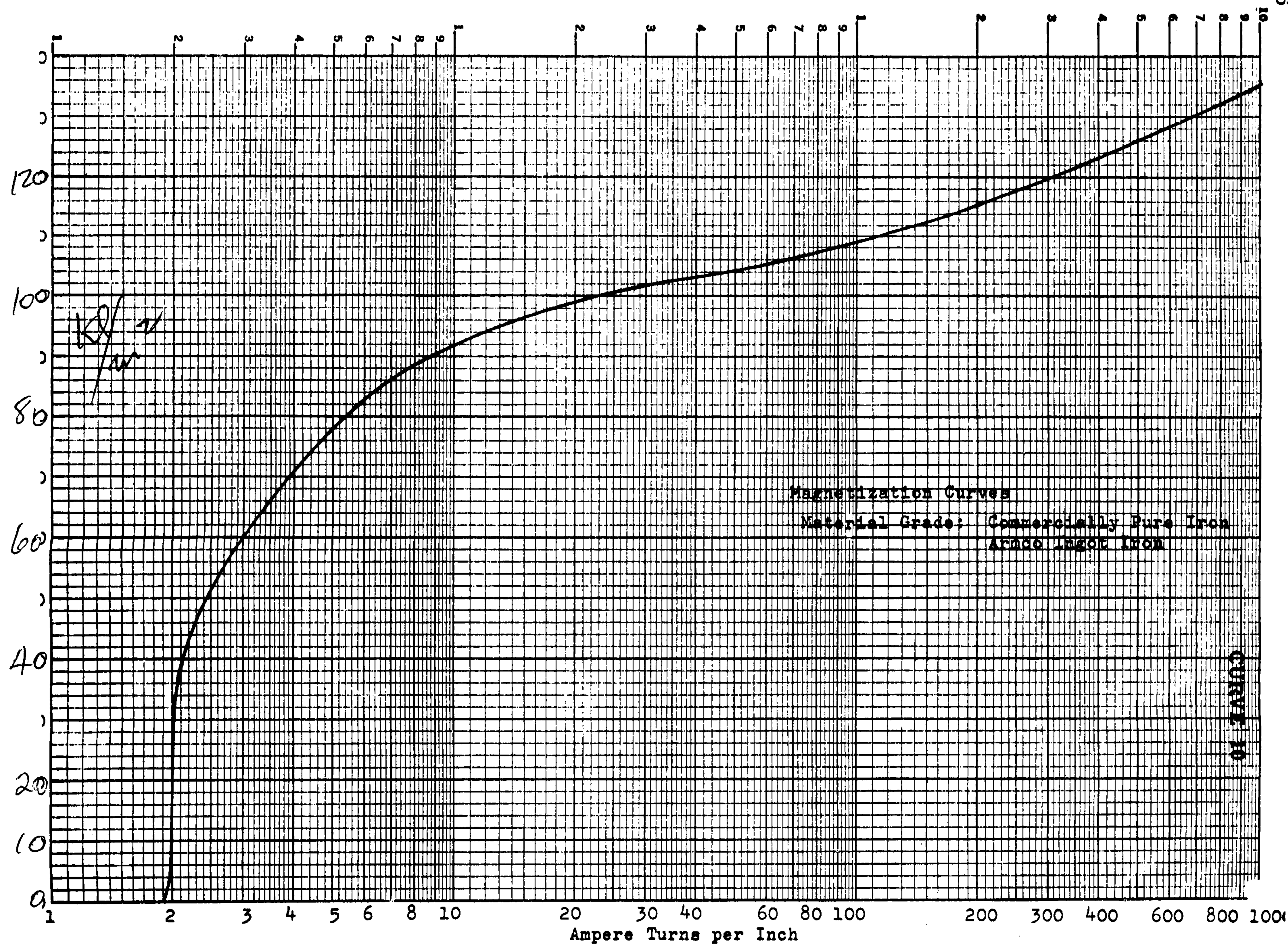
From E. I. Pollard "Calculation of No-Load Damper Winding Loss in Synchronous Machines", AIEE Vol. 51, 1932, pp 477-81.

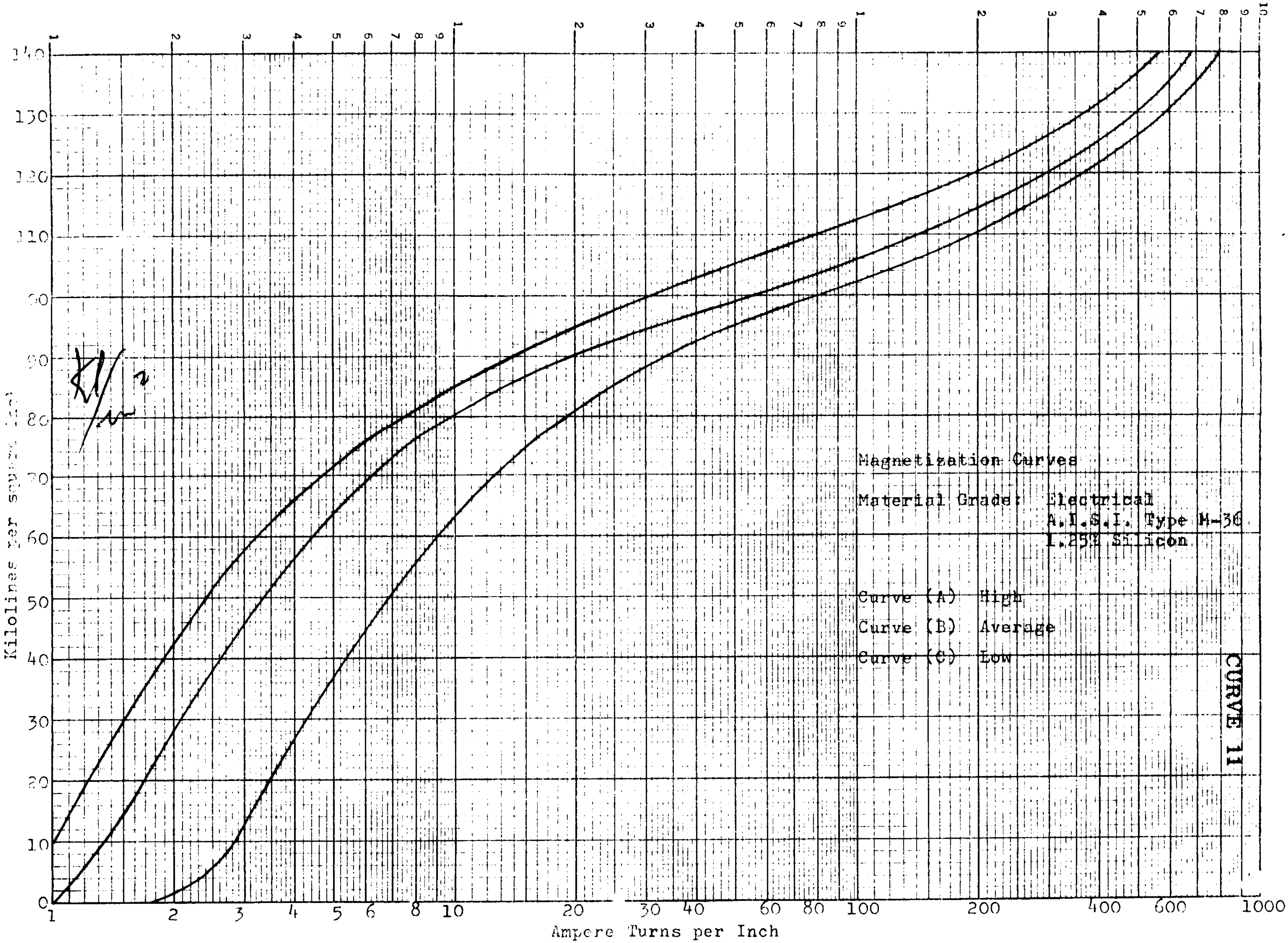
NO-LOAD DAMPER LOSS CURVE 8

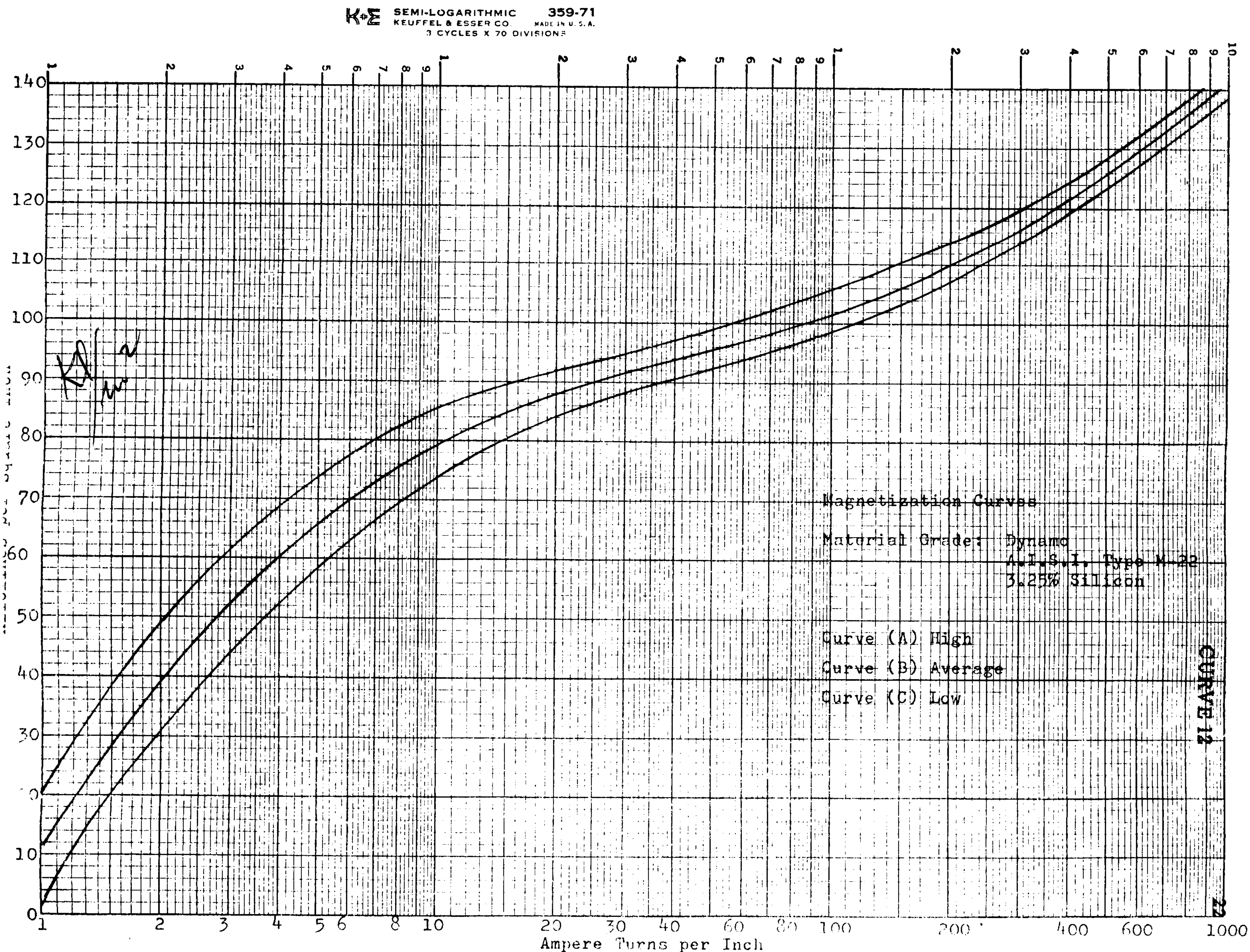


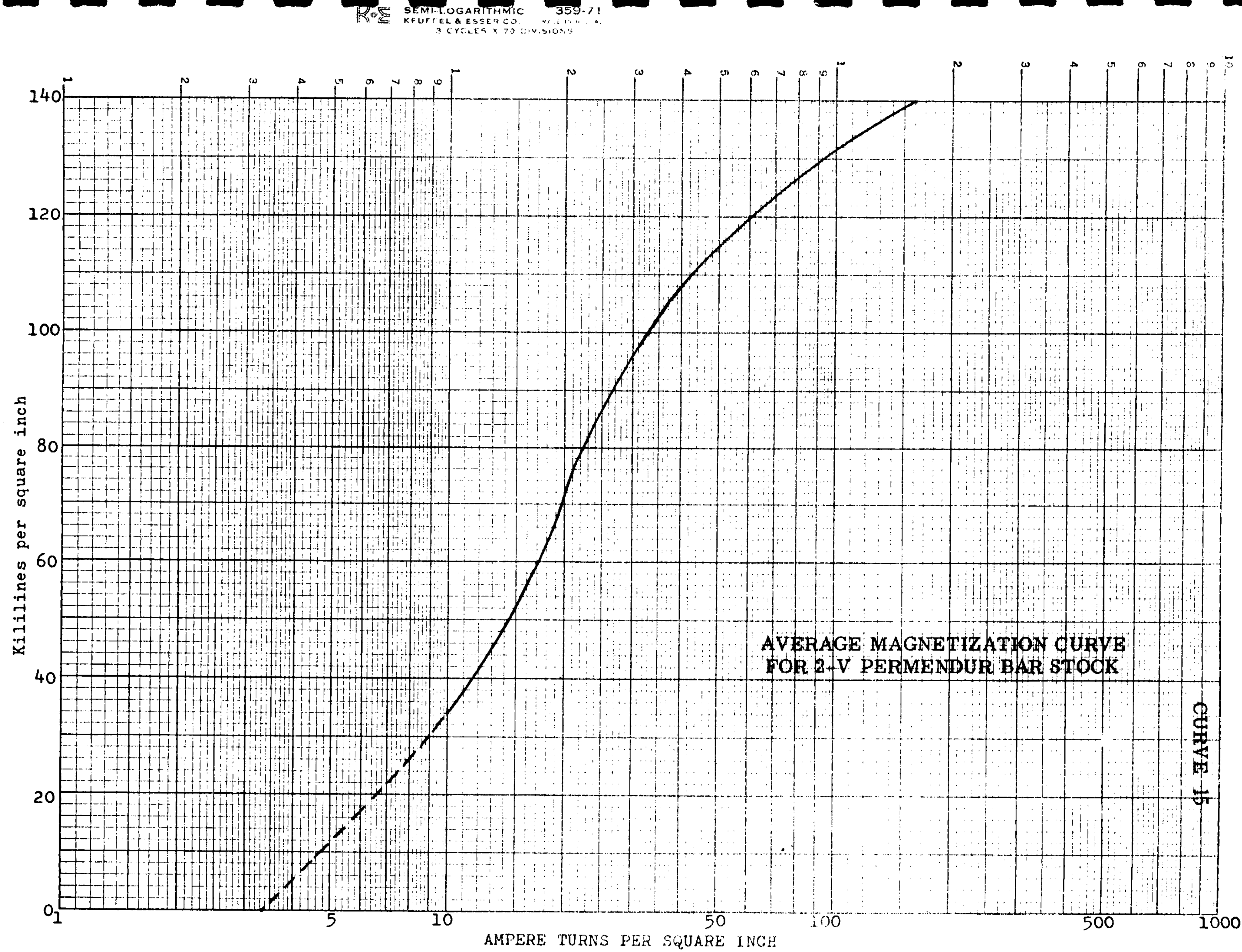


SALIENT POLE MACHINE CONSTANTS — C_m $C_{q'}$

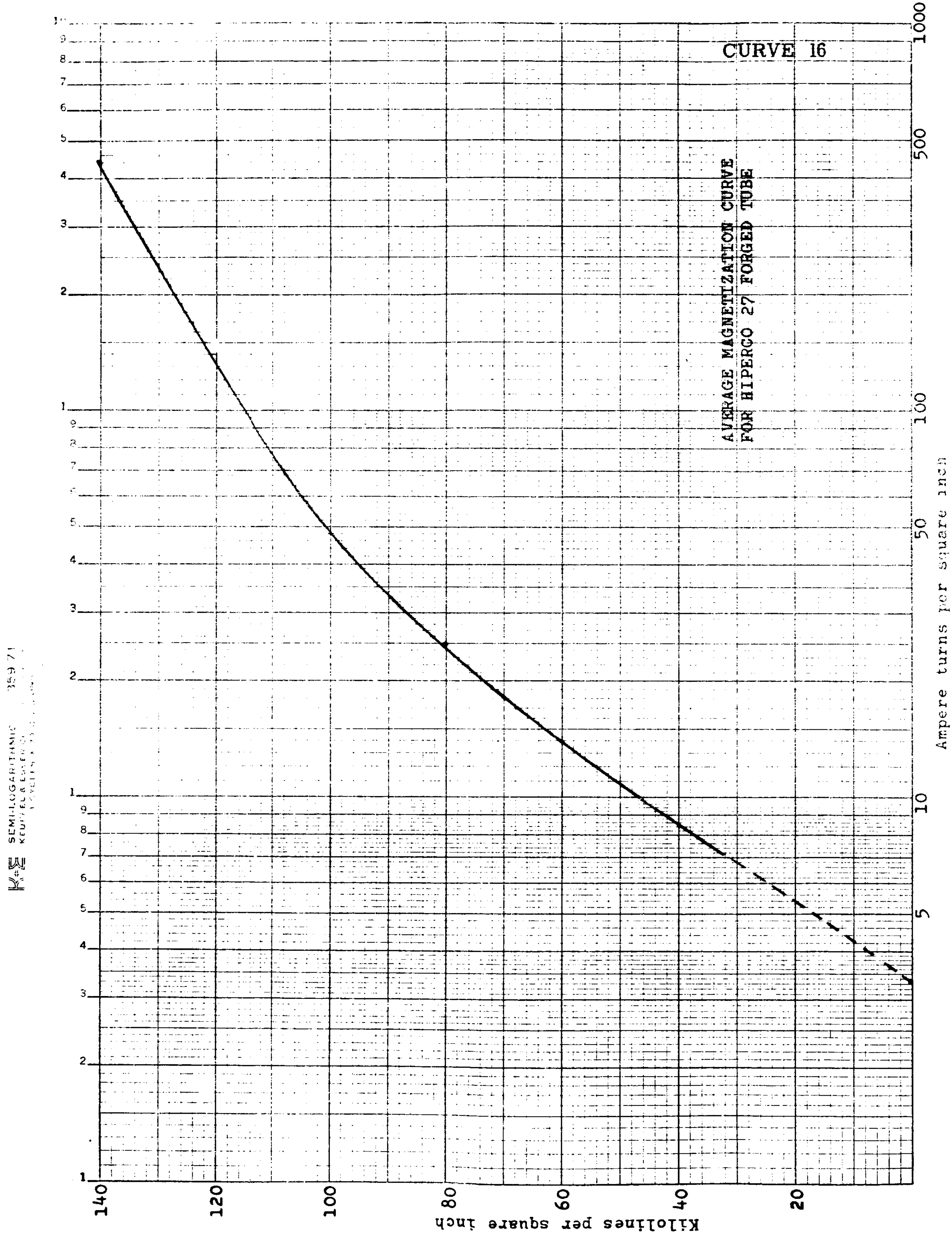

CURVE 10
Magnetization Curves
Material Grade: Commercially Pure Iron
Armco Ingot Iron
Ampere Turns per Inch
1 2 3 4 5 6 8 10 20 30 40 60 80 100 200 300 400 600 800 1000
0 10 20 40 60 80 100 120


CURVE 11
Magnetization Curves
Material Grade: Electrical
A.I.S.I. Type M-36
1.25% Silicon
Curve (A) High
Curve (B) Average
Curve (C) Low
Kilolines per square inch
Ampere Turns per Inch
Kl/in²

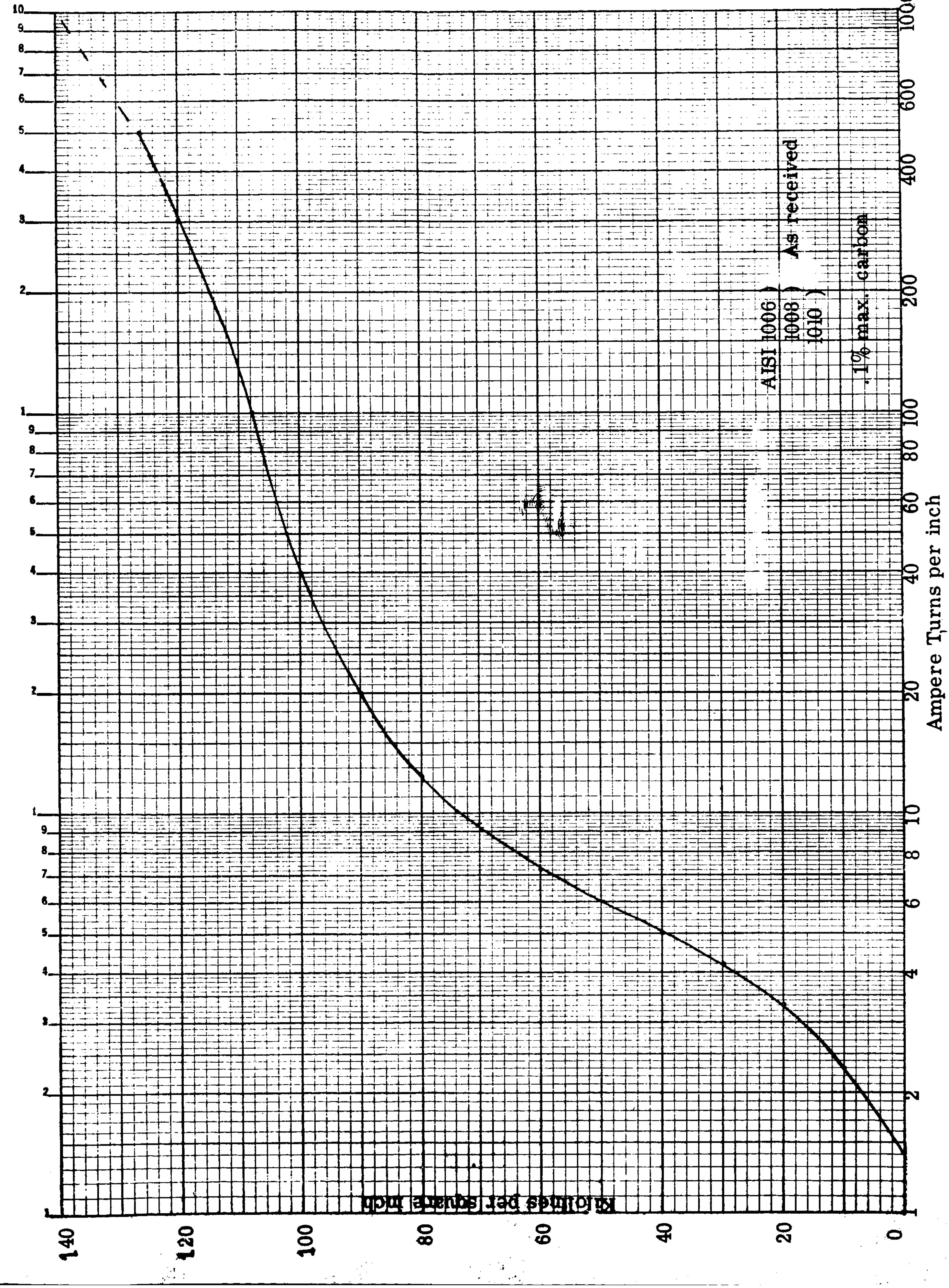

K+E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS
CURVE 12
Magnetization Curves
Material Grade: Dynamo
A.I.S.I. Type M-22
3.25% Silicon
Curve (A) High
Curve (B) Average
Curve (C) Low
Ampere Turns per Inch
22



SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO.
3 CYCLES X 70 DIVISIONS
CURVE 15
AVERAGE MAGNETIZATION CURVE
FOR 2-V PERMENDUR BAR STOCK
Kililines per square inch
AMPERE TURNS PER SQUARE INCH


CURVE 16
AVERAGE MAGNETIZATION CURVE
FOR HIPERCO 27 FORGED TUBE
Ampere turns per square inch
Kilolines per square inch
140
120
100
80
60
40
20
1000
500
100
50
10
5
SEMI-LOGARITHMIC
KEUFFEL & ESSER CO.
359 71

CURVE 17


Kilolines per square inch
Ampere Turns per inch
AISI 1006
1008
1010
As received
.1% max. carbon
SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO.
3 CYCLES X 70 DIVISIONS
0
20
40
60
80
100
120
140
1
2
4
6
8
10
20
40
60
80
100
200
400
600
1000

DISC-TYPE SYNCHRONOUS GENERATOR

STATOR

STATOR I.D. ______
STATOR O.D. ______
CORE LENGTH ______
DBS x 2 ______
SLOTS ______
CARTER COEFF. ______

TYPE WDGE ______
THROW ______
SKEW & DIST FACT. ______
CHORD FACTOR ______
COND. PER SLOT ______
TOTAL EFF. COND. ______
COND. SIZE ______
COND. AREA ______
CURRENT DENSITY ______

WDG. CONST. ______ C_1 ______
TOTAL FLUX ______
GAP AREA ______
GAP DENSITY ______
POLE CONST. ______
FLUX PER POLE ______
SHAFT FLUX ______

TOOTH PITCH ______
TOOTH DENSITY ______
CORE DENSITY ______
GRADE IRON ______

$\frac{1}{2}$ MEAN TURN ______
RES/PHASE @ ______ ° ______
EDDY FACT TOP ______
E.F. AVE ______ EFF. BOT. ______

DEMAG. FACT. C_m ______ C_q ______
AMP COND. PER IN ______
REACT. FACTOR ______
COND. PERM. ______
END PERM. ______
LEAKAGE REACT. ______
AIR GAP PERM. ______
REACT. OF ARM. X_{ad} ______ X_{aq} ______

WT. OF COPPER ______
WT. OF IRON ______

ROTOR

SINGLE GAP ______ g_e ______
ROTOR O.D. ______ I.D. ______
PERIPHERAL SPEED ______
POLE PITCH ______ α ______
POLE AREA-OUTER ______
POLE AREA-INNER ______
ROTOR LEAKAGE ______
POLE DENSITY ______
ROTOR IRON ______

DAMPER BARS Nº ______
BAR SIZE ______
BAR PITCH ______ h_0 ______ b_0 ______

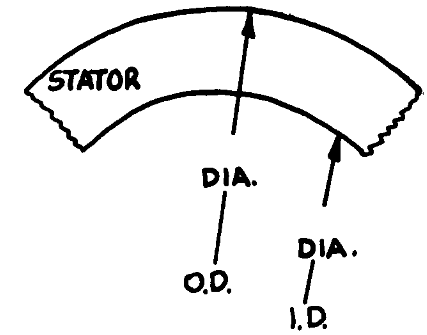
FIELD COIL TURNS ______
COND. SIZE ______
COND AREA ______
MEAN TURN ______
RES@ ______ ° ______

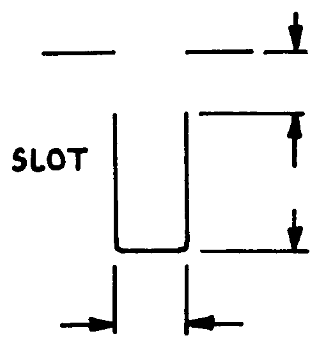
% LOAD				
P.F.				
AMPS				
VOLTS				
I^2R				
AMPS/IN.^{2}				

FIELD SELF IND. ______
DAMP. LEAK XD_d ______ XD_q ______

REACTION-TIME CONSTANT

SYNCH. X_d ______ X_q ______
UNSAT. TRANS. ______
SAT. TRANS. ______
SUBTRANS. X_c'' ______ X_q'' ______
NEG. SEQUENCE ______
ZERO SEQUENCE ______
OPEN CIR. TIME CON. ______
ARM. TIME CON. ______
TRANS. TIME CON. ______
SUBTRANS. TIME CON. ______





SATURATION

AIR GAP AT ______
STATOR AT ______
POLE AT ______
NO LOAD AT ______
RATED LOAD AT ______
OVERLOAD AT ______
SHORT CIRCUIT AT ______

LOSSES-EFFICIENCY

% LOAD ______

F & W ______
STA. TEETH ______
STA. CORE ______
POLE FACE ______
DAMPER ______
STA. I^2R ______
EDDY ______
FIELD I^2R ______

Σ LOSSES ______
RATING ______

RATING & LOSSES ______
% LOSSES ______
% EFF. ______

W.O. ______ FOR ______ COOLING ______
______ KVA ______ % P.F. ______ VOLTS ______ AMPS ______ PHASE
______ CYCLES/SEC. ______ POLES ______ RPM BY ______

AXIAL AIR-GAP, LUNDELL TYPE A. C. GENERATOR, DESIGN MANUAL

(1)	--	DESIGN NUMBER - To be used for filing purposes.
(2)	KVA	GENERATOR KVA
(3)	E	LINE VOLTS
(4)	E_{PH}	PHASE VOLTS - For 3 phase, Y [redacted] connected generator $E_{PH} = \frac{\text{(Line Volts)}}{\sqrt{3}} = \frac{(3)}{\sqrt{3}}$ For 3 phase, Δ [redacted] connected generator $E_{PH} = \text{(Line Volts)} = (3)$
(5)	m	PHASES - number of
(5a)	f	FREQUENCY - In cycles per second
(6)	P	POLES - Number of
(7)	RPM	SPEED - In revolutions per minute
(8)	I_{PH}	PHASE CURRENT - In amperes at rated load
(9)	PF	POWER FACTOR - Given in per unit
(9a)	K_c	ADJUSTMENT FACTOR - When PF = 0. to .95 set $K_c = 1$; when PF = .95 to 1. set $K_c = 1.05$

(10a)	d	<u>STATOR EQUIVALENT DIAMETER</u> $d = \frac{(O.D.) + (I.D.)}{2} = \frac{(12) + (11)}{2}$
(11)	I.D.	<u>STATOR I.D.</u> - The inside diameter of the stator toroid in inches.
(12)	O.D.	<u>STATOR O.D.</u> - The outside diameter of the stator toroid in inches
(13)	ℓ	<u>GROSS CORE LENGTH</u> - In inches $\ell = \frac{(O.D.)-(I.D.)}{2} = \frac{(12)-(11)}{2}$
(16)	K_i	STACKING FACTOR - This factor allows for the coating (core plating) on the punchings, and the looseness of the ribbon. Approximate values are giver in Table IV.

THICKNESS OF LAMINATIONS (INCHES)	GAGE	K_i
.C14	29	0.92
.018	26	0.93
.025	24	0.95
.028	23	0.97
.063	--	0.98
.125	--	0.99

TABLE IV

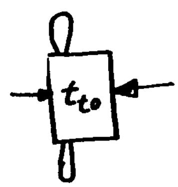
(17)	ℓ_s	SOLID CORE LENGTH - The solid length is the gross length times the stacking factor. $$\ell_s = (K_i) \times (\ell) = (16) \times (13)$$
(18)	--	MAGNETIZATION CURVES are to be available for stator, pole and yoke.
(19)	k	WATTS/LB - Core loss per lb of lamination material. Must be given at the density specified in (20).
(20)	B	DENSITY - This value must correspond to the density used in Item (19) to pick the watts/lb. The density that is usually used is 77.4 kilolines/in^2.
(21)		TYPE OF STATOR SLOT - Refer to Figure 1 for type of slot.
(22)	b_0, b_1, b_2, b_3, b_s, h_0, h_1, h_2, h_3, h_s, h_t, h_w	ALL SLOT DIMENSIONS - Given in inches per Figure 1. Note: For Type (c) slot $$b_s = \frac{(b_1) + (b_3)}{2} = \frac{(22) + (22)}{2}$$

(23) | Q | STATOR SLOTS - number of

(24) | h_c | DEPTH BELOW SLOTS - The depth of the stator core below the slots.

$$h_c = t_{to} - h_s = (24) - (22)$$

Where t_{to} is the thickness of the stator core.



(25) | q | SLOTS PER POLE PER PHASE

$$q = \frac{(Q)}{(P)\ (m)} = \frac{(23)}{(6)\ (5)}$$

(26) | τ_s | STATOR SLOT PITCH (average)

$$\tau_s = \frac{\pi (d)}{Q} = \frac{\pi (10a)}{(23)}$$

(27) | $\tau_{s\,1/3}$ | STATOR SLOT PITCH - 1/3 distance up from narrowest section of tooth.

$$\tau_{s\,1/3} = \tau_s = (26)$$

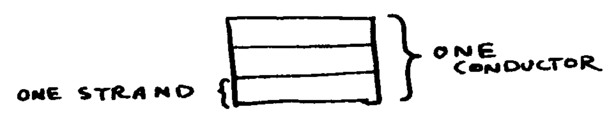
(28) | -- | TYPE OF WINDING - Record whether the connection is "wye" of "delta".

(29) | -- | TYPE OF COIL - Record whether random wound or formed coils are used.

(30)	n_s	CONDUCTORS PER SLOT - The actual number of conductors per slot. For random wound coils use a space factor of 75% to 80%. Where space factor is the percent of the total slot area that is available for insulated conductors after **all** other insulation areas have been subtracted out.
(31)	γ	THROW - Number of slots spanned. For example, with a coil side in slot 1 and the other coil side in slot 10, the throw is 9.
(31a)		PERCENT OF POLE PITCH SPANNED - Ratio of the number of slots spanned to the number of slots in a pole pitch $= \frac{(\gamma)}{(m)(q)} = \frac{(31)}{(5)(25)}$
(32)	C	PARALLEL PATHS, no. of - Number of parallel circuits per phase
(33)	--	STRAND DIA OR WIDTH - In inches. For round wire, use strand diameter. For rectangular wire, use strand width.

(34) N_{ST} NUMBER OF STRANDS PER CONDUCTOR IN DEPTH -

Applies to rectangular wire. In order to have a more flexible conductor and reduce eddy current loss a stranded conductor is often used. For example, when the space available for one conductor is .250 width x .250 depth, the actual conductor can be made up of 2 or 3 strands in depth as shown.



For a more detailed explanation refer to section titled "Effective Resistance and Eddy Factor" in the Derivations in Appendix.

(35) d_b DIAMETER OF BENDER PIN in inches - This pin is used in forming coils

(36) ℓ_{e2} COIL EXTENSION BEYOND CORE in inches - Straight portion of coil that extends beyond stator core.

(37) h_{ST} HEIGHT OF UNINSULATED STRAND in inches

(38) h'_{st} DISTANCE BETWEEN CENTERLINES OF STRANDS IN DEPTH in inches.

(39) -- STATOR COIL STRAND THICKNESS in inches - For rectangular conductors only. For round wire use 0.

(40) τ_{SK} SKEW - Stator slot skew in inches at main air gap. To be measured at the stator O.D. as the deviation from a radial line at that point.

(41) τ_P POLE PITCH in inches (average)

$$\tau_P = \frac{\pi(d)}{(P)} = \frac{\pi(10a)}{(6)}$$

(42) K_{SK} SKEW FACTOR - The skew factor is the ratio of the voltage induced in the coils to the voltage that would be induced if there were no skew

$$K_{SK} = \frac{\sin\left[\dfrac{\pi(\tau_{SK})}{2(\tau_P)}\right]}{\dfrac{\pi(\tau_{SK})}{2(\tau_P)}} = \frac{\sin\left[\dfrac{\pi(40)}{2(41)}\right]}{\dfrac{\pi(40)}{2(41)}}$$

(43) K_d DISTRIBUTION FACTOR - The distribution factor is the ratio of the voltage induced in the coils to the voltage that would be induced in the coils if the winding were concentrated in a single slot. See Table 2 for compilation of distribution factors for the various harmonies.

$$K_d = \frac{\sin\left[\dfrac{(q)(\alpha_s)}{2}\right]}{(q)\sin\dfrac{(\alpha_s)}{2}} \quad \text{where } \alpha_s = \frac{180^\circ}{(m)(q)}$$

$$\therefore K_d = \frac{\sin\left[90^\circ/(m)\right]}{(q)\sin\left[90^\circ/(m)(q)\right]} = \frac{\sin\left[90^\circ/(5)\right]}{(25)\sin\left[90^\circ/(5)\times(25)\right]} \text{ For (25) = Integer}$$

or

$$K_d = \frac{\sin\left[N\alpha(m)/2\right]}{N\sin\left[\alpha(m)/2\right]} \text{ where } N \neq \text{Integer} = \frac{(Q)}{(m)(P)} \times \text{Integer \& } \alpha m = \frac{180^\circ}{N \times (m)}$$

$$\therefore K_d = \frac{\sin\left[90^\circ/(m)\right]}{N\sin\left[90^\circ/N(m)\right]} = \frac{\sin\left[90^\circ/(5)\right]}{N\sin\left[90^\circ/N\times(5)\right]} \text{ For (25) = Integer}$$

(44) K_P PITCH FACTOR - The ratio of the voltage induced in the coil to the voltage that would be induced in a full pitched coil. See Table 1 for compilation of the pitch factors for the various harmonics.

$$K_P = \sin\left[\frac{(Y)}{(m)(q)} \times 90^\circ\right] = \sin\left[\frac{(31)}{(5)(25)} \times 90^\circ\right]$$

(45) n_e TOTAL EFFECTIVE CONDUCTORS - The actual number of effective series conductors in the stator winding taking into account the pitch and skew factors but not allowing for the distribution factor.

$$n_e = \frac{(Q)(n_s)(K_P)(K_{SK})}{(C)} = \frac{(23)(30)(44)(42)}{(32)}$$

(46) a_c CONDUCTOR AREA OF STATOR WINDING in $(\text{inches})^2$ - The actual area of the conductor taking into account the corner radius on square and rectangular wire. See the following table for typical values of corner radii

If (39) = 0 then $a_c = .25\pi(\text{Dia})^2 = .25\pi(33)^2$

If (39) $\neq$ 0 then $a_c = (N_{ST})\left[\text{(strand width) (strand depth)} - (.858\, r_c^{\,2})\right]$

$$= (34)\left[(33)\,(39) - \{.858\, r_c^{\,2}\}\right]$$

where $.858\, r_c^{\,2}$ is obtained from Table V below.

(39)	(33) .188	.189 (33) .75	(33) .751
.050	.000124	.000124	.000124
.072	.000210	.000124	.000124
.125	.000210	.00084	.000124
.165	.000840	.00084	.003350
.225	.001890	.00189	.003350
.438	--	.00335	.007540
.688	--	.00754	.01340
--	--	.03020	.03020

TABLE V

(47) S_S CURRENT DENSITY - Amperes per square inch of stator conductor

$$S_S = \frac{(I_{PH})}{(C)(a_c)} = \frac{(8)}{(32)(46)}$$

(48) L_E END EXTENSION LENGTH in inches

When (29) = 0 then:

$$L_E = \frac{.5 + K_T \pi (Y)[O.D.]}{Q} = \frac{.5 + \begin{bmatrix} 1.3 \text{ if } (6) = 2 \\ 1.5 \text{ if } (6) = 4 \\ 1.7 \text{ if } (6) > 4 \end{bmatrix} \pi (31) \left[(12)\right]}{(23)}$$

When (29) = 1. then:

$$L_E = 2\, l_{e2} + \pi\left[\frac{(\text{dia})}{2}\right] + Y\left[\frac{\tau_s^{\,2}}{\tau_s^{\,2} - b_s^{\,2}}\right]$$

$$= 2 \times (36) + \pi\left[\frac{(35)}{2}\right] + (31)\left[\frac{(26)^2}{(26)^2 - (22)^2}\right]$$

(49) ℓ_t **1/2 MEAN TURN** - The average length of one conductor in inches

$$\ell_t = (\ell) + (L_E) = (13) + (44)$$

(50) $X_s\,^{o}C$ **STATOR TEMP ^{o}C.** - Input temp at which F. L. losses will be calculated. No load losses and cold resistance will be calculated at 20^{o}C.

(51) ρ_s **RESISTIVITY OF STATOR WINDING** - In micro ohm-inches @ 20^{o}C. If tables are available using units other than that given above, use Table VI for conversion to ohm-inches.

ρ	ohm-cm	ohm-in	ohm-cir mil/ft
1 ohm-cm =	1.000	0.3937	6.015×10^{6}
1 ohm-in =	2.540	1.000	1.528×10^{7}
1 ohm-cir mil/ft =	1.662×10^{-7}	6.545×10^{-8}	1.000

TABLE VI
Conversion Factors for Electrical Resistivity

(52) ρ_{S} (hot) **RESISTIVITY OF STATOR WINDING** - Hot at $X_s\,^{o}C$ in micro ohm-inches

$$\rho_{S(hot)} = (\rho_S)\left[\frac{(X_s\,^{o}C) + 234.5}{254.5}\right] = (51)\left[\frac{(50) + 234.5}{254.5}\right]$$

(53)	R_{SPH} (cold)	STATOR RESISTANCE/PHASE - Cold @ 20^{o}C in ohms
		$$R_{SPH(cold)} = \frac{(\rho_s)(n_s)(Q)(\ell_t)\times 10^{-6}}{(m)(a_c)(C)^2} = \frac{(51)(30)(23)(49)}{(5)(46)(32)^2}\times 10^{-6}$$
(54)	R_{SPH} (hot)	STATOR RESISTANCE/PHASE - Calculated @ X^{o}C in ohms
		$$R_{SPH(hot)} = \frac{(\rho_{s\ hot})(n_s)(Q)(\ell_t)\times 10^{-6}}{(m)(a_c)(C)^2} = \frac{(52)(30)(23)(49)}{(5)(46)(32)^2}\times 10^{-6}$$
(55)	EF (top)	EDDY FACTOR TOP - The eddy factor of the top coil. Calculate this value at the expected opera·ing temperature of the machine.
		$$EF_{top} = 1 + \left\{.584 + \left[\frac{N_{st}^2 - 1}{16}\right]\left[\frac{h'_{st}\,\ell}{h_{st}\,\ell_t}\right]^2\right\} 3.35 \times 10^{-3} \left[\frac{(h_{st})(n_s)(f)(a_c)}{(b_s)(\rho_{S\,hot})\,x}\right]^2$$
		$$= 1 + \left\{.584 + \left[\frac{(34)^2 - 1}{16}\right]\left[\frac{(38)(13)}{(37)(49)}\right]^2\right\} 3.35 \times 10^{-3} \left[\frac{(37)(30)(5a)(46)}{(22)(52)}\right]^2$$
(56)	EF (bot)	EDDY FACTOR BOTTOM - The eddy factor of the bottom coil at the expected operating temperature of the machine
		$$EF_{(bot)} = (EF_{(top)}) - 1.677 \left[\frac{(h_{st})(n_s)(f)(a_c)}{(b_s)(\rho_{S\,hot})}\right]^2 \times 10^{-3}$$

$= (55)$ … (30) … 46) / 22)(52 …] 10^{-3}

(57) b_{tm} <u>STATOR T</u>… - 1/2 way down tooth in inches-

… type (a), (b), (d) and (e), Figure I

$= (\Upsilon_s) - (b_s) = (26) - (22)$

For slot type (c), Figure I

$b_{tm} = (\Upsilon_s) - (b_3) = (26) - (22)$

(57a) $b_{t\,1/3}$ <u>STATOR TOOTH WIDTH</u> - 1/3 distance up from narrowest section

For slots type (a), (b) and (e)

$b_{t\ 1/3} = (\Upsilon_{s\ 1/3}) - (b_s) = (27) - (22)$

For slot type (c)

$b_{t\ 1/3} = b_{tm} = (57)$

For slot type (d)

$$b_{t\ 1/3} = (\Upsilon_{1/3}) - \frac{2\sqrt{2}}{3}(b_s) = (27) - .94(22)$$

(58) b_t <u>TOOTH WIDTH AT STATOR</u> - Main air gap in inches

For partially closed slot

$$b_t = \frac{\pi(d)}{(Q)} - b_0 = \frac{\pi(10a)}{(23)} - (22)$$

For open slot

$$b_t = \frac{\pi(d)}{(Q)} - b_s = \frac{\pi(10a)}{(23)} - (22)$$

(59)	g	MAIN AIR GAP - given in inches
(59a)	g_2	AUXILIARY AIR GAP (g_2) - given in inches
(59c)	g_3	AUXILIARY AIR GAP (g_3) - given in inches
(60)	C_X	REDUCTION FACTOR - Used in calculating conductor permeance and is dependent on the pitch and distribution factor. This factor can be obtained from Graph 1 with an assumed K_d of .955 or calculated as shown $$C_X = \frac{(K_X)}{(K_P)^2\,(K_d)^2} = \frac{(61)}{(44)^2\,(43)^2}$$
(61)	K_X	FACTOR TO ACCOUNT FOR DIFFERENCE in phase current in coil sides in same slot.

For 60^o phase belt winding, i.e. when (42a) = 60

$$K_X = 1/4\left[\frac{3(y)}{(m)(q)} + 1\right] \text{ where } 2/3 \leqq (y)/(m)(q) \leqq 1.0$$

$$K_X = 1/4\left[\frac{3(31)}{(5)(25)} + 1\right] \text{ where } 2/3 \leqq (31a) \leqq 1.0$$

or

$$K_X = 1/4\left[\frac{6(y)}{(m)(q)} - 1\right] \text{ where } 1/2 \leqq (31a) \leqq 2/3$$

$$K_X = 1/4\left[\frac{6(31)}{(5)(25)} - 1\right] \text{ where } 1/2 \leqq (31a) \leqq 2/3$$

For 120° phase belt winding, i.e. when (42a) = 120

$$K_X = .75 \text{ when } 2/3 \leqq (y)/(m)(q)$$

$$K_X = .75 \text{ when } 2/3 \leqq (31a)$$

or

$$K_X = .05 \left[\frac{24(y)}{(m)(q)} - 1\right] \text{ where } 1/2 \leqq \frac{(y)}{(m)(q)} \leqq 2/3$$

$$K_X = .05 \left[\frac{24(31)}{(3)(25)} - 1\right] \text{ where } 1/2 \leqq (31a) \leqq 2/3$$

(62) | λ_i | CONDUCTOR PERMEANCE - The specific permeance for the portion of the stator current that is embedded in the iron. This permeance depends upon the configuration of the slot.

(a) For open slots

$$\lambda_i = (C_X)\frac{20}{(m)(q)}\left[\frac{(h_2)}{(b_s)} + \frac{(h_1)}{3(b_s)} + \frac{(b_t)^2}{16(\tau_s)(g)} + \frac{.35(b_t)}{(\tau_s)}\right]$$

$$\lambda_i = (60)\frac{20}{(5)(25)}\left[\frac{(22)}{(22)} + \frac{(22)}{3(22)} + \frac{(58)^2}{16(26)(59)} + \frac{.35(58)}{(26)}\right]$$

(b) For partially closed slots with constant slot width

$$\lambda_i = (C_X)\frac{20}{(m)(q)}\left[\frac{(h_o)}{(b_o)} + \frac{2(h_t)}{(b_o)+(b_s)} + \frac{(h_w)}{(b_s)} + \frac{(h_1)}{3(b_s)} + \frac{(b_t)^2}{16(\tau_s)(g)} + \frac{.35(b_t)}{(\tau_s)}\right]$$

$$\lambda_i = (60)\frac{20}{(5)(25)}\left[\frac{(22)}{(22)} + \frac{2(22)}{(22)+(22)} + \frac{(22)}{(22)} + \frac{(22)}{3(22)} + \frac{(58)^2}{16(26)(59)} + \frac{.35(58)}{(26)}\right]$$

(c) For partially closed slots (tapered sides)

$$\lambda_i = (C_X)\frac{20}{(m)(q)}\left[\frac{(h_o)}{(b_o)} + \frac{2(h_t)}{(b_o)+(b_1)} + \frac{2(h_w)}{(b_1)+(b_2)} + \frac{(h_1)}{3(b_2)} + \frac{(b_t)^2}{16(\tau_s)(g)} + \frac{.35(b_t)}{(\tau_s)}\right]$$

$$\lambda_i = (60)\frac{20}{(5)(25)}\left[\frac{(22)}{(22)} + \frac{2(22)}{(22)+(22)} + \frac{2(22)}{(22)+(22)} + \frac{(22)}{3(22)} + \frac{(58)^2}{16(26)(59)} + \frac{.35(58)}{(26)}\right]$$

(d) For round slots

$$\lambda_i = (C_X)\,\frac{20}{(m)(q)}\left[.62 + \frac{(h_o)}{(b_o)}\right]$$

$$\lambda_i = (60)\,\frac{20}{(5)(25)}\left[.62 + \frac{(22)}{(22)}\right]$$

(e) For open slots with a winding of one conductor per slot

$$\lambda_i = (C_X)\,\frac{20}{(m)(q)}\left[\frac{(h_2)}{(b_s)} + \frac{(h_1)}{3(b_s)} + .6 + \frac{(g)}{2(\tau_s)} + \frac{(\tau_s)}{4(g)}\right]$$

$$\lambda_i = (60)\,\frac{20}{(5)(25)}\left[\frac{(22)}{(22)} + \frac{(22)}{3(22)} + .6 + \frac{(59)}{2(26)} + \frac{(26)}{4(59)}\right]$$

(63) K_E **LEAKAGE REACTIVE FACTOR** for end turn

$$K_E = \frac{\text{Calculated value } (L_E)}{\text{Value } (L_E) \text{ from Graph 1}} \quad \text{(For machines where } \cancel{(11)} > 8''\text{)}$$

where L_E = (48) and abscisa of Graph 1 = $(\gamma)(\tau_s)$ = (31)(26)

$$K_E = \sqrt{\frac{\text{Calculated value of } (L_E)}{\text{Value } (L_E) \text{ from Graph 1}}} \quad \text{(For machines where (11)} < 8''\text{)}$$

(64) λ_E **END WINDING PERMEANCE** - The specific permeance for the end extension portion of the stator winding

$$\lambda_E = \frac{6.28(K_E)}{(\ell)(K_d)^2}\left[\frac{\phi_E\,L_E}{2n}\right] = \frac{6.28(63)}{(13)(43)^2}\left[\frac{Q_E\,L_E}{2n}\right]$$

The term $\left[\frac{\phi_E\,L_E}{2n}\right]$ is obtained from Graph 1.

The symbols used in this (term) do not apply to those of this design manual. Reference information for the symbol origin is included on Graph 1.

(65) -- **WEIGHT OF COPPER** - the weight of stator copper in lbs.

$$\#\text{'s copper} = .321(n_s)(Q)(a_c)(\ell_t) = .321(30)(23)(46)(49)$$

(66) -- **WEIGHT OF STATOR IRON** - in lbs.

$$\#\text{'s iron} = .283\left\{(b_{tm})(Q)(\ell_s)(h_s) + \pi(d) \quad (h_c)(\ell_s)\right\}$$

$$= .283\left\{(57)(23)(17)(22) + \pi(10a) \quad (24)(17)\right\}$$

(67) K_s **CARTER COEFFICIENT**

$$K_s = \frac{(\tau_s)\left[5(g)+(b_s)\right]}{(\tau_s)\left[5(g)+(b_s)\right] - (b_s)^2} \quad \text{(For open slots)}$$

$$K_s = \frac{(26)\left[5(59)+(22)\right]}{(26)\left[5(59)+(22)\right] - (22)^2}$$

$$K_s = \frac{\tau_s\left[4.44(g)+.75(b_o)\right]}{\tau_s\left[4.44(g)+.75(b_o)\right] - (b_o)^2} \quad \text{(For partially closed slots)}$$

$$K_s = \frac{(26)\left[4.44(59)+.75(22)\right]}{(26)\left[4.44(59)+.75(22)\right] - (22)^2}$$

(68) A_g **MAIN AIR GAP AREA** - The area of the gap surface at the stator bore

$$A_g = \frac{\pi}{4}\left[(O.D.)^2 - (I.D.)^2\right] = \frac{\pi}{4}\left[(12)^2 - (11)^2\right]$$

(69) g_e **EFFECTIVE AIR GAP (MAIN)**

$$g_e = (K_s)(g) = (67)(59)$$

(70) A_{g2} AREA OF OUTER AUXILIARY AIR GAP (g_2) - Calculate from layout. This gap must be uniform circumferentially with no saturated sections if parasitic losses in the gap surfaces are to be prevented.

(70a) A_{g3} AREA OF THE INNER AUXILIARY GAP (g_3) - The same comment applies to g_3 as to g_2 above. Avoid discontinuity in the circumferential flux pattern.

(71) C_1 THE RATIO OF MAXIMUM FUNDAMENTAL of the field form to the actual maximum of the field form.

For pole heads with only one radius, C_1 is obtained from Curve #4. The abscissa is "pole embrace" (α) = (77). The graphical flux plotting method of determining C_1 is explained in the section titled "Derivations" in the Appendix.

(72) C_W WINDING CONSTANT - The ratio of the RMS line voltage for a full pitched winding to that which would be introduced in all the conductors in series if the density were uniform and equal to the Maximum value.

$$C_W = \frac{(E)(C_1)(K_d)}{\sqrt{2}(E_{PH})(m)} \quad \frac{(3)(71)(43)}{\sqrt{2}(4)(5)}$$

Assuming $K_d = .955$, then $C_W = .225\ C_1$ for three phase delta machines and $C_W = .390\ C_1$ for three phase star machines.

(73) C_p POLE CONSTANT - The ratio of the average to the maximum value of the field form. C_p is obtained from Curve #4. Note the correction factor at the top of the curve.

(74) C_M DEMAGNETIZING FACTOR - direct axis.

$$C_M = \frac{(\alpha)\pi + \sin\left[(\alpha)\pi\right]}{4\sin\left[(\alpha)\pi/2\right]} = \frac{(77)\pi + \sin\ (77)}{4\sin\left[(77)\ \pi/2\right]}$$

(75) C_q CROSS MAGNETIZING FACTOR - quadrature axis

$$\left.\begin{aligned} C_q &= \frac{1/2\cos\left[(\alpha)\ \pi/2\right] + (\alpha)\pi - \sin\left[(\alpha)\pi\right]}{4\sin\left[(\alpha)\ \pi/2\right]} \\ &= \frac{1/2\cos\left[(77)\ \pi/2\right] + (77)\pi - \sin\left[(77)\pi\right]}{4\sin\left[(77)\ \pi/2\right]} \end{aligned}\right\}\ \text{valid for concentric poles.}$$

C_q can also be obtained from Curve 9.

(76) -- POLE DIMENSIONS LOCATIONS per Figure 2b

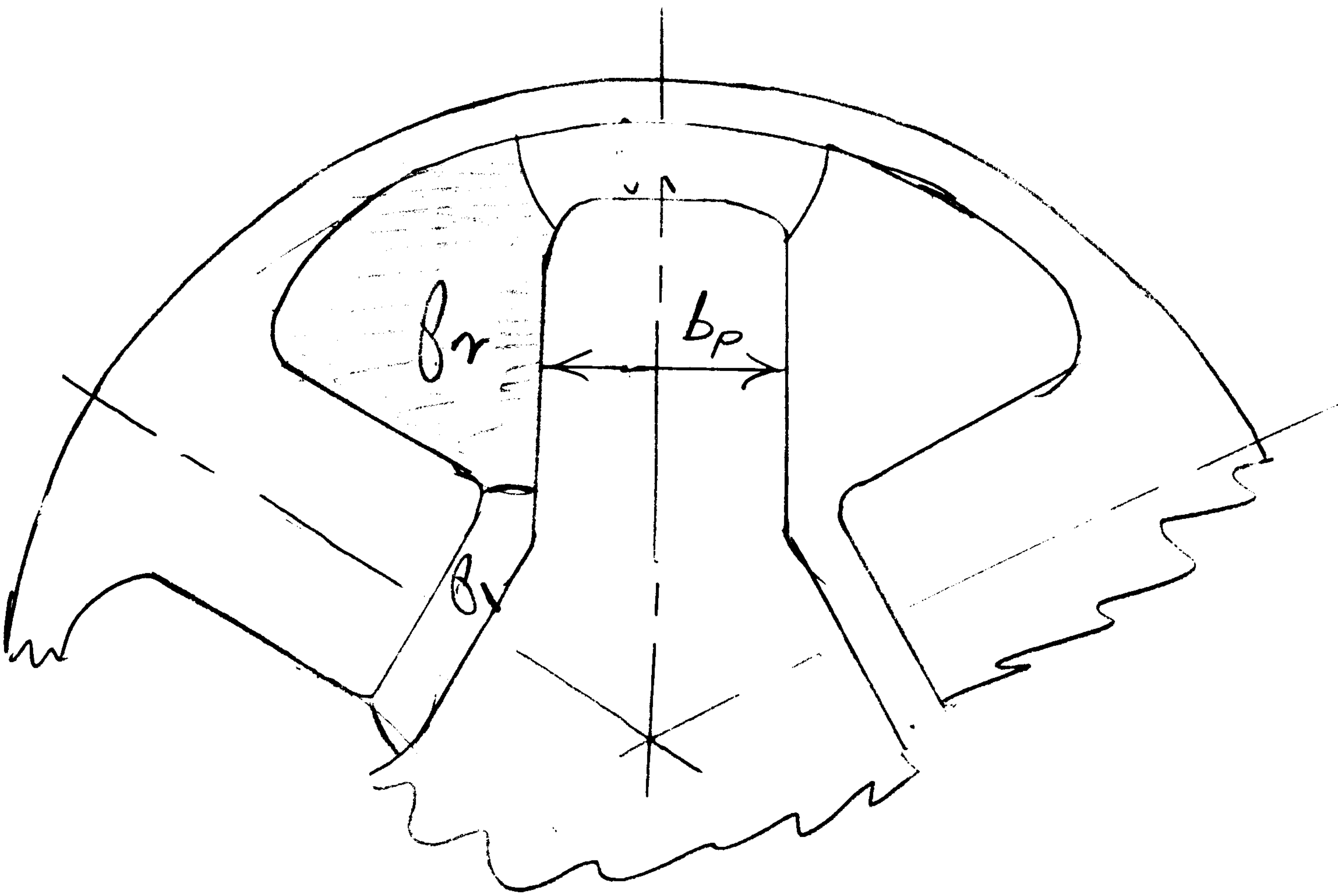
b_{p1} = minimum width of pole (usually at tip) measured at the edge of the stator toroid.

b_{p2} = maximum width of pole (usually at entering edge) at edge of stator toroid.

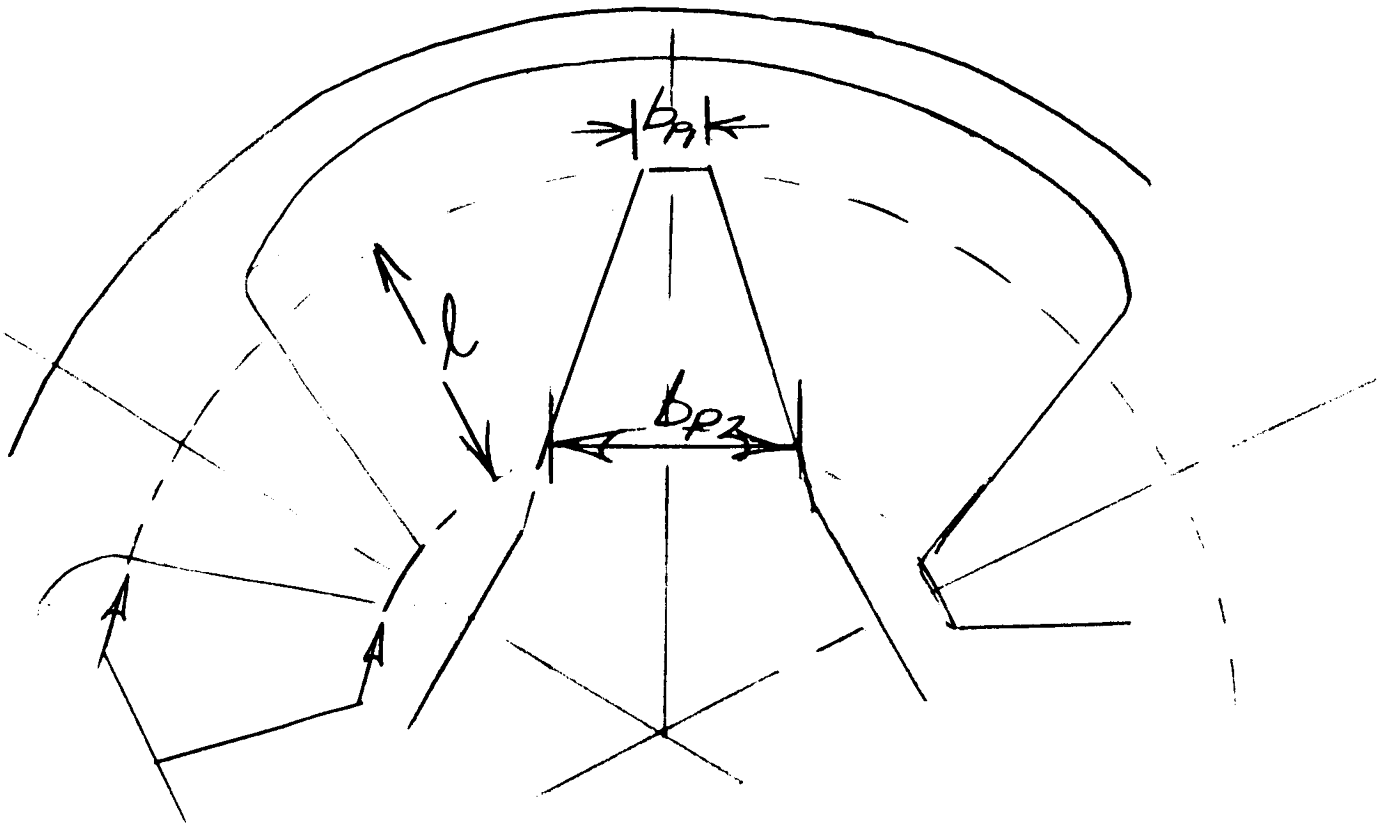
b_p = average width of pole

$$b_p = \frac{b_{p1} + b_{p2}}{2}$$

(79)	A_{po}	<u>AREA OF POLE AT ENTERING EDGE OF STATOR TOROID</u> (outer pole) - Obtain from layout.
(79a)	A_{pi}	<u>AREA OF POLE AT ENTERING EDGE OF STATOR TOROID</u> (inner pole) - Obtain from layout.



UNIFORM POLE WIDTH FIGURE 2a



EDGES OF STATOR TOROID

$$b_p = \frac{b_{p1} + b_{p2}}{2} = \text{EFFECTIVE WIDTH OF POLE}$$

FIGURE 2b

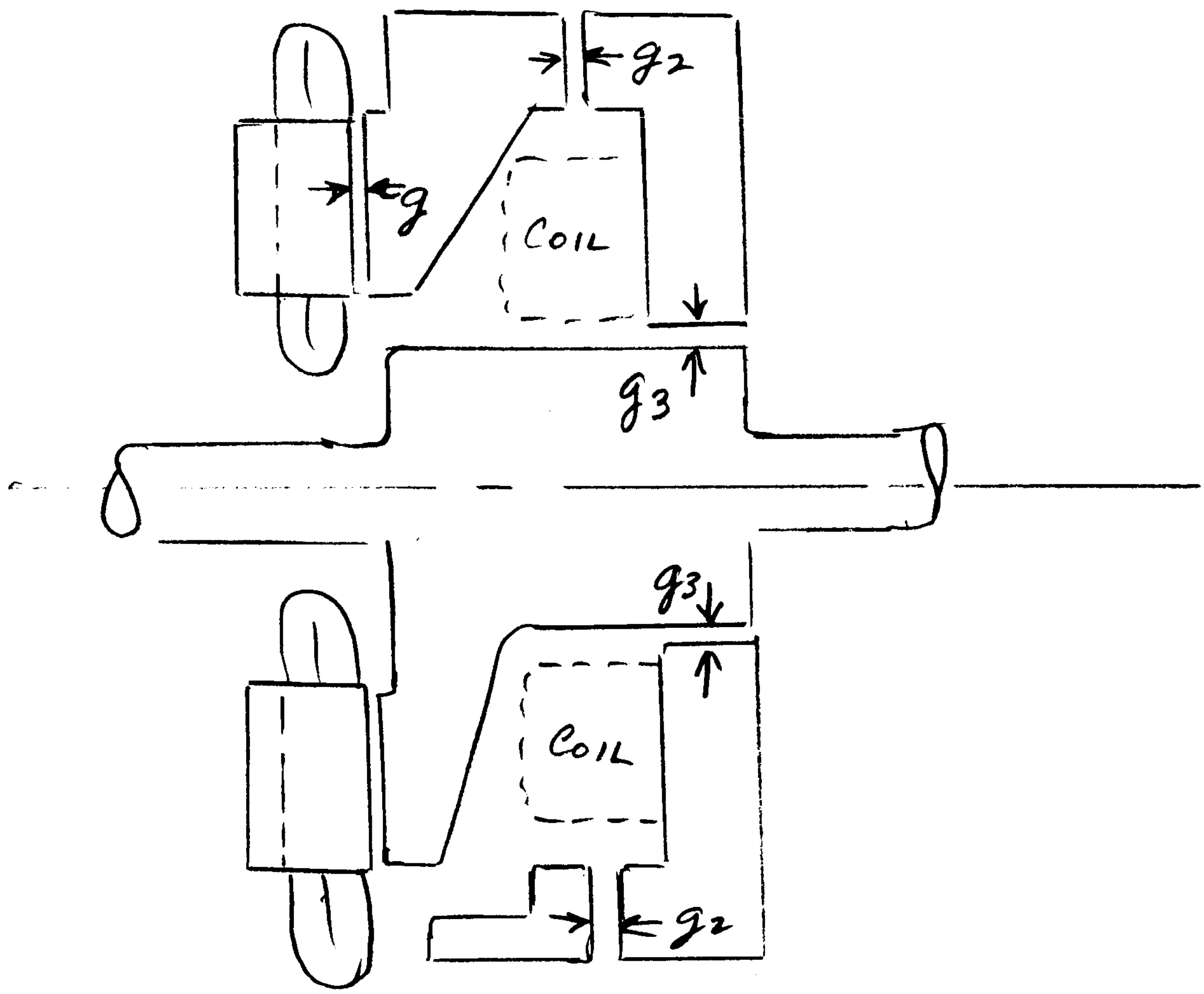


FIG 3

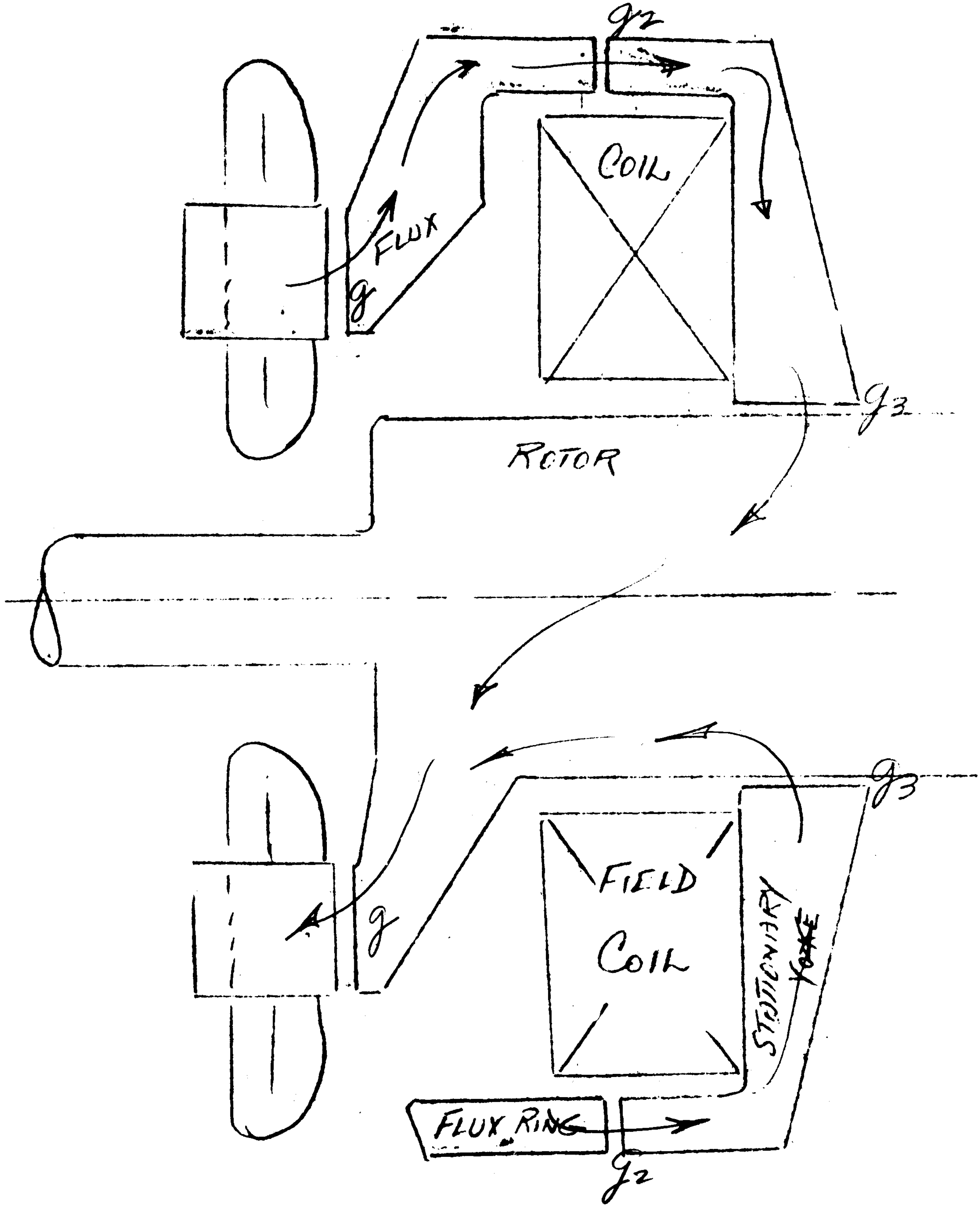

g2
COIL
FLUX
g
g3
ROTOR
g3
FIELD COIL
g
STATIONARY ~~YOKE~~
FLUX RING
g2

FIG 4

(80) P_1 POLE TIP TO ROTOR LEAKAGE PERMEANCE - Add the leakage permeance from the inside pole to the outer flux ring and the outside pole to the shaft section. PER FIG 5

$$P_1 = \left[\frac{\mu a_1}{\ell_1} + \frac{\mu a'_1}{\ell'_1}\right]\left[\frac{P}{2}\right]$$

(81) P_2 SIDE LEAKAGE FROM POLE-TO-POLE

$$P_2 = \frac{\mu a}{\ell} \qquad \text{PER FIG 5}$$

a = area of leakage path between poles x poles

ℓ = median length of leakage path between a pair of poles

(82) P_3 LEAKAGE PERMEANCE FROM UNDERSIDE OF POLE TO ROTOR.

Add the leakage permeance from inner pole to outer flux ring and from outer pole to shaft. Multiply this sum by $\frac{P}{2}$ PER FIG 6 & 7

$$P_3 = \left[\frac{\mu a_3}{\ell_3} \quad \frac{\mu a'_3}{\ell'_3}\right]\frac{P}{2}$$

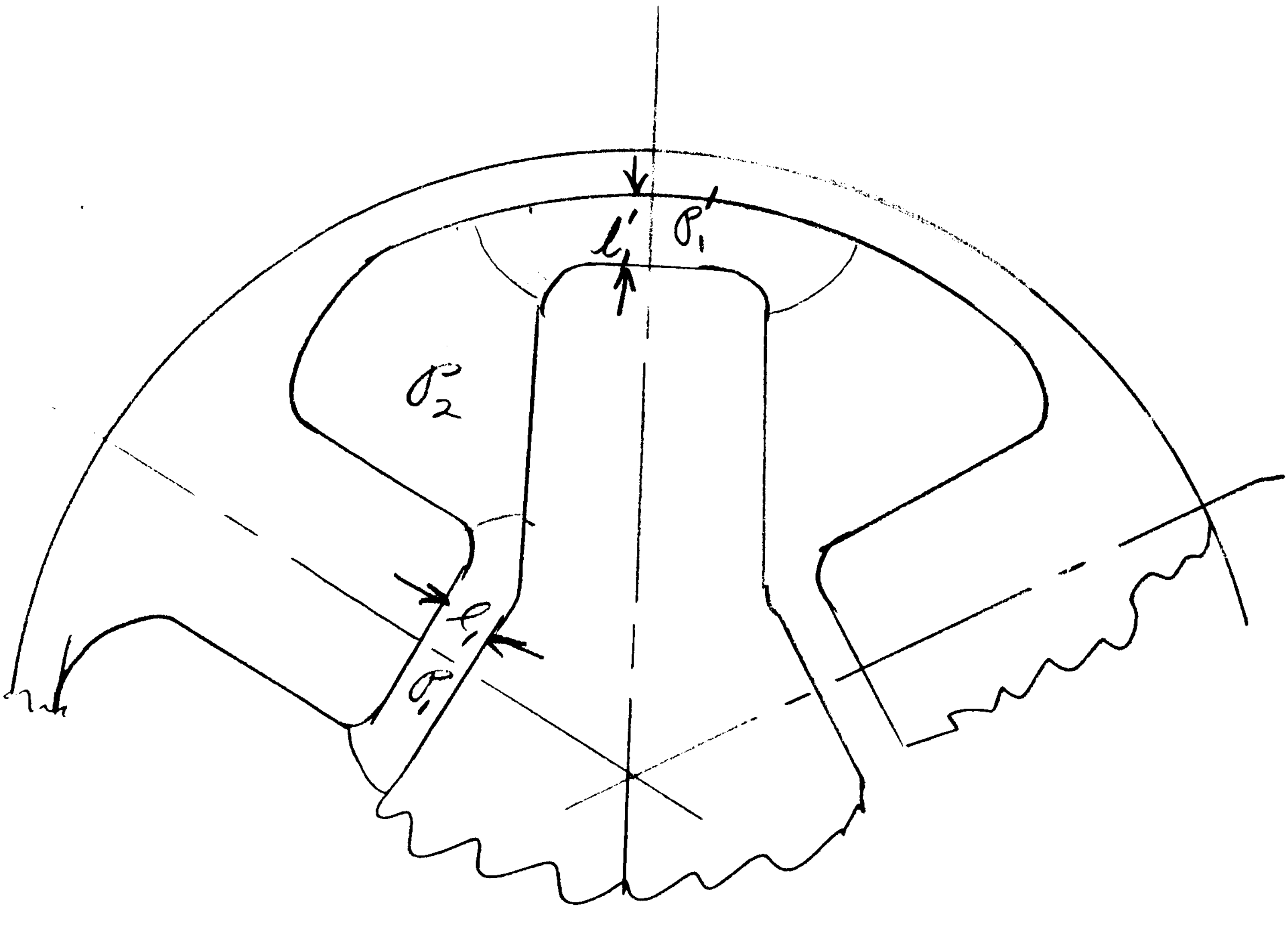


FIG 5

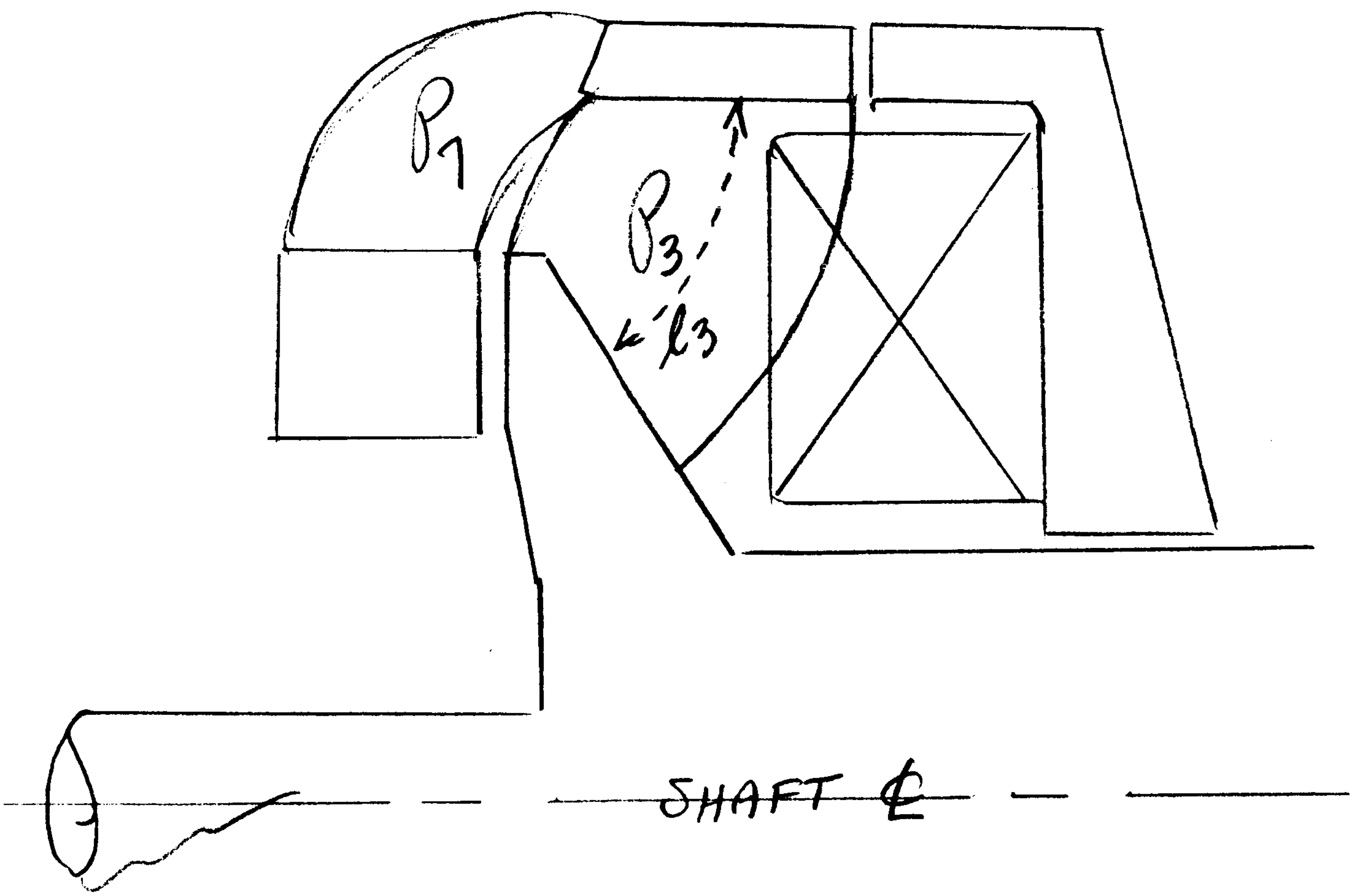


FIG 6

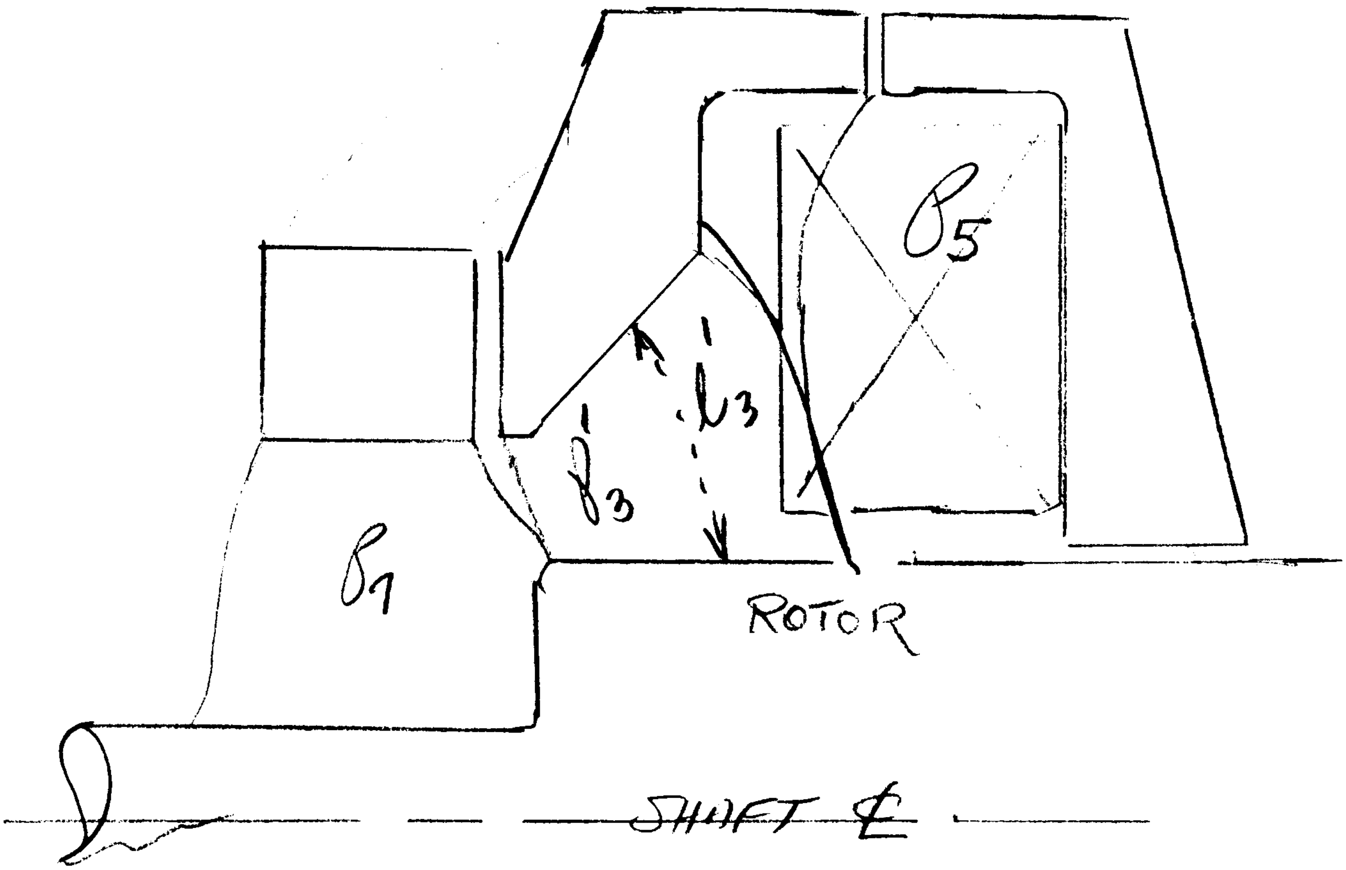
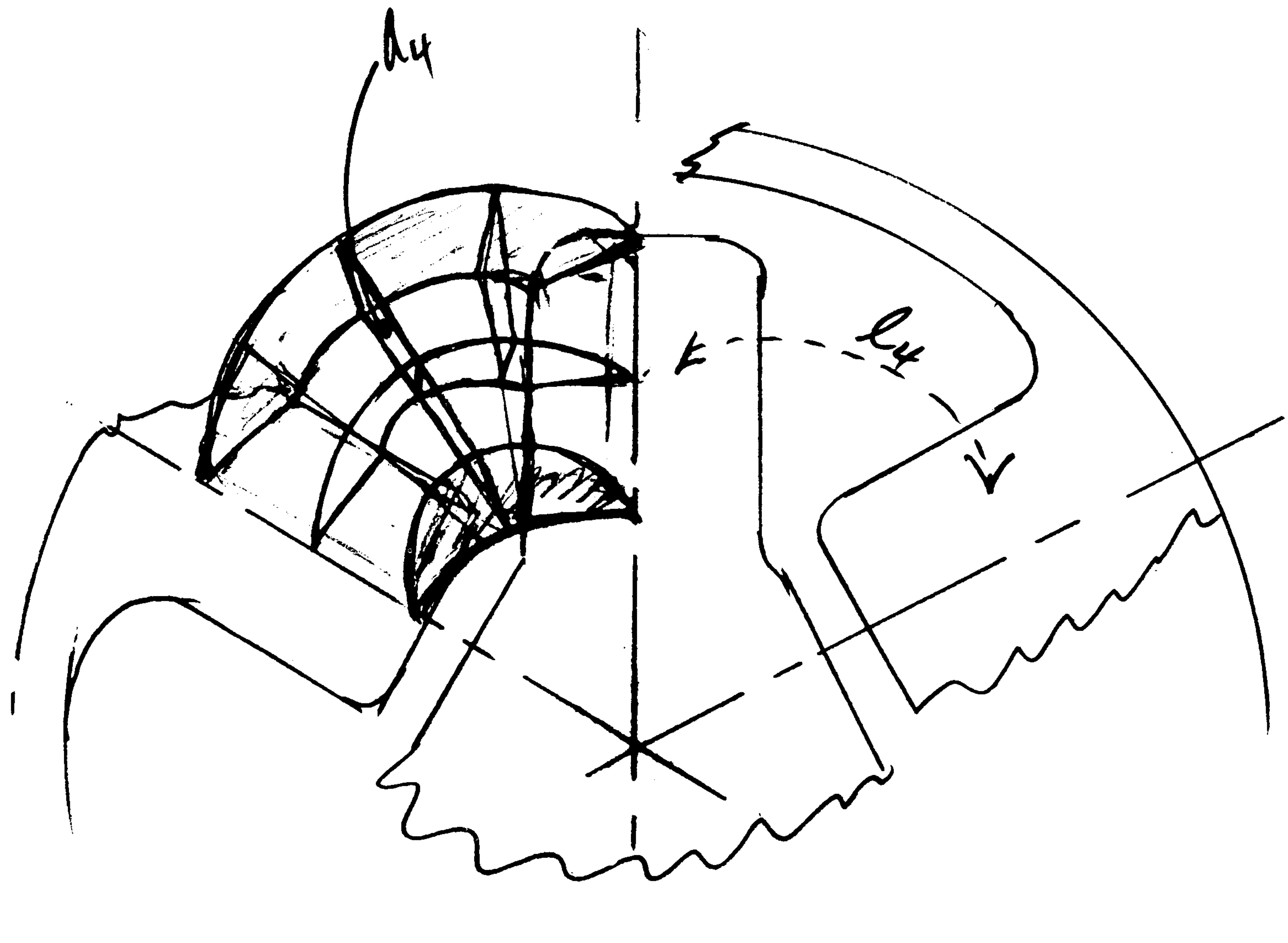


FIG 7

(83)	P_4	<u>LEAKAGE PERMEANCE FROM UNDERSIDE OF POLE TO UNDERSIDE OF POLE</u> -
		$P_4 = \left[\frac{\mu a_4}{l_4}\right] (P)$ PER FIG 8



LEAKAGE FLUX FROM UNDERSIDE OF POLE TO UNDERSIDE OF POLE, $\mathcal{P}_4$

FIG 8

(84) P_5 **LEAKAGE PERMEANCE THROUGH FIELD COIL**

$$P_5 = \frac{\mu\, a_5}{\ell_5} \qquad \text{PER FIG 7}$$

Where $a_5 = \pi(d_c)(b_c)$ inches^2

Where b_c = width of field coil

Where d_c = field coil diameter

$$= \frac{\text{Coil O.D.} + \text{Coil I.D.}}{2} \text{ inches}$$

Where $$\ell_5 = \frac{\text{Coil O.D.} - \text{Coil I.D.}}{2} \text{ inches}$$

$\mu = 3.19$

(86) P_7 **STATOR TO FLUX RING AND SHAFT LEAKAGE**

PER FIG 7

(88) ϕ_T **TOTAL FLUX** in Kilolines

$$\phi_T = \frac{6(E)10^6}{(C_W)(n_e)(RPM)} = \frac{6(3)10^6}{(72)(45)(17)}$$

(89) $\phi_{\ell 7}$ **LEAKAGE FLUX FROM STATOR TO SHAFT AND OUTER FLUX RING**

$$\phi_{\ell 7} = \frac{P_7\left[2(F_T) + 2(F_c) + (F_{g2}) + (F_{g3}) + (F_{po}) + (F_{pi})\right] \times 10^{-3}}{2}$$

$$= (86)\left[2(97) + 2(98) + (123) + (120) + (104) + (104b)\right] \times 10^{-3}$$

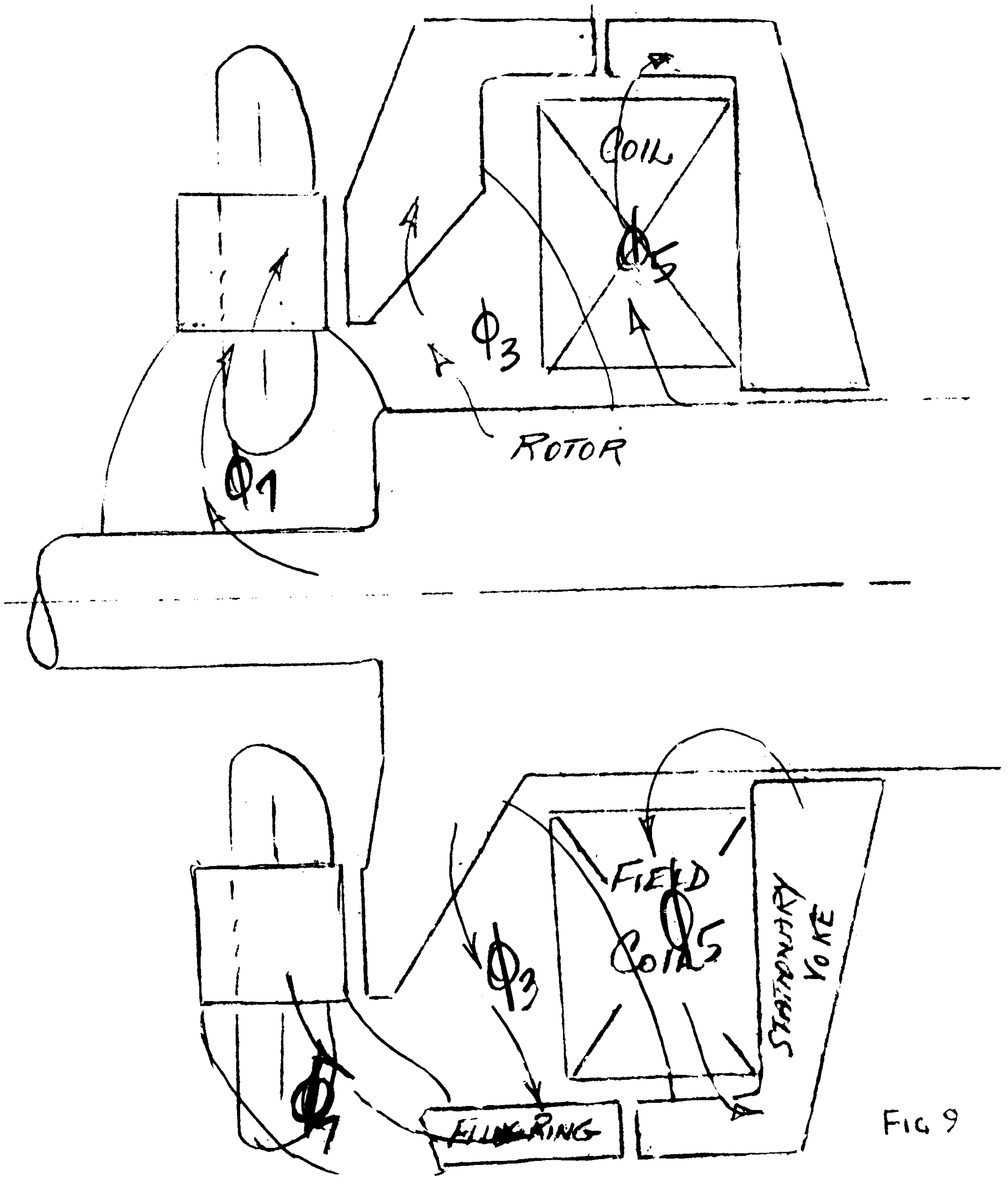

COIL
Φ_5
Φ_3
ROTOR
Φ_1
FIELD
COIL
Φ_5
Φ_3
STATIONARY
YOKE
Φ_1
FILLING RING

FIG 9

(91)	B_t	TOOTH DENSITY in Kilolines/in^2 - The flux density in the stator tooth at 1/3 of the distance from the minimum section. $$B_t = \frac{\phi_T}{(Q)(\ell_s)(b_t\ 1/3)} = \frac{(88)}{(23)(17)(57a)}$$
(92)	ϕ_P	FLUX PER POLE in Kilolines $$\phi_P = \frac{(\phi_T)(C_P)}{(P)} = \frac{(88)(73)}{(6)}$$
(94)	B_c	CORE DENSITY in Kilolines/in^2 - The flux density in the stator core $$B_c = \frac{(\phi_P)}{2(h_c)(\ell_s)} = \frac{(92)}{2(24)(17)}$$
(95)	B_g	GAP DENSITY in Kilolines/in^2 - The maximum flux density in the air gap $$B_g = \frac{(\phi_T)}{(A_g)} = \frac{(88)}{(68)}$$
(96)	F_g	AIR GAP AMPERE TURNS - The field ampere turns per pole required to force flux across the air gap when operating at no load with rated voltage. $$F_g = \frac{(B_g)(g_e)}{3.19} \times 10^3 = \frac{(95)(69)}{3.19} \times 10^3$$

(97) F_T **STATOR TOOTH AMPERE TURNS**

$$F_T = (h_s) \left[\text{NI/inch at density } (B_t) \right]$$

$$= (22) \left[\begin{array}{l} \text{look up on stator magnetization curve} \\ \text{given in (18) at density (91)} \end{array} \right.$$

(98) F_c **STATOR CORE AMPERE TURNS**

$$F_c = \frac{\pi(d)}{4(P)} \left[\text{NI/inch at density } (B_c) \right]$$

$$F_c = \frac{\pi(10a)}{4(6)} \left[\begin{array}{l} \text{Look up on stator magnetization curve} \\ \text{at density (94)} \end{array} \right]$$

(100) ϕ_ℓ **LEAKAGE FLUX** - at no load

$$\phi_\ell = \left[(P_1)+(P_2)+(P_3)+(P_4) \right] \left[2(F_T)+2(F_c)+(F_{g2})+(F_{g3}) \right] \times 10^{-3}$$

$$= \left[(80)+(81)+(82)+(83) \right] \left[2(97)+2(98)+(123)+(120) \right] \times 10^{-3}$$

(102) ϕ_{pt} **TOTAL FLUX PER POLE** - at no load

$$\phi_{pt} = \phi_p + \frac{\phi_\ell}{P} = (92) + \frac{(100)}{(6)}$$

(103) B_{po} **FLUX DENSITY IN OUTER POLE (NL)**

$$B_{po} = \frac{(\phi_{pt})}{(a_{po})} = \frac{(102)}{(79)}$$

(104)	F_{po}	**AMPERE TURN DROP THROUGH OUTER POLE @ N. L.**
		$F_{po} = (\ell_{po}) \left[\text{NI/inch at density } (B_{po}) \right]$
		$= (104) \left[\begin{array}{l} \text{Look up on pole magnetization curve} \\ \text{at density (103).} \end{array} \right]$
		Where ℓ_{po} = length of outer pole.
(104a)	B_{pi}	**FLUX DENSITY IN INNER POLE @ N. L.**
		$B_{pi} = \frac{\phi_{pt}}{A_{pi}} = \frac{(102)}{(79a)}$
(104b)	F_{pi}	**AMPERE TURN DROP THROUGH THE INNER POLE @ N. L.**
		$F_{pi} = (\ell_{pi}) \left[\text{NI/inch at density } (B_{pi}) \right]$
		$= (104b) \left[\begin{array}{l} \text{Look up on pole magnetization curve at} \\ \text{density (104a)} \end{array} \right]$
		Where (ℓ_{pi}) = length of inner pole
(104c)	ϕ_r	**FLUX IN ROTATING OUTER FLUX RING AT NO LOAD**
		$\phi_r = \phi_{g2} = \phi_{g3} = \phi_{sh}$
		$= (108) = (118a) = (111)$

(104d) B_r — FLUX DENSITY IN ROTATING OUTER RING at no load

$$B_r = \frac{(\phi_r)}{(A_r)} = \frac{(104c)}{(104d)}$$

Where A_r = ring cross-section area adjacent to the outer pole (P_o)

(104e) F_r — AMPERE TURN DROP IN RING at no load.

$$F_r = (\ell_r) \left[\text{NI/inch at density } (B_r) \right]$$

$$= (104e) \left[\begin{array}{l} \text{Look up on ring magnetization curve} \\ \text{at density } (104d) \end{array} \right]$$

Where ℓ_4 = length of ring

(108) ϕ_{g2} — FLUX IN AUXILIARY GAP at no load

$$\phi_{g2} = \phi_{g3} = \phi_r = \phi_{sh} = \phi_{pt} \frac{(P)}{2} + \phi_7$$

$$= 102 \frac{(6)}{2} + (89)$$

(111) ϕ_{sh} — FLUX IN SHAFT at no load

$$\phi_{sh} = \phi_{g2} = \phi_r = \phi_{g3}$$

$$= (108) = (104c) = (118a)$$

(112) A_{sh} <u>AREA OF SHAFT</u> (cross-sectional to flux)

(113) B_{sh} <u>FLUX DENSITY IN SHAFT</u> at no load

$$B_{sh} = \frac{\phi_{sh}}{A_{sh}} = \frac{(111)}{(112)}$$

(114) F_{sh} <u>AMPERE TURN DROP IN SHAFT</u> at no load

$$F_{sh} = \ell_{sh} \left[\text{NI/inch at density } (B_{sh}) \right]$$

$$= (114) \left[\begin{array}{l} \text{Look up on shaft magnetization curve} \\ \text{at density (113)} \end{array} \right]$$

Where ℓ_{sh} = effective length of shaft

(118) $\phi_{\ell 5}$ <u>LEAKAGE FLUX ACROSS THE FIELD COIL</u> in Kilolines

$$\phi_{\ell 5} = (P_5) \left[(F_{g2}) + (F_{g3}) + 2(F_t) + 2(F_c) + (F_{po}) + (F_{pi}) + (F_r) + (F_{sh}) \right] \times 10^{-3}$$

$$= (84) \left[(123) + (120) + 2(97) + 2(98) + (104) + (104b) + (104e) + (114) \right] \times 10^{-3}$$

(118a) ϕ_{g3} <u>FLUX IN AUXILIARY GAP</u> g_3

$$\phi_{g3} = \phi_{g2} = (108)$$

(119) B_{g3} <u>FLUX DENSITY IN AUXILIARY GAP</u> g_3

$$B_{g3} = \frac{(\phi_{g3})}{(A_{g3})} = \frac{(118a)}{(70a)}$$

(120)	F_{g3}	<u>AMPERE TURN DROP ACROSS THE AUXILIARY AIR GAP</u> g_3
		$F_{g3} = \frac{(B_{g3})}{3.19}(g_3) \times 10^3 = \frac{(119)}{3.19}(59c) \times 10^3$
(122)	B_{g2}	<u>FLUX DENSITY IN AUXILIARY AIR GAP</u>
		$B_{g2} = \frac{(\phi_{g2})}{(A_{g2})} = \frac{(108)}{(70)}$
(123)	F_{g2}	<u>AMPERE TURN DROP ACROSS AUXILIARY GAP</u> (g_2)
		$F_{g2} = \frac{(B_{g2})(g_2)}{3.19} \times 10^3 = \frac{(122)(59a)}{3.19} \times 10^3$
(126a)	ϕ_y	<u>FLUX IN YOKE</u>
(126b)	B_y	<u>YOKE DENSITY</u>
		$B_y = \frac{(\phi_y)}{(A_y)} = \frac{(126a)}{(126b)}$
		Where a_y = yoke cross-sectional area
(126c)	F_y	<u>AMPERE TURN DROP IN YOKE</u> at no load
		$F_y = \ell_y$ [NI/inch at density (B_y)]
		= (126c) [Look up on yoke magnetization curve at density (126b)]
		Where ℓ_y = length of yoke

(127)	F_{NL}	TOTAL AMPERE TURNS at no load
		$F_{NL} = \left[2(F_c)+2(F_T)+(F_{po})+(F_{pi})+(F_r)+(F_{sh})+(F_{g2})+(F_{g3})+(F_y)\right]$
		$= 2(98)+2(97)+(104)+(104b)+(104e)+(114)+(123)+(120)+(126c)$
(127a)	I_{FNL}	FIELD CURRENT - at no load
		$I_{FNL} = (F_{NL})/(N_F) = 127)/(146)$
(127b)	E_{FNL}	FIELD VOLTS - at no load. This calculation is made with cold field resistance at 20°C for no load condition.
		$E_F = (I_{FNL})(R_{f\ cold}) = (127a)(154)$
(127c)	S_F	CURRENT DENSITY - at no load. Amperes per square inch of field conductor.
		$S_F = (I_{FNL})/(a_{cf}) = (127)/(153)$
(128)	A	AMPERE CONDUCTORS per inch - The effective ampere conductors per inch of stator periphery. This factor indicates the "specific loading" of the machine. Its value will increase with the rating and size of the machine and also will increase with the number of poles. It will decrease with increases in voltage or frequency. A is generally higher in single phase machines than in polyphase ones.
		$A = \dfrac{(I_{PH})(n_s)(K_P)}{(C)\ (\Upsilon_s)} = \dfrac{(8)(30)(44)}{(32)(26)}$

(129) X **REACTANCE FACTOR** - The reactance factor is the quantity by which the specific permeance must be multiplied to give percent reactance. It is the percent reactance for unit specific permeance, or the percent of normal voltage induced by a fundamental flux per pole per inch numerically equal to the fundamental armature ampere turns at rated current. Specific permeance is defined as the average flux per pole per inch of core length produced by unit ampere turns per pole.

$$X = \frac{100(A)(K_d)}{\sqrt{2}\,(C_1)(B_g) \times 10^3} = \frac{100\,(128)(43)}{\sqrt{2}\,(71)\,(95) \times 10^3}$$

(130) X_ℓ **LEAKAGE REACTANCE** - The leakage reactance of the stator for steady state conditions. When (5) = 3, calculate as follows:

$$X_\ell = X\left[(\lambda_i) + (\lambda_E)\right] = (79)\left[(62) + (64)\right]$$

In the case of two phase machines a component due to belt leakage must be included in the stator leakage. reactance. This component is due to the harmonics caused by the concentration of the MMF into a small number of phase belts per pole and is negligible for three phase machines. When (5) = 2, calculate as follows:

$$\lambda_B = \frac{0.1(d)}{(P)(g_e)}\left[\frac{\sin\left[\frac{3(y)}{(m)(q)}\right]90^o}{(K_P)}\right] = \frac{0.1(11)}{(6)(69)}\left[\frac{\sin\left[\frac{3(31)}{(5)(25)}\right]90^o}{(44)}\right]$$

$$X_\ell = X\left[(\lambda_i) + (\lambda_E) + (\lambda_B)\right] \text{ where } \lambda_B = 0 \text{ for 3 phase machines.}$$

$$X_\ell = (79)\left[(62) + (64) + (80)\right]$$

(131)	X_{ad}	<u>REACTANCE</u> - direct axis - This is the fictitious reactance due to armature reaction in the direct axis. $X_{ad} = (X)(\lambda_a)(C_1)(C_M) = (129)(70)(71)(74)$
(132)	X_{aq}	<u>REACTANCE</u> - quadrature axis - This is the fictitious reactance due to armature reaction in the quad. axis. $X_{aq} = (X)(C_q)(\lambda_a) = (129)(75)(70)$
(133)	X_d	<u>SYNCHRONOUS REACTANCE</u> - direct axis - The steady state short circuit reactance in the direct axis. $X_d = (X_\ell) + (X_{ad}) = (130) + (131)$
(134)	X_q	<u>SYNCHRONOUS REACTANCE</u> - quadrature axis - The steady state short circuit reactance in the quadrature axis. $X_q = (X_\ell) + (X_{aq}) = (130) + (132)$
(145)	V_r	<u>PERIPHERAL SPEED</u> - The velocity of the rotor surface in feet per minute $V_r = \frac{\pi(d_r)(RPM)}{12} = \frac{\pi(11a)(7)}{12}$
(146)	N_F	<u>NUMBER OF FIELD TURNS</u>
(147)	ℓ_{tF}	<u>MEAN LENGTH OF FIELD TURN</u>
(148)	--	<u>FIELD CONDUCTOR DIA OR WIDTH</u> in inches
(149)	--	<u>FIELD CONDUCTOR THICKNESS</u> in inches - Set this item = 0. for round conductor.

(150)	$X_f{}^oC$	FIELD TEMP IN ^oC - Input temp at which full load field loss is to be calculated.
(151)	ρ_f	RESISTIVITY of field conductor @ 20^{o}C in micro ohm-inches. Refer to table given in item (51) for conversion factors.
(152)	ρ_f (hot)	RESISTIVITY of field conductor at $X_f{}^oC$ $\rho_{f\ (hot)} = \rho_f \left[\frac{(X_f{}^oC) + 234.5}{254.5}\right] = (104)\left[\frac{(150) + 234.5}{254.5}\right]$
(153)	a_{cf}	CONDUCTOR AREA OF FIELD WINDING - Calculate same as stator conductor area (46) except substitute (149) for (39) (148) for (33)
(154)	R_f (cold)	COLD FIELD RESISTANCE @ 20^{o}C $R_f\ (cold) = (\rho_f)\frac{(N_f)(\ell_{tf})}{(a_{cf})} = (151)\frac{(146)\ (147)}{(153)}$
(155)	R_f (hot)	HOT FIELD RESISTANCE - Calculated at $X_f{}^oC$ (103) $R_f\ (hot) = (\rho_{f\ hot})\frac{(N_f)(\ell_{tf})}{(a_{cf})} = (152)\frac{(146)\ (147)}{(153)}$
(156)	--	WEIGHT OF FIELD COIL in lbs. #'s of copper $= .321(N_f)(\ell_{tf})(a_{cf})$ $= .321(146)(6)(147)(153)$

(157)	--	WEIGHT OF ROTOR IRON - Because of the large number of different pole shapes, one standard formula cannot be used for calculating rotor iron weight. Therefore, the computer will not calculate rotor iron weight. The space is allowed on the input sheet for record purposes only. By inserting 0. in the space allowed for rotor iron weight, the computer will show "0". on the output sheet. If the rotor iron weight is available and inserted on input sheet, then the output sheet will show this same weight on the output sheet.
(160)	X_F	FIELD LEAKAGE REACTANCE $$X_F = (X_{ad})\left[1 - \frac{\left[(C_1)/(C_m)\right]}{2(C_p) + \frac{4(\lambda F)}{\pi(\lambda_a)}}\right]$$ $$= (81)\left[1 - \frac{\left[(71)/(74)\right]}{2(73) + \frac{4(160c)}{\pi(70)}}\right]$$
(160a)	P_e	ROTOR LEAKAGE PERMEANCE $$P_e = P\left[P_1 + P_2 + P_3 + P_4\right] + P_5$$ $$= (6)\left[(80) + (81) + (82) + (83)\right] + (84)$$
(160c)	λ_F	ROTOR LEAKAGE PERMEANCE per inch of stator stack $$\lambda_F = \frac{P_e}{\ell} = \frac{(160a)}{(13)}$$
(161)	L_f	FIELD SELF INDUCTANCE $$L_f = (N_f)^2 (\ell_p)\left[(C_P)(\lambda_a)\frac{\pi}{2} + (\lambda_f)\right] \times 10^{-8}$$ $$= (99)^2 (76)\left[(73)(70)\frac{\pi}{2} + (160c)\right] \times 10^{-8}$$

(166) X'_{du} UNSATURATED TRANSIENT REACTANCE

$$X'_{du} = (X_\ell) + (X_f) = (130) + (160)$$

(167) X'_d SATURATED TRANSIENT REACTANCE

$$X'_d = .88(X'_{du}) = .88(166)$$

(168) X''_d SUBTRANSIENT REACTANCE in direct axis

$$X''_d = (X'_d) = (167)$$

(169) X''_q SUBTRANSIENT REACTANCE in quadrature axis

$$X''_q = (X_q) = (134)$$

(170) X_2 NEGATIVE SEQUENCE REACTANCE - The reactance due to the field which rotates at synchronous speed in a direction opposite to that of the rotor.

$$X_2 = .5\left[X''_d + X''_q\right] = .5\left[(168) + (169)\right]$$

(172) X_0 ZERO SEQUENCE REACTANCE - The reactance drop across any one phase (star connected) for unit current in each of the phases. The machine must be star connected for otherwise no zero sequence current can flow and the term then has no significance.

If $(28) = 0$, then $X_0 = 0$

If $(28) \neq 0$, then

$$X_o = X\left\{\frac{(K_{xo})}{(K_{x1})}\left[(\lambda_i) + (\lambda_{Bo})\right] + \frac{1.667\left[(h_1) + 3(h_3)\right]}{(m)(q)(K_p)^2(K_d)^2(b_s)} + .2(\lambda_F\right.$$

$$= (79)\left\{\frac{(173)}{(174)}\left[(62) + (123c)\right] + \frac{1.667\left[(22) + 3(22)\right]}{(5)(25)(44)^2(43)^2(22)} + .2\right.$$

(173) | K_{xo}

If (30) = 1 Then $K_{xo} = 1$

If (30) $\neq$ 1 Then $K_{xo} = \frac{3(\gamma)}{(m)(q)} - 2$

$$= \frac{3(31)}{(5)(25)} - 2$$

(174) | K_{xl}

If (30) = 1 Then $K_{x1} = 1$

If (30) $\neq$ 1 Then:

$$K_{x1} = \left[\frac{3(\gamma)}{4(m)(q)} + \frac{1}{4}\right] = \left[\frac{3(31)}{4(5)(25)} + \frac{1}{4}\right] \quad \text{If } (31a) \geq .667$$

$$K_{x1} = \left[\frac{3(\gamma)}{4(m)(q)} - \frac{1}{4}\right] = \left[\frac{3(31)}{4(5)(25)} - \frac{1}{4}\right] \quad \text{If } (31a) < .667$$

(175) | λ_{Bo}

$$\lambda_{Bo} = \frac{(K_{xo})}{(K_p)^2}\left[.07(\lambda_a)\right] = \frac{(173)}{(44)^2}\left[.07(70)\right]$$

(176) | T'_{do}

OPEN CIRCUIT TIME CONSTANT - The time constant of the field winding with the stator open circuited and with negligible external resistance and inductance in the field circuit. Field Resistance at room temperature (20^oC) is used in this calculation.

$$T'_{do} = \frac{L_F}{R_F} = \frac{(161)}{(154)}$$

(177)	T_a	ARMATURE TIME CONSTANT - Time constant of the D.C. component. In this calculation stator resistance at room temperature (20°C) is used.

$$T_a = \frac{X_2}{200\pi(f)(r_a)} = \frac{(170)}{200\pi(5a)(177)}$$

$$\text{Where} \quad r_a = \frac{(m)(I_{PH})^2(R_{SPH\ cold})}{\text{Rated KVA} \times 10^3} = \frac{(5)(8)^2(53)}{(2) \times 10^3}$$

(178)	T'_d	TRANSIENT TIME CONSTANT - The time constant of the transient reactance component of the alternating wave.

$$T'_d = \frac{(X'_d)}{(X_d)}(T'_{do}) = \frac{(167)}{(133)}(176)$$

(179)	T''_d	SUBTRANSIENT TIME CONSTANT - The time constant of the subtransient component of the alternating wave. This value has been determined empirically from tests on large machines. Use following values:

$$T''_d = .035 \text{ second at 60 cycle}$$

$$T''_d = .005 \text{ second at 400 cycle}$$

(180)	F_{SC}	SHORT CIRCUIT AMPERE TURNS - The field ampere turns required to circulate rated stator current when the stator is short circuited.

$$F_{SC} = (X_d)(F_g) = (133)(96)$$

(181)	SCR	SHORT CIRCUIT RATIO - The ratio of the field current to produce rated voltage on open circuit to the field current required to produce rated current on short circuit. Since the voltage regulation depends on the leakage reactance and the armature reaction, it is closely related to the current which the machine produces under short circuit conditions and, therefore, is directly related to the SCR. $SCR = (F_{NL})/(F_{SC}) = (127)/(180)$
(182)	I^2R_F	FIELD I^2R - at no load. The copper loss in the field winding is calculated with cold field resistance at 20^oC for no load condition. $\text{Field } I^2R = (I_{FNL})^2 \ (R_{f\ cold}) = (127a)^2 \ (154)$
(183)	F&W	FRICTION & WINDAGE LOSS - The best results are obtained by using existing data. For ratioing purposes, the loss can be assumed to vary approximately as the 5/2 power of the rotor diameter and as the 3/2 power of the RPM. When no existing data is available, the following calculation can be used for an approximate answer. Insert 0. when computer is to calculate F&W. Insert actual F&W when available. Use same value for all load conditions. $F\&W = 2.52 \times 10^{-6} \ (d_r)^{2.5} \ (\ell_p) \ (RPM)^{1.5}$ $= 2.52 \times 10^{-6} \ (11a)^{2.5} \ (76) \ (7)^{1.5}$

(184) W_{TNL} STATOR TEETH LOSS - at no load. The no load loss (W_{TNL}) consists of eddy current and hysteresis losses in the iron. For a given frequency the no load tooth loss will vary as the square of the flux density.

$$W_{TNL} = .453(b_{t\ 1/3})(Q)(\ell_s)(h_s)(K_Q)$$

$$= .453(57a)(23)(17)(22)(184)$$

$$\text{Where } K_Q = (k)\left[\frac{(B_t)}{(B)}\right]^2 = (19)\left[\frac{(91)}{(20)}\right]^2$$

(185) W_c STATOR CORE LOSS - The stator core losses are due to eddy currents and hysteresis and do not change under load conditions. For a given frequency the core loss will vary as the square of the flux density (B_c).

$$W_c = 1.42\left[(D) - (h_c)\right](h_c)(\ell_s)(K_Q)$$

$$= 1.42\left[(12) - (24)\right](24)(17)(185)$$

$$\text{Where } K_Q = (k)\left[\frac{(B_c)}{(B)}\right]^2 = (19)\left[\frac{(94)}{(20)}\right]^2$$

(186) W_{NPL} POLE FACE LOSS - at no load. The pole surface losses are due to slot ripple caused by the stator slots. They depend upon the width of the stator slot opening, the air gap, and the stator slot ripple frequency. The no load pole face loss (W_{PNL}) can be obtained from Graph 2. Graph 2 is plotted on the bases of open

slots. In order to apply this curve to partially open slots, substitute b_o for b_s. For a better understanding of Graph 2, use the following sample:

K_1 is given in Graph 2 is derived empirically and depends on lamination material and thickness. Those values given on Graph 2 have been used with success. K_1 is an input and must be specified. See Item (187) for values of K_1.

K_2 is shown as being plotted as a function of $(B_G)^{2.5}$. Also note that upper scale is to be used. Another note in the lower right hand corner of graph indicates that for a solid line (_______), the factor is read from the left scale, and for a broken or dashed line (____ _ _____ _), the right scale should be read. For example, find K_2 when B_G = 30 kilolines. First locate 30 on upper scale. Read down to the intersection of solid line plot of $K_2 = f(B_G)^{2.5}$. At this intersection read the left scale for K_2. $K_2 = .28$. Also refer to Item (188) for K_2 calculations.

K_3 is shown as a solid line plot as a function of $(F_{SLT})^{1.65}$. The note on this plot indicates that the upper scale X 10 should be used. Note F_{SLT} = slot frequency. For an example, find K_3 when F_{SLT} = 1000. Use upper scale X 10 to locate 1000. Read down to intersection of solid line plot of $K_3 = f(F_{SLT})^{1.65}$. At this intersection read the left scale for K_3. $K_3 = 1.35$. Also refer to Item (189) for K_3 calculations.

For K_4 use same procedure as outlined above except use lower scale. Do not confuse the dashed line in this plot with the note to use the right scale. The note does not apply in this case. Read left scale. Also refer to Item (190) for K_4 calculations.

For K_5 use bottom scale and substitute b_o for b_s when using partially closed slot. Read left scale when using solid plot. Use right scale when using dashed plot. Also refer to Item (191) for K_5 calculations.

For K_6 use the scale attached for C_1 and read K_6 from left scale. Also refer to Item (192) for K_6 calculations.

The above factors (K_2), (K_3), (K_4), (K_5), (K_6) can also be calculated as shown in (188), (189), (190), (191), (192) respectively.

$$W_{PNL} = \pi(d)(\ell)(K_1)(K_2)(K_3)(K_4)(K_5)(K_6)$$

$$= \pi(11)(13)(187)(188)(189)(180)(199)(192)$$

(187) | K_1 | K_1 is derived empirically and depends on lamination material and thickness. The values used successfully for K_1 are shown on Graph 2. They are:

$K_1 = 1.17$ for .028 lam thickness, low carbon steel

$= 1.75$ for .063 lam thickness, low carbon steel

$= 3.5$ for .125 lam thickness, low carbon steel

$= 7.0$ for solid core

K_1 is an input and must be specified on input sheet.

(188) K_2 — K_2 can be obtained from Graph 2 (see Item 186 for explanation of Graph 2) or it can be calculated as follows:

$$K_2 = f(B_G) = 6.1 \times 10^{-5} (B_G)^{2.5}$$

$$= 6.1 \times 10^{-5} (95)^{2.5}$$

(189) K_3 — K_3 can be obtained from Graph 2 (see Item 186 for explanation of Graph 2) or it can be calculated as follows:

$$K_3 = f(F_{SLT}) = 1.5147 \times 10^{-5} (F_{SLT})^{1.65}$$

$$= 1.5147 \times 10^{-5} (189)^{1.65}$$

$$\text{Where } F_{SLT} = \frac{(RPM)}{60} (Q)$$

$$= \frac{(7)}{60} (23)$$

(190) K_4 — K_4 can be obtained from Graph 2 (see Item 186 for explanation of Graph 2) or it can be calculated as follows:

For $\gamma_s \leq .9$

$$K_4 = f(\gamma_s) = .81(\gamma_s)^{1.285}$$

$$= .81(26)^{1.285}$$

For $.9 \leqq \tau_s \leqq 2.0$

$$K_4 = f(\tau_s) = .79(\tau_s)^{1.145}$$

$$= .79(26)^{1.145}$$

For $\tau_s > 2.0$

$$K_4 = f(\tau_s) = .92(\tau_s)^{.79}$$

$$= .92(26)^{.79}$$

(191)	K_5	K_5 can be obtained from Graph 2 (see item 186 for explanation of Graph 2) or it can be calculated as follows:

For $(b_s)/(g) = 1.7$

$$K_5 = f(b_s/g) = .3\left[(b_s)/(g)\right]^{2.31}$$

$$= .3\left[(22)/(59)\right]^{2.31}$$

NOTE: For partially open slots substitute b_o for b_s in equations shown.

For $1.7 < (b_s)/(g) \leqq 3$

$$K_5 = f(b_s)/(g) = .35\left[(b_s)/(g)\right]^{2}$$

$$= .35\left[(22)/(59)\right]^{2}$$

For $3 < (b_s)/(g) \leqq 5$

$$K_5 = f(b_s)/(g) = .625\left[(b_s)/(g)\right]^{1.4}$$

$$= .625\left[(22)/(59)\right]^{1.4}$$

For $(b_s)/(g) > 5$

$$K_5 = f\,(b_s)\,/(g) = 1.38\left[(b_s)\,/\,(g)\right]^{.965}$$

$$= 1.38\left[(22)/(59)\right]^{.965}$$

(192) | K_6 | K_6 can be obtained from Graph 2 (see Item 186 for explanation of Graph 2) or it can be calculated as follows:

$$K_6 = f(C_1) = 10^{\left[.9323(C_1) - 1.60596\right]}$$

$$= 10^{\left[.9323(71) - 1.60596\right]}$$

(194) | I^2R | <u>STATOR I^2R</u> - at no load. This item = 0. Refer to Item (245) for 100% load stator I^2R.

(195) | -- | <u>EDDY LOSS</u> - at no load. This item = 0. Refer to Item (246) for 100% load eddy loss.

(196) | -- | <u>TOTAL LOSSES</u> - at no load. Sum of all losses.

Total losses = (Field I^2R) + (F&W) + (Stator Teeth Loss) + (Stator Core Loss) + (Pole Face Loss)

= (182) + (183) + (184) + (185) + (186)

NOTE: The output sheet shows the next items to be: (Rating), (Rating + Losses), (% Losses), (% Efficiency). These items do not apply to the no load calculation since the rating is zero. Refer to Items (175), (176), (177), (178) for these calculations under load.

The no load calculations should all be repeated now for 100% load.

(196a) $\phi_{\ell\ell}$ ROTOR LEAKAGE FLUX PER POLE at 100% load

$$\phi_{\ell\ell} = \phi_\ell \left\{ \frac{(e_d)(F_g) + \left[1 + \cos(\theta)\right](F_T) + (F_C)}{(F_g) + (F_T) + (F_C)} \right\}$$

$$= (100) \left\{ \frac{(198)(96) + \left[1 + \cos(198a)\right](97) + (98)}{(96) + (97) + (98)} \right\}$$

(198) e_d Where $e_d = \cos\epsilon + (X_d) \sin\psi$

$$= \cos(198a) + (83) \sin(198b)$$

(198a) θ Where $\theta = \cos^{-1}\left[(\text{Power Factor})\right]$

$$= \cos^{-1}\left[(9)\right]$$

Where $\psi = \tan^{-1}\left[\frac{\sin(\theta) + (X_q)/(100)}{\cos(\theta)}\right]$

$$= \tan^{-1}\left[\frac{\sin(198a) + (134)/(100)}{\cos(198a)}\right]$$

Where $\epsilon = \psi - \theta = (198a) - (198a)$

(207) ϕ_{7L} STATOR TO ROTOR FLUX LEAKAGE at full load

$$\phi_{7L} = P_7 \frac{\left[2(F_c) + 2(F_T)\left[1 + \cos(\theta)\right] + (F_{g2L}) + (F_{g3L}) + (F_{p0L}) + (F_{piL})\right]}{2} \times 10^{-3}$$

$$= (86) \frac{\left[2(98) + 2(97)\left[(1 + \cos(198a)\right] + (225) + (231) + (222a) + (222c)\right]}{2} \times 10^{-3}$$

(213) ϕ_{PL} FLUX PER POLE at 100% load

For P.F. 0 to .95

$$\phi_{PL} = (\phi_P)\left[(e_d) - \frac{.93(X_{ad})}{100} \sin(\psi)\right]$$

$$= (92)\left[(198a) - \frac{.93(131)}{100} \sin(198a)\right]$$

For P. F. .95 to 1.0

$$\phi_{PL} = (\phi_P)(K_c) = (126)(9a)$$

(213a) ϕ_{PTL} TOTAL FLUX PER POLE at 100% load

$$\phi_{PTL} = \phi_{PL} + \frac{\phi_{g\ell}}{P} = (213) + \frac{(196a)}{(6)}$$

(221) ϕ_{g2L} AUXILIARY GAP (g_2) FLUX

$$\phi_{g2L} = (\phi_{g3L}) = (\phi_{rL}) = (\phi_{shL}) = (\phi_{pL})\frac{P}{2} + (\phi_{7L})$$

$$= (213)\frac{(6)}{2} + (207)$$

(222) B_{poL} FLUX DENSITY IN OUTER POLE at full load

$$B_{po} = \frac{\phi_{PTL}}{A_{po}} = \frac{(213a)}{(79)}$$

(222a) F_{poL} AMPERE TURN DROP THROUGH OUTER POLE at full load

$$F_{poL} = (\ell_{po}) \left[\text{NI/inch at density } (B_{poL})\right]$$

$$= (104) \left[\begin{array}{l}\text{Look up on pole magnetization curve} \\ \text{at density (222)}\end{array}\right]$$

(222b) B_{piL} FLUX DENSITY IN INNER POLE at full load

$$B_{pi} = \frac{\phi_{PTL}}{A_{pi}} = \frac{(213a)}{(79a)}$$

(222c)	F_{piL}	AMPERE TURN DROP THROUGH INNER POLE at full load
		$F_{piL} = \ell_{pi} \left[\text{NI/inch at density } (B_{piL}) \right]$
		$= (104b) \left[\text{Look up on pole magnetization curve at density (222b)} \right]$
(222d)	B_{rL}	FLUX DENSITY IN ROTATING OUTER RING at no load
		$B_{rL} = \dfrac{\phi_{rL}}{A_r} = \dfrac{(221)}{(104d)}$
(222e)	F_{rL}	AMPERE TURN DROP IN RING at full load
		$F_{rL} = (\ell_r) \left[\text{NI/inch at density } (B_r) \right]$
		$= (104e) \left[\text{Look up on ring magnetization curve at density (222d)} \right]$
(224)	B_{g2L}	FLUX DENSITY IN AUXILIARY GAP under load
		$B_{g2L} = \dfrac{\phi_{g2L}}{A_{g2}} = \dfrac{(221)}{(70)}$
(225)	F_{g2L}	AMPERE TURN DROP IN AUXILIARY GAP (g_2)
		$F_{g2L} = \dfrac{(B_{g2L})}{3.19} (g_2) \times 10^3$
		$= \dfrac{(224)}{3.19} (59a) \times 10^3$

(226)	ϕ_{5L}	<u>LEAKAGE ACROSS FIELD COIL</u>
		$\phi_{5L} = P_5 \left[2(F_c) + 2(F_T) \left[1 + \cos(\theta) \right] (F_{g2}) + (F_{g3}) + (F_{poL}) + (F_{piL}) + (F_{rL}) + (F_{shL}) \right] \times 10^{-3}$
		$= (84) \left[2(98) + 2(97) \left[1 + \cos(198a) \right] + (225) + (231) + (222a) + (222 \ldots) + (222e) + (233) + (229c) \right] \times 10^{-3}$
(229a)	ϕ_{yL}	<u>FLUX IN YOKE BACK OF COIL</u> at full load
(229b)	B_{yL}	<u>FLUX DENSITY IN YOKE BACK OF COIL</u> at full load
		$B_y = \frac{(\phi_y)}{(A_y)} = \frac{(227a)}{(126b)}$
(229c)	F_{yL}	<u>AMPERE TURN DROP IN YOKE</u> at full load
		$F_{yL} = \ell_y \left[\text{NI/inch at density } (B_{yL}) \right]$
		$= (123c)$ [Look up on yoke magnetization curve at density (229b)]
(230)	B_{g3L}	<u>GAP DENSITY IN AUXILIARY GAP (g_3)</u> at full load
		$B_{g3L} = \frac{(\phi_{g3L})}{(A_{g3})} = \frac{(221)}{(70a)}$
(231)	F_{g3L}	<u>AMPERE TURN DROP ACROSS GAP</u> at full load
		$F_{g3} = \frac{(B_{g3L})}{3.19} (g_3) \times 10^3$
		$= \frac{(230)}{3.19} (59c) \times 10^3$

(232)	B_{shL}	<u>SHAFT DENSITY</u> at full load
		$$B_{shL} = \frac{(\emptyset_{shL})}{(A_{sh})} = \frac{(221)}{(112)}$$
(233)	F_{shL}	<u>SHAFT AMPERE TURN DROP</u>
		$F_{shL} = (\ell_{sh}) \left[\text{NI/inch at density } (B_{sh}) \right]$
		$= (114) \left[\begin{array}{l} \text{Look up on shaft magnetization curve} \\ \text{at density (232)} \end{array} \right]$
(236)	F_{FL}	<u>TOTAL AMPERE TURNS</u> at full load
		$F_{FL} = 2(F_c)+2(F_T)\left[1+\cos(\theta)\right]+(F_{g2L})+(F_{g3L})+(F_{poL})+(F_{piL})+(F_{rL})+(F_{shL})+(F_{yL})$
		$= 2(98)+2(97)\left[1+\cos(198a)\right]+(225)+(231)+(222a)+(222c)+(222e)+(233)+(229c)$

(237)	I_{FFL}	FIELD CURRENT at 100% load $I_{FFL} = (F_{FL})/(N_F) = (236)/(146)$
(239)	--	CURRENT DENSITY at 100% load Current Density $= (I_{FFL})/(a_{cf}) = (237)/(153)$
(238)	E_{FFL}	FIELD VOLTS at 100% load - This calculation is made with ho field resistance at expected temperature at 100% load. Field Volts $= (I_{FFL})(R_{f\ hot}) = (237)(155)$
(241)	I^2R_{FL}	FIELD I^2R at 100% load - The copper loss in the field winding is calculated with hot field resistance at expected temperature for 100% load condition. Field $I^2R = (I_{FFL})^2(R_{f\ hot}) = (237)^2(155)$
(242)	W_{TFL}	STATOR TEETH LOSS at 100% load - The stator tooth loss under load increases over that of no load because of the parasitic fluxes caused by the ripple due to the rotor damper bar slot openings. $W_{TFL} = \left\{ 2\left[.27\frac{(X_d)}{100}\,\frac{(\%\ \text{Load})}{100} \right]^{1.8} + 1 \right\}(W_{TNL})$ $= \left\{ 2\left[.27\frac{(133)}{100}\ 1 \right]^{1.8} + 1 \right\}(148)$

(243) W_{PFL} POLE FACE LOSS at 100% load

$$W_{PFL} = \left\{\left[\frac{(K_{sc})(I_{PH})\frac{(\%\ Load)}{100}(n_s)}{(C)(F_g)}\right]^2 + 1\right\}(W_{PNL})$$

$$= \left\{\left[\frac{(242)(8)\ 1\ (30)}{(32)(96)}\right]^2 + 1\right\}(186)$$

(K_{sc}) is obtained from Graph 3

(245) I^2R_L STATOR I^2R at 100% load - The copper loss based on the D.C resistance of the winding. Calculate at the maximum expected operating temperature.

$$I^2R = (m)(I_{PH})^2\ (R_{SPH}\ hot)\ \frac{(\%\ Load)}{100}$$

$$= (5)(8)^2\ (54)\ 1$$

(246) -- EDDY LOSS - Stator I^2R loss due to skin effect

$$\text{Eddy Loss} = \left[\frac{(EF\ top) + (EF\ bot)}{2} - 1\right](\text{Stator } I^2R)$$

$$= \left[\frac{(55) - (56)}{2} - 1\right](245)$$

(247) -- TOTAL LOSSES at 100% load - sum of all losses at 100% load

Total Losses = (Field I^2R) + (F&W) + (Stator Teeth Loss)
+ (Stator Core Loss) + (Pole Face Loss)
+ (Stator I^2R) + (Eddy Loss)

= (241) + (183) + (242) + (185) + (243) + (245) + (246)

(248) -- **RATING IN KILOWATTS** at 100% load

$$\text{Rating} = 3(E_{PH})(I_{PH}) \quad - \quad (\text{P.F.}) \frac{(\%\ \text{Load})}{100} \times 10^{-3}$$

$$= 3(4)(8) \qquad (9)(1.) \times 10^{-3}$$

(249) -- **RATING AND LOSSES** $= (248) + (247) \times 10^{-3}$

(250) -- **% LOSSES** $= \left[\Sigma \text{LOSSES} \times 10^{-3} / (\text{Rating} + \Sigma \text{Losses})\right] 100$

$$= \left[(247) \times 10^{-3} / 249\right] 100$$

(251) -- **% EFFICIENCY** = 100% - % Losses

= 100% - (250)

These items can be recalculated for any load condition by simply inserting the values that correspond to the % load being calculated. The factor $\frac{(\%\ \text{Load})}{100}$ takes care of (I_{PH}) as it changes with load.

Note that values for F&W (183) and W_C (Stator Core Loss) (185) do not change with load, therefore, they can be calculated only once.

AXIAL AIR GAP LUNDELL GENERATORS

Axial air gap synchronous generators have definite design limits that allow the prediction of generator output from stator O.D. and RPM.

Discussion

When the axial air gap machine is worked to definite limits of current loading and air gap density, simply specifying the speed and the KVA output determines the diameter of the stator.

To determine the size of a specific type of generator at different speeds and ratings, a stator current loading limit and an air gap density limit should be assumed. For the determination of the size of the axial gap generators, an ampere loading of 900 ampere-conductors per inch of circumference of the stator has been used. This circumference is at the average diameter = $\frac{OD + ID}{2}$. The gap density has been fixed at 40 Kl/in^2. This density is the actual maximum of the flux wave under each pole. The equations used assume a sine wave of flux, or that the maximum fundamental of the pole flux wave is equal to the actual maximum of the flux wave. For concentric poles (square flux waves), this occurs at a pole embrace of 55%.

40 Kl/in^2 gap density and 900 amp-conductors per inch are set as the design limits.

The following discussion explains the derivation of the output equation used to determine generator sizes.

The output of a three phase generator is $KVA = \frac{I_{LL} E_{LL} \sqrt{3}}{10^3}$

The voltage is defined by -

$$E_{LL} = \frac{\phi_T \text{ (RPM) } Cw\ Ne}{60 \times 10^5}$$

See derivation elsewhere in report.

Where ϕ_T = Hypothetical total flux in air gap in Kilolines,

$$Cw = \frac{E_{LL}}{M\ Eph} \cdot \frac{C_1 K_d}{\sqrt{2}} = .39\ C_1$$

m = No. of phases

C_1 = Ratio $\frac{\text{maximum fundamental}}{\text{actual maximum}}$ of the flux wave = 1.0 for sine wave

Ne = Total effective conductors in the machine

The basic voltage equation is substituted in the output equation.

$$KVA = E_{LL}\ I_L\ \frac{\sqrt{3}}{10^3} = \frac{\phi_T\ RPM\ C_w\ Ne\ I\ \sqrt{3}}{60 \times 10^5 \times 10^3}$$

$$\phi_T = B_{Gap}\ A_{Gap} \text{ and}$$

$$I = \frac{A\pi D}{Ne} \text{ where}$$

A = Ampere wire/inch loading of stator based on the average diameter of stator

$$KVA = \frac{Bg\ (\pi D \ell_c)\ RPM\ (.39\ C_1)\ Ne\ \sqrt{3}}{60 \times 10^8}\ \frac{A\pi D}{Ne}$$

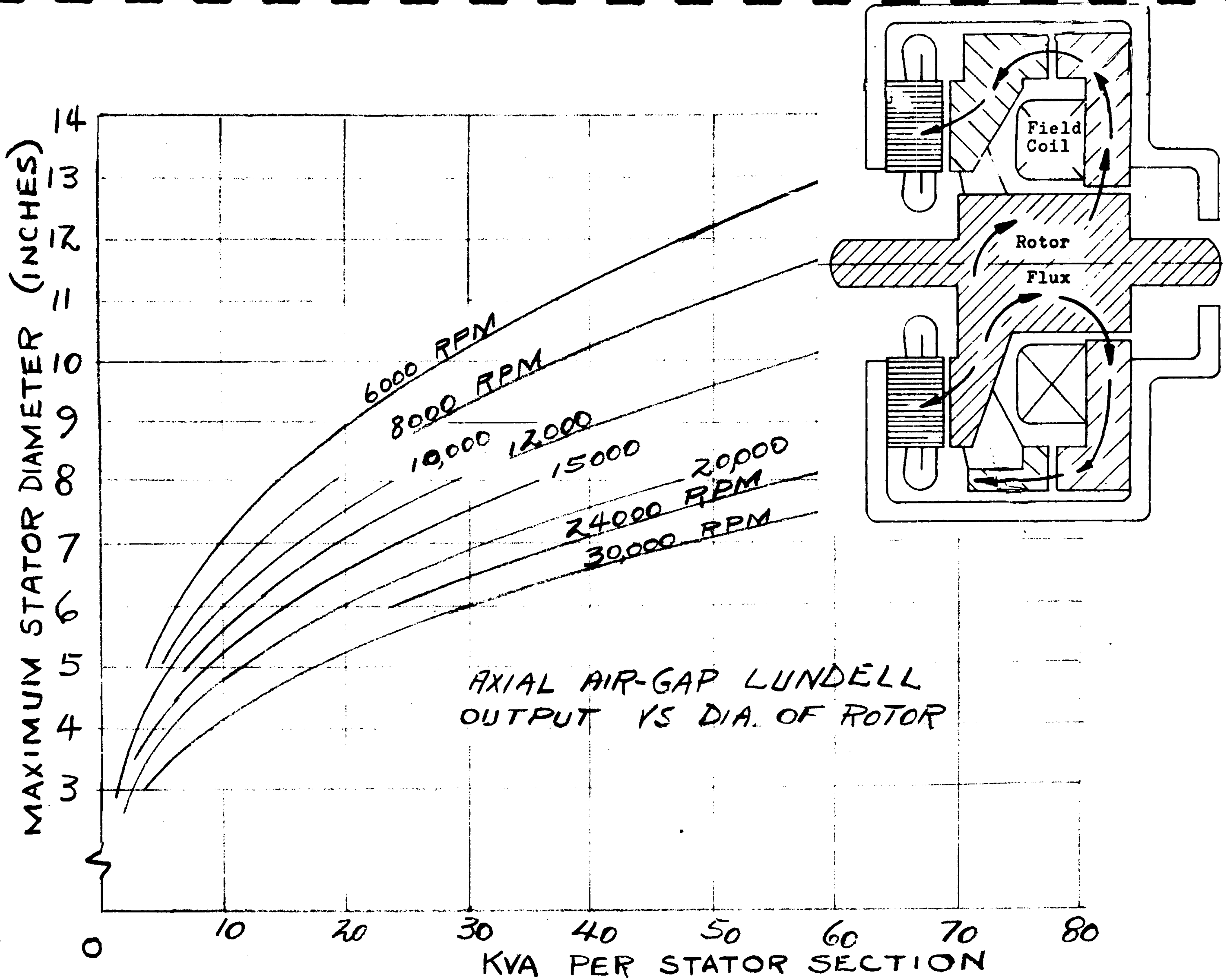
$$KVA = \frac{Bg\ 9.85\ D^2\ \ell_c\ RPM\ .39\ C_1\ A\ \sqrt{3}}{60 \times 10^8}$$

$$KVA = \frac{Bg\ D^2 \ell_c\ RPM\ A}{90 \times 10^7} \quad \text{Basic eqn.}$$

For axial air gap machines

D = average diameter of stator

$$= \frac{OD + ID}{2}$$


Field Coil
Rotor
Flux
6000 RPM
8000 RPM
10,000
12,000
15000
20000
24000 RPM
30,000 RPM
AXIAL AIR-GAP LUNDELL
OUTPUT VS DIA. OF ROTOR
MAXIMUM STATOR DIAMETER (INCHES)
14
13
12
11
10
9
8
7
6
5
4
3
0
10
20
30
40
50
60
70
80
KVA PER STATOR SECTION

For radial air gap machines

D = rotor diameter.

To allow a general treatment of the axial air gap machine, the typical design will be considered to have the following characteristics:

Stator OD = $\frac{3}{2}$ stator ID

Gap density = Bg = 40 Kl/in^2

Ampere loading = A = 900 $\frac{\text{Amp. Cond.}}{\text{in.}}$

C_1 = Field form factor = 1.0 (assume sine wave)

Then,

$$KVA = \frac{Bg\, D^2_{avg}\, \ell c\ RPM\ A}{90 \times 10^7}$$

$$D_{avg} = \frac{OD + \frac{2}{3}\,OD}{2} = \frac{5}{6}\,OD$$

$$\ell c = \frac{1}{6}\,OD$$

and,

$$KVA = 40\ \frac{25}{36}\,(OD)^2\ \frac{1}{6}\,(OD)\ \frac{RPM\ (900)}{90 \times 10^7}$$

$$KVA = \frac{4.63\ (OD)^3\ RPM}{10^6}$$

Area of stator = .437 (OD)^2

Total flux = 40 (Area of stator) = 17.5 (OD)^2 Kl.

ϕ_P = approx. $\frac{\phi_T}{\text{Poles}} \times .56$

ϕ shaft = approx. $\phi_T \times .27$

When the ID of the stator is fixed at $\frac{2}{3}$ OD, the diameter which divides the stator into two equal areas is .85 OD. The average diameter is $\frac{5}{6}$ OD or .835 OD -- a 1.75% difference. This difference will be ignored and the average diameter $\frac{5}{6}$ OD will be used as the design diameter for all calculations.

For axial air gap generators

$$KVA = \frac{4.63}{10^6} (OD)^3 \text{ RPM}$$

OD	$(OD)^3$	$\times \frac{4.63}{10^6}$	x 6000	12000
3	27	.125 (10^{-3})	.75	1.5
4	64	.296 (10^{-3})	1.77	3.54
5	125	.58 (10^{-3})	3.48	6.96
6	216	1.00 (10^{-3})	6.00	12.0
7	343	1.59 (10^{-3})	9.55	19.1
8	512	2.37 (10^{-3})	14.2	28.4
9	730	3.38 (10^{-3})	20.3	40.6
10	10^3	4.63 (10^{-3})	27.7	55.4
11	1330	6.16 (10^{-3})	37.0	74.0
12	1728	8.0 (10^{-3})	48.0	96.0
13	2197	10.15 (10^{-3})	61.0	122.0
14	2744	12.7 (10^{-3})	76.3	152.6

KVA for RPM Shown

OD	6000	8000	10,000	12,000	15,000	20,000	24,000
3	.75	1.0	1.25	1.5	1.88	2.5	3.0
4	1.77	2.36	2.95	3.54	4.42	5.9	7.08
5	3.48	4.63	5.8	6.96	8.7	11.6	13.92
6	6.00	8.0	10.0	12.0	15.0	20.0	24.0
7	9.55	12.7	15.9	19.1	23.9	31.8	38.2
8	14.2	18.9	23.6	28.4	35.5	47.2	56.8
9	20.3	27.1	33.8	40.6	50.7	67.6	81.2
10	27.7	36.9	46.2	55.4	69.2	92.4	110.8
11	37.0	49.3	61.7	74.0	92.5	123.4	148.0
12	48.0	64.0	80.0	96.0	120.0	160.0	192.0
13	61.0	81.3	102.0	122.0	153.0	204.0	244.0
14	76.3	101.5	127.0	152.6	191.0	254.0	305.2

ESTIMATING THE COIL WEIGHT AND I^2R LOSS

Based on 80% ratio of $\frac{\text{Field Copper Area}}{\text{Total Coil Area}}$ for field coils and a current density of 5000 amps/in^2, 4000 ampere turns would require one square inch of coil cross section.

Assume a square copper coil in all cases then using the coil inner diameter; d_c:

$$WT = \left[d_c + \frac{AT}{4000} \times \frac{AT}{4000}\right] .321 \text{ lbs.} \qquad (1)$$

Where d_c = coil inner dia, inches

Here the weight of coil hangers and insl. is estimated at 20% of the total.

$$\frac{I^2R\ \text{Loss}}{\text{Lb. Cu.}} = \left(\frac{\text{Amps}}{\text{In.}^2}\right)^2 \frac{\rho}{\#/\text{in}^3} = 25\ \frac{(10^6)}{.321}\ \rho(.8)$$

Where ρ = Coil resistivity in microhm inches

at 400°F and 5000 amps/in^2 in CU, use Wt. obtained in equation (1) x .8

$$I^2R\ \text{Loss} = \frac{25\ (1.17)}{.321}\ .8\ \frac{(\text{WT Coil})}{\text{Gross}} = 73\ (\text{WT.})\ \text{Watts} \quad (2)$$

@ 400°F (240°C)

ROTOR STRESSES

The speed and rating obtainable from a generator are functions of allowable rotor stresses.

The brushless generator rotors can be made into composite cylinders and preliminary stress treatment can be on the basis of a cylinder of homogeneous material.

Maximum stress in a homogeneous solid disk is -

$$\text{Max. } S_r = \text{MAX } S_t = \frac{1}{8}\ \frac{\gamma W^2}{386.4}\ (3 + v)\ R^2$$

R = disk outside radius, inches ; W = RAD/SEC

v = Poisson's Ratio = .26 for steel (general approximation)

γ = density LB/in^3 = .283 for steel ; N = REV/MIN

$$S = \frac{.283}{8}\ \frac{(2\pi)^2\ N^2}{386.4\ (3600)}\ (3.26)\ R^2 \qquad ; S = PSI$$

$$= \frac{3.26}{10^6}\ N^2\ R^2$$

A more realistic condition usually is represented by a cylinder with a hole in the center.

The maximum stress in a homogeneous circular disk or a cylinder with a small hole in the center is:

$$\text{Max } S_t = \frac{1}{4} \frac{\gamma W^2}{386.4} \left[(3 + v) R^2 + (1 - v) R_o^{\ 2} \right]$$

R = disk outside radius, inches

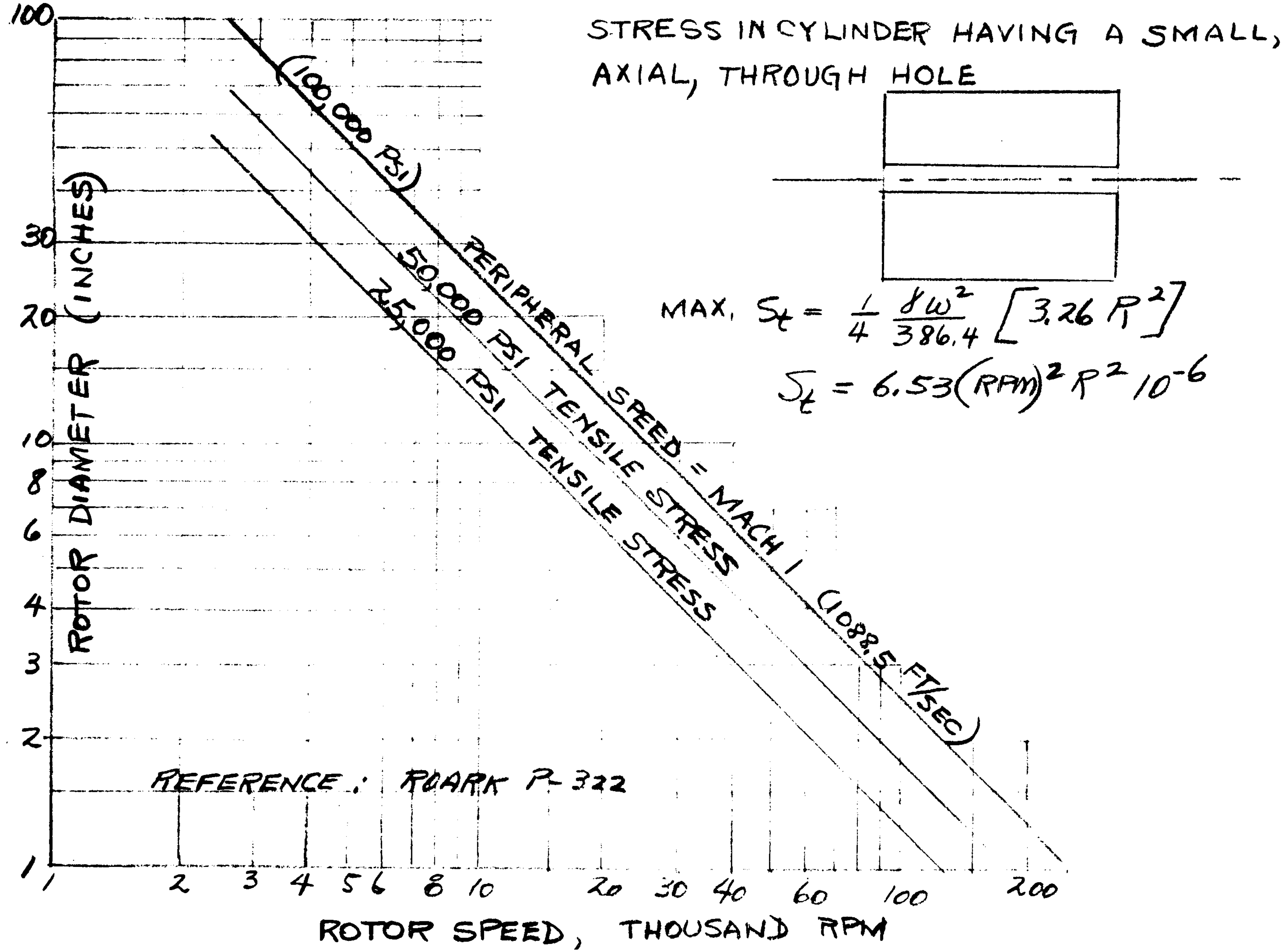
R_o = disk inside radius, inches

If R_o is small and insignificant

$$\text{Max } S_t = \frac{1}{4} \frac{\gamma W^2}{386.4} (3 + v) R^2 \quad \text{PSI}$$

This equation gives twice the stress calculated in a disk without a hole or -

$$S_t = \frac{6.52}{10^6} N^2 R^2$$


STRESS IN CYLINDER HAVING A SMALL, AXIAL, THROUGH HOLE
MAX. $S_t = \frac{1}{4}\frac{\gamma\omega^2}{386.4}\left[3.26 R_1^2\right]$
$S_t = 6.53(RPM)^2 R^2 10^{-6}$
PERIPHERAL SPEED = MACH 1 (1088.5 FT/SEC)
(100,000 PSI)
50,000 PSI TENSILE STRESS
25,000 PSI TENSILE STRESS
REFERENCE: ROARK P-322
ROTOR DIAMETER (INCHES)
ROTOR SPEED, THOUSAND RPM
100
30
20
10
8
6
4
3
2
1
1
2
3
4
5
6
8
10
20
30
40
60
100
200

EQUIVALENT CIRCUITS FOR

SYNCHRONOUS GENERATORS

Introduction

In the statement of work describing this study, an equivalent circuit is requested. The description in part reads: "The circuit and parameters chosen and evaluated should be capable of completely describing both steady state and transient performances including various overloading and short-circuit capabilities."

"Parameters for the equivalent circuit are to be derived and evaluated."

"Transfer functions and time constants are to be derived and evaluated."

"All applicable reactances are to be derived and evaluated e.g., synchronous, positive and negative sequence, transient and subtransient, direct and quadrature axes, armature, leakage, armature reaction, etc."

This section contains a derivation of an equivalent circuit submitted to satisfy the requirement for a circuit describing steady state and transient performance. This circuit also describes the performance of the generator when subjected to unbalanced loading.

The derivation of the equivalent circuit described here is an original work by Liang Liang.

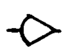
Overloading, short-circuit capabilities, time constants and reactance are derived and calculated elsewhere in the study.

The equivalent circuits themselves are on Pages 31, 33, 72 and 73. The symbols are on Pages 2 and 3. Derivations and explanations are given step-by-step.

Nomenclature

Symbol		Subscript	
τ	torque	R	resultant
ℓ	distance	α, β	reference frames
v	velocity	em	electromagnetic
F	force	d	direct axis
e	voltage	q	quadratic axis
i	current	a	armature
ψ	flux linkage	f	excitation field
ω	frequency in rad/sec	md	direct axis magnetizing component
R	resistance	mq	quadratic axis magnetizing component
L	inductance	Dd	direct axis damper bar
θ	power factor angle	Dq	quadratic axis damper bar
x	reactance	g	generator
p	power	$a\ell$	armature leakage
J	moment of inertia	$f\ell$	field leakage
D	damping factor	o	zero sequence
K	conversion constant	s	shaft
f	frequency in cycles per sec	t	terminal
P	number of poles	L	load
M	mutual inductance	a	phases
N	turns of winding	b	phases
Z	impedance	c	phases
s	Laplace operator	A	load of phase
$G(s)$	transfer function	B	load of phase b
$\Phi(s)$	power density spectrum	C	load of phase c
$\phi(t)$	correlation function	i	input

Symbol

- T - time constant
- A - amplifier
- C - capacitor
- SJ - summing junction
- t - time
- $E(s)$ - voltage
- $\mathcal{E}$ - error signal
-  - integrator
-  - operation amplifier
-  - multiplier
-  - square root
- ○ - potentiometer
-  - high gain amplifier
- $\boxed{SQ}$ - square
- $\underline{x}$ - state vector
- $\underline{m}$ - control vector
- $\underline{n}$ - disturbance vector
- A - coefficient matrix
- B - driving matrix
- Exp - exponential
- $\ln$ - natural logarithm
- S - sensitivity

Subscript

- ab - between phase a and b
- bc - between phase b and c
- ca - between phase c and a
- r - rated
- fb - feedback
- g - generator
- e - excitation
- ss - steady state

I FUNDAMENTALS

1. Assumptions

(a) Symmetrical three phase, delta or Y-connected machine with field structure symmetrical about the axis of the field winding and interpolar space.

(b) Armature phase mmf in effect, sinusoidally distributed.

(c) Magnetic and electric materials are rigidly connected.

(d) Neglect eddy current in armature iron.

(e) Neglect hysteresis effect.

(f) Neglect magnetic saturation (optional).

(g) Rotor considered as stationary reference frame.

(h) Parameters are time invariant.

2. Classical Approach

For all electric machines, the dynamic equation of Lagrange applies (in tensor):

$$\tau^{\alpha} = \frac{d}{dt}\left(\frac{\partial T_R}{\partial v^{\alpha}}\right) - \frac{\partial T_R}{\partial \ell^{\alpha}} + \frac{\partial F_{em}}{\partial v^{\alpha}} \qquad (1)$$

The stator and the rotor of the machine are considered as reference frames respectively. This holonomic expression has to be transformed into unholonomic before the two-reaction theory can be applied. That is, to choose an arbitrary frame (stator or rotor) as stationary and the other considers it as reference. Thus -

$$\tau^{\alpha} = \frac{d}{dt}\left(\frac{\partial T_R}{\partial v^{\alpha}}\right) - \frac{\partial T_R}{\partial \ell^{\alpha}} + \frac{\partial F_{em}}{\partial v^{\alpha}} + \frac{\partial T_R}{\partial v^{\alpha}} v^{\beta} Q^{\delta}_{\alpha\beta} \qquad (2)$$

$$Q^{\delta}_{\alpha\beta} = \left(\frac{\partial C^{\delta}_{k}}{\partial \ell^{n}} - \frac{\partial C^{\delta}_{n}}{\partial \ell^{k}}\right) C^{k}_{\alpha}\, C^{n}_{\beta}$$

= non-holonomic object

C^{k}_{α}, C^{n}_{β} = transformation tensors

The complexity in solving the problem directly is obvious; therefore, other approaches are used.

3. Basic Equations

By means of the two-reaction method and by choosing the rotor as the stationary reference frame, the representation of the dynamic behavior of synchronous generators can be written in set of ordinary differential equations. The reference frame is resolved into direct and quadratic axis.

Armature -

$$e_d = -R_a i_d + \frac{d}{dt}\Psi_d - \Psi_q \omega_g \qquad (3)$$

$$e_q = -R_a i_q + \frac{d}{dt}\Psi_q + \Psi_d \omega_g \qquad (4)$$

$$e_t = \sqrt{e_d^2 + e_q^2} \qquad (5)$$

$$\Psi_d = L_{md} i_f - (L_{md} + L_{al}) i_d + L_{md} i_{Dd} \qquad (6)$$

$$\Psi_q = -(L_{mq} + L_{al}) i_q + L_{mq} i_{Dq} \qquad (7)$$

Field -

$$e_f = R_f i_f + \frac{d}{dt}\Psi_f \qquad (8)$$

$$\Psi_f = (L_{md} + L_{fl}) i_f - L_{md} i_d + L_{md} i_{Dd} \qquad (9)$$

Damper bar -

$$e_{Dd} = R_{Dd} i_{Dd} + \frac{d}{dt}\Psi_{Dd} = 0 \qquad (10)$$

$$e_{Dq} = R_{Dq} i_{Dq} + \frac{d}{dt}\Psi_{Dq} = 0 \qquad (11)$$

$$\Psi_{Dd} = L_{md} i_f - L_{md} i_d + (L_{md} + L_{Dd}) i_{Dd} \qquad (12)$$

$$\Psi_{Dq} = -L_{mq} i_q + (L_{mq} + L_{Dq}) i_{Dq} \qquad (13)$$

Zero sequence -

$$e_o = -R_o i_o + \frac{d}{dt}\Psi_o \qquad (14)$$

$$\Psi_o = -L_o i_o \qquad (15)$$

Electromagnetic torque -

$$\tau_{em} = \psi_d i_q - \psi_q i_d \qquad (16)$$

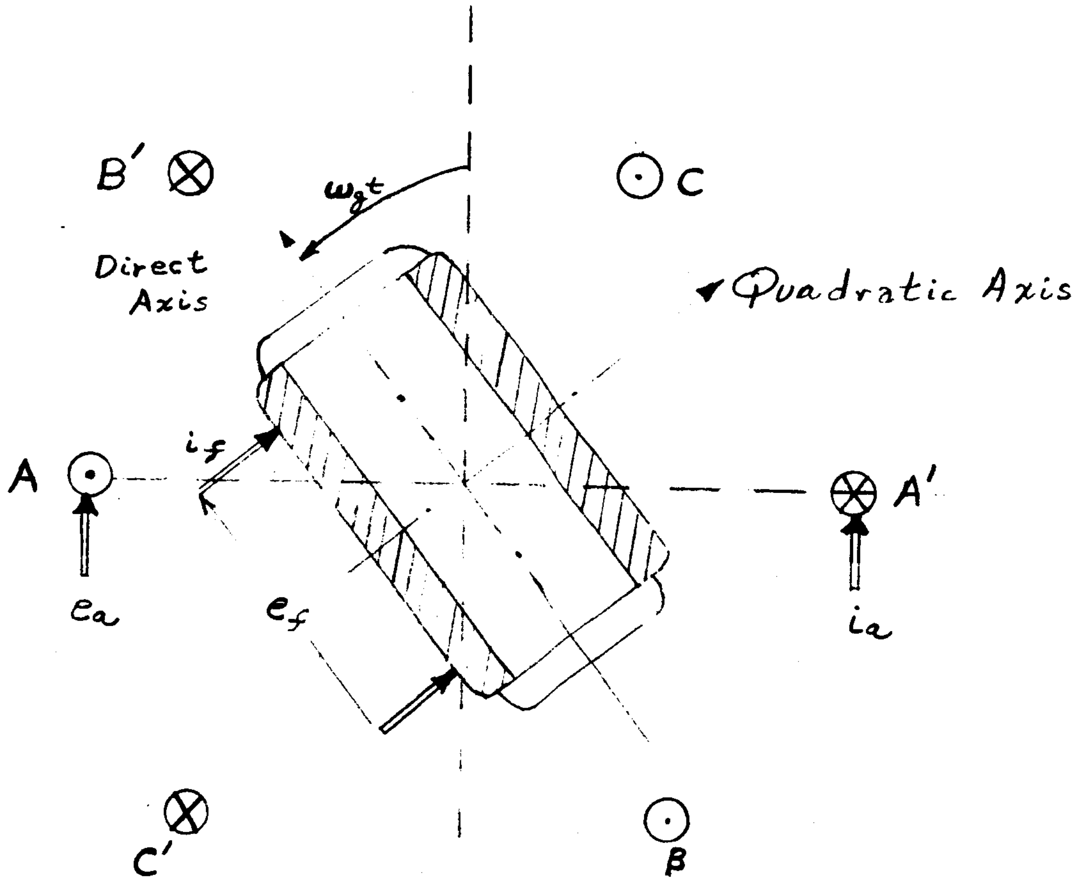


Fig. 1 Physical Arrangement

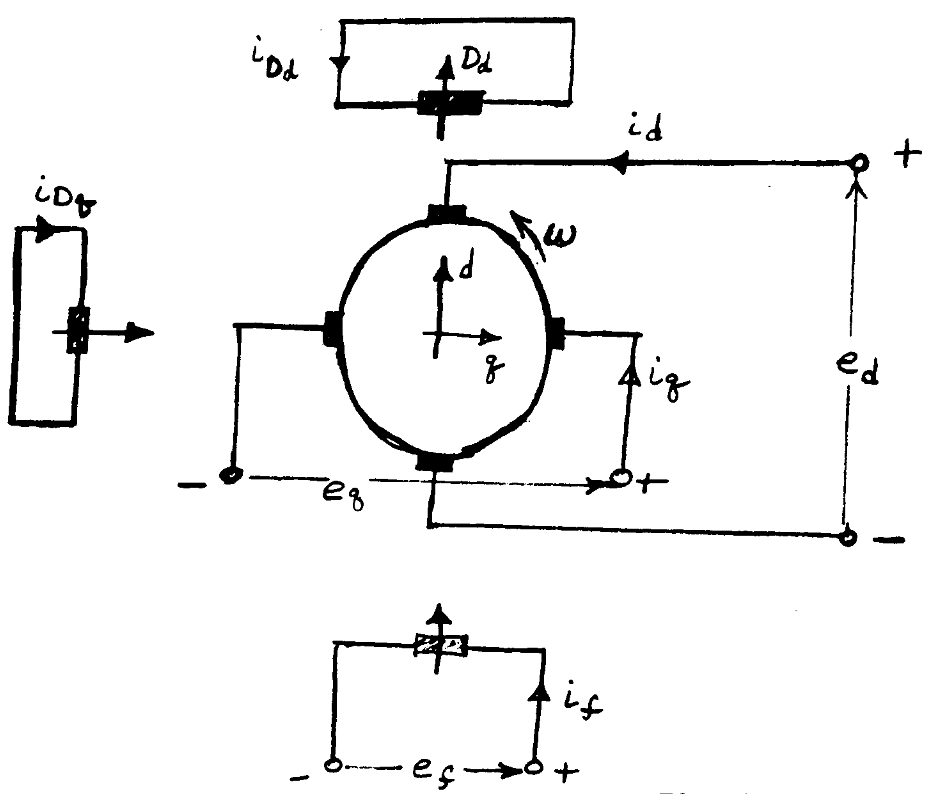
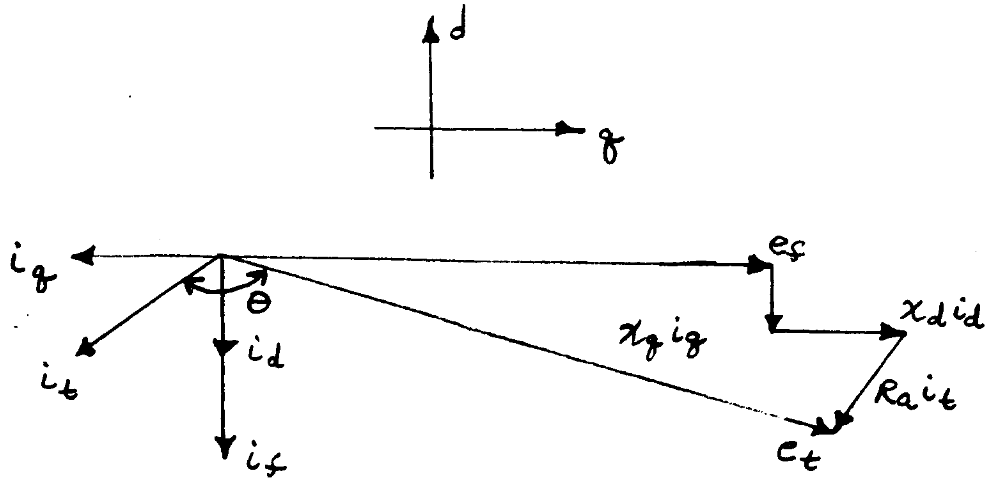
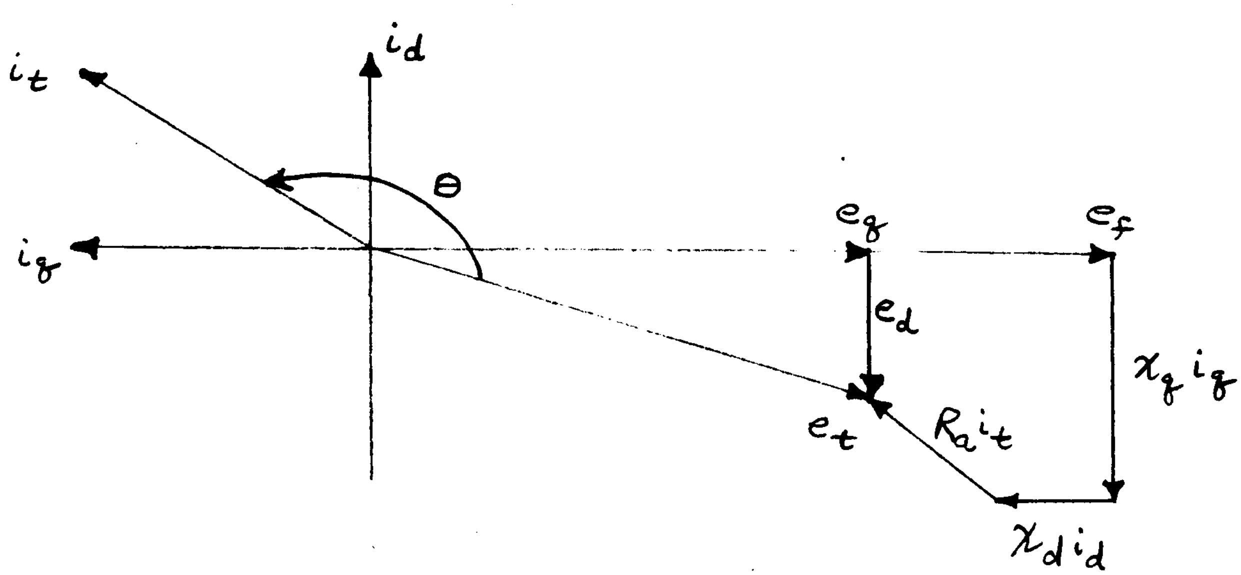


Fig. 2 Representation



(I) Under excited



(II) Over excited

Fig. 3 Steady state vector representation

4. Simplification

If detailed accuracy is not essential, it can be traded for simplification. Damper bar, armature resistances and leakage reactances have relatively small effects on voltage, current and phase relationship in a normal steady state operation. Therefore, they can be ignored.

$$e_f = R_f i_f + \frac{d}{dt}\psi_f \qquad (17)$$

$$e_d = \frac{d}{dt}\psi_d - \psi_q \omega_g \qquad (18)$$

$$e_q = \frac{d}{dt}\psi_q + \psi_d \omega_g \qquad (19)$$

$$\psi_f = L_{md}(i_f - i_d) \qquad (20)$$

$$\psi_d = \psi_f \qquad (21)$$

$$\psi_q = -L_{mq} i_q \qquad (22)$$

$$\tau_{em} = \psi_d i_q - \psi_q i_d \qquad (23)$$

Additional simplification can be made in a situation where only the steady state condition of a synchronous generator is considered in a complex system. Since all the time dependent variables become constant as the transient settles down, their rates of change approach to zero. A set of algebraic equations is derived below.

$$e_f = R_f i_f \qquad (24)$$

$$e_d = -\psi_q \omega_g \qquad (25)$$

$$e_q = \psi_d \omega_g \qquad (26)$$

$$\psi_f = L_{md}(i_f - i_d) \qquad (27)$$

$$\psi_d = \psi_f \qquad (28)$$

$$\psi_q = -L_{mq} i_q \qquad (29)$$

$$\tau_{em} = \psi_d i_q - \psi_q i_d \qquad (30)$$

5. Inputs and Outputs

(a) Most literature in discussing the synchronous generator choose the frequency ω_g and the field excitation voltage e_f as inputs and the terminal voltage which is resolved into two-axis components as outputs. They are applied to the balanced loads while the direct and quadratic currents feedback to the generator.

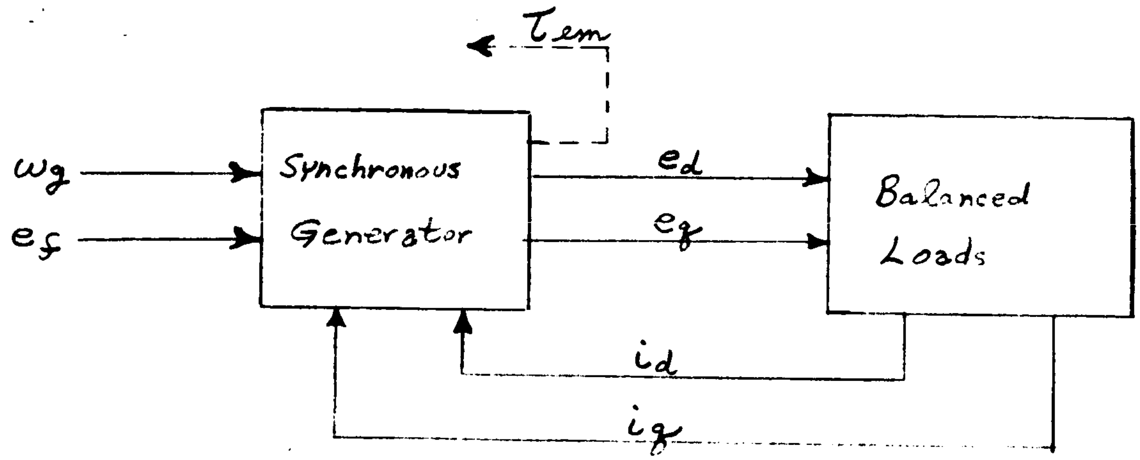


Fig. 4 Balanced load simulation

It should be recognized that the functions of e_d, e_q and i_d, i_q can be reversed.

(b) For a more detail representation, electric-mechanical relation can be included. Thus, the fluctuations of the frequency and of its dependent variables can be observed. Otherwise, the shaft speed ws has to be assumed well regulated to stand against any disturbance. Consider the shaft is rigid.

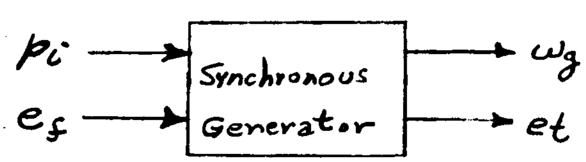


Figure 5

$$p_i = \tau_i \omega_s \quad (31)$$

$$\tau_i - \tau_{em} = J \frac{d\omega_s}{dt} + D\omega_s \quad (32)$$

$$\omega_g = \frac{P}{2} \cdot \frac{\omega_s}{60} \quad (33)$$

Where pi is the input power, τ_i input torque, and P, number of poles.

The moment of inertia J should include that of the prime mover. The damping factor D is a non-linear element which consists of mechanical losses like friction and windage. The latter is proportional to square of shaft speed ω_s.

(c) The power supply for the field excitation of a synchronous generator ideally comes from a battery. In practice, it is either from a DC generator or by means of static excitation for the purpose of regulation.

(i) The transfer function of the output voltage and the excitation voltage of a DC generator in frequency domain is

$$\frac{E_g(s)}{E_e(s)} = \frac{K_g}{R_f + L_f s} \qquad (34)$$

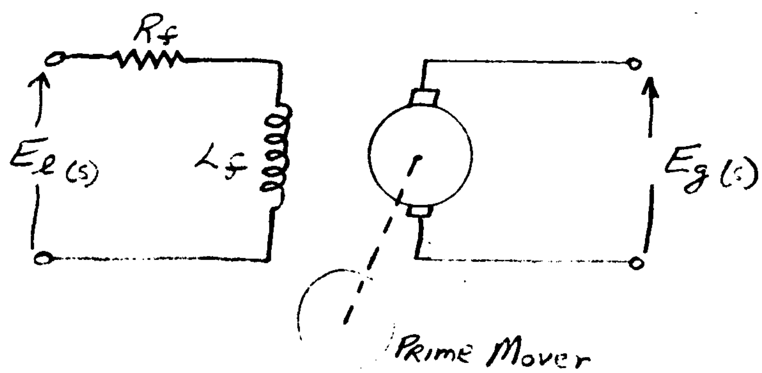


Fig. 6 DC generator

(ii) Static excitation for synchronous generator becomes widely accepted for obvious reasons like faster response and the elimination of rotating excitation machine. A typical approach is stated as follows: Excitation is provided to the generator from load currents through current transformers and rectifiers. The voltage regulator plays the role of no-load excitation and regulation of terminal voltage under different load conditions.

Such a method can be applied, for instance, for a two-coil Lundell generator with both the armature and the field stationary.

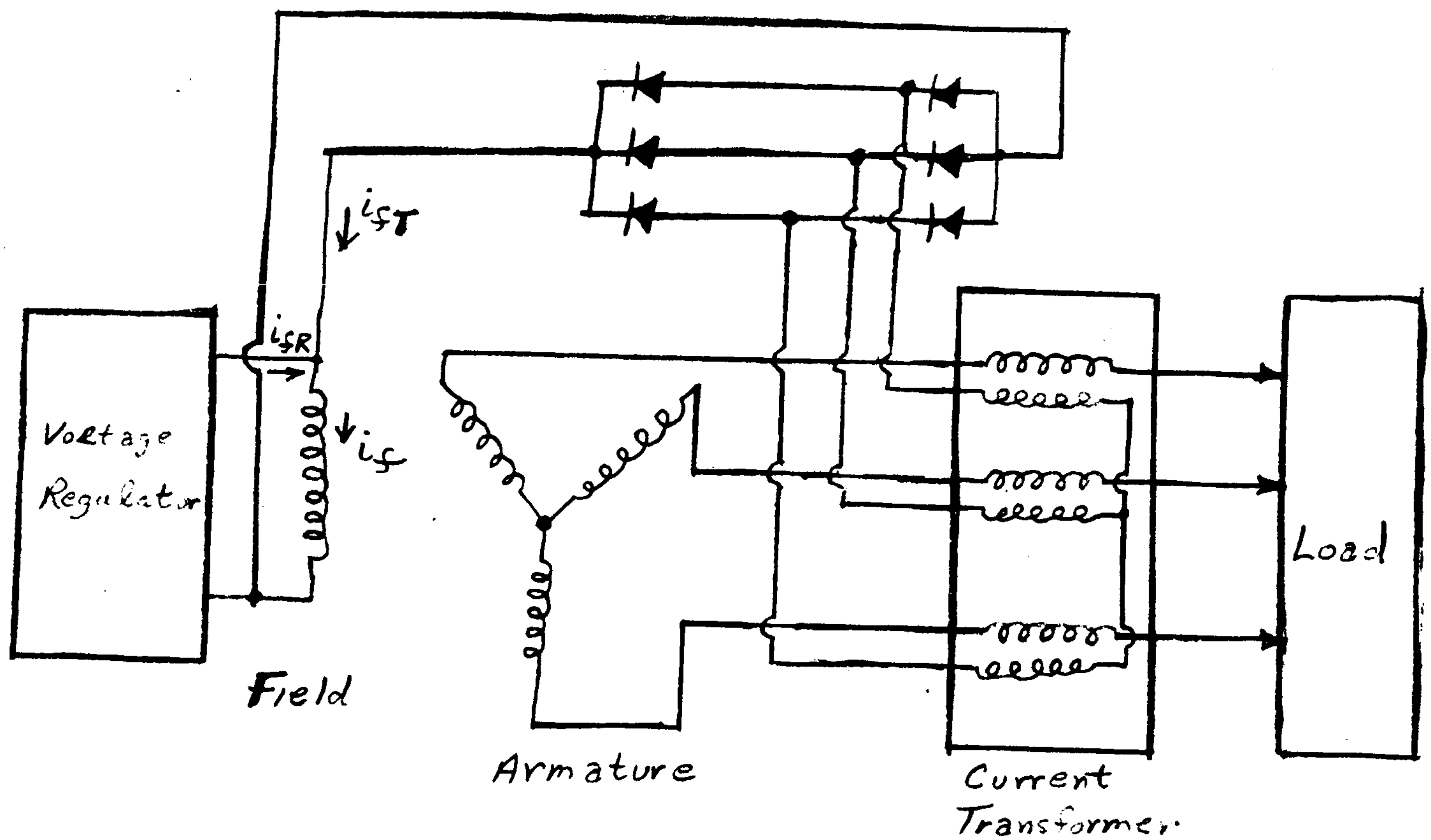


Fig. 7
<u>Static Excitation</u>

$$e_f = R_f i_f + \frac{d}{dt}\psi_f \qquad (35)$$

$$i_f = i_{fT} + i_{fR} \qquad (35a)$$

$$\psi_f = \psi_{fT} + \psi_{fR} \qquad (36)$$

$$i_{fT} = K i_L \qquad (37)$$

$$\psi_{fT} = (L_{md} + L_{f\ell})\, i_{fT} \qquad (38)$$

(d) For a detail study of synchronous generator, unbalanced load simulation is suggested. The affects of all kinds of faults due to the load can be pictured simply by adjusting the load parameters. Balanced load condition is only a special case. The major feature of an analog simulation is to convert DC representing voltages of e_d and e_q into three phase AC components e_a, e_b and e_c which are applied to the unbalanced load. The AC components i_a, i_b and i_c are converted back into DC level before feeding back to the generator. Certainly the price to pay for is complexity.

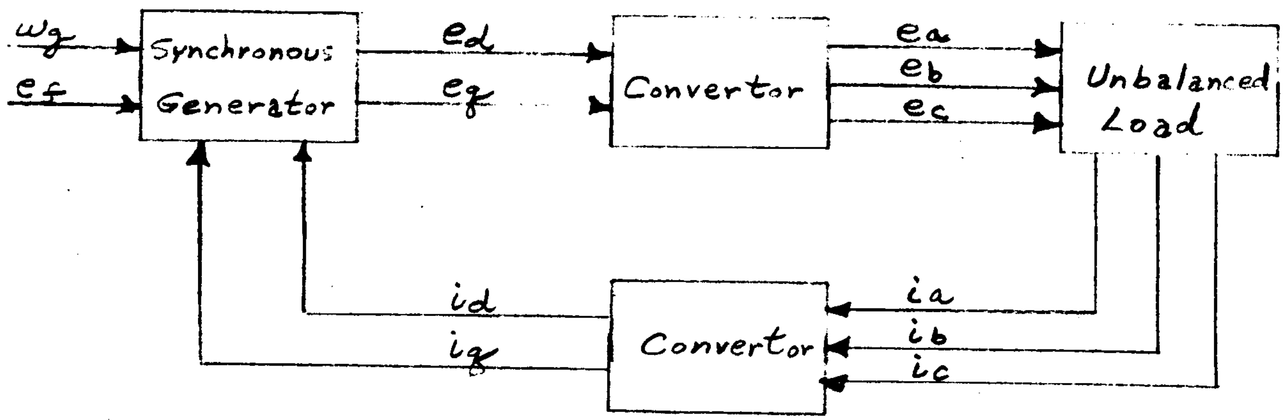


Fig. 8 Unbalanced load analog simulation

6. Conversion

For the unbalanced load simulation, the direct and quadratic output voltages of the generator have to be converted into corresponding three phases before applying to the load. Similarly, the currents from the load have to be converted back into direct and quadratic components before returning to the machine.

$$e_a = e_d \sin(\omega_g t) - e_q \cos(\omega_g t) + e_o \tag{39}$$

$$e_b = e_d \sin\left(\omega_g t - \frac{2\pi}{3}\right) - e_q \cos\left(\omega_g t - \frac{2\pi}{3}\right) + e_o \tag{40}$$

$$e_c = e_d \sin\left(\omega_g t - \frac{4\pi}{3}\right) - e_q \cos\left(\omega_g t - \frac{4\pi}{3}\right) + e_o \tag{41}$$

$$i_d = -\frac{2}{3}\left[i_a \cos(\omega_g t) + i_b \cos\left(\omega_g t - \frac{2\pi}{3}\right) + i_c \cos\left(\omega_g t - \frac{4\pi}{3}\right) \right] \tag{42}$$

$$i_q = \frac{2}{3}\left[i_a \sin(\omega_g t) + i_b \sin\left(\omega_g t - \frac{2\pi}{3}\right) + i_c \sin\left(\omega_g t - \frac{4\pi}{3}\right) \right] \tag{43}$$

$$e_o = \frac{1}{3}(e_a + e_b + e_c) \tag{44}$$

$$i_o = \frac{1}{3}(i_a + i_b + i_c) \tag{45}$$

The load is normally expressed in Y-connection. If delta load is used, proper connection of load can be made as in the analog simulation or convert them into Y-connection by using the following equations:

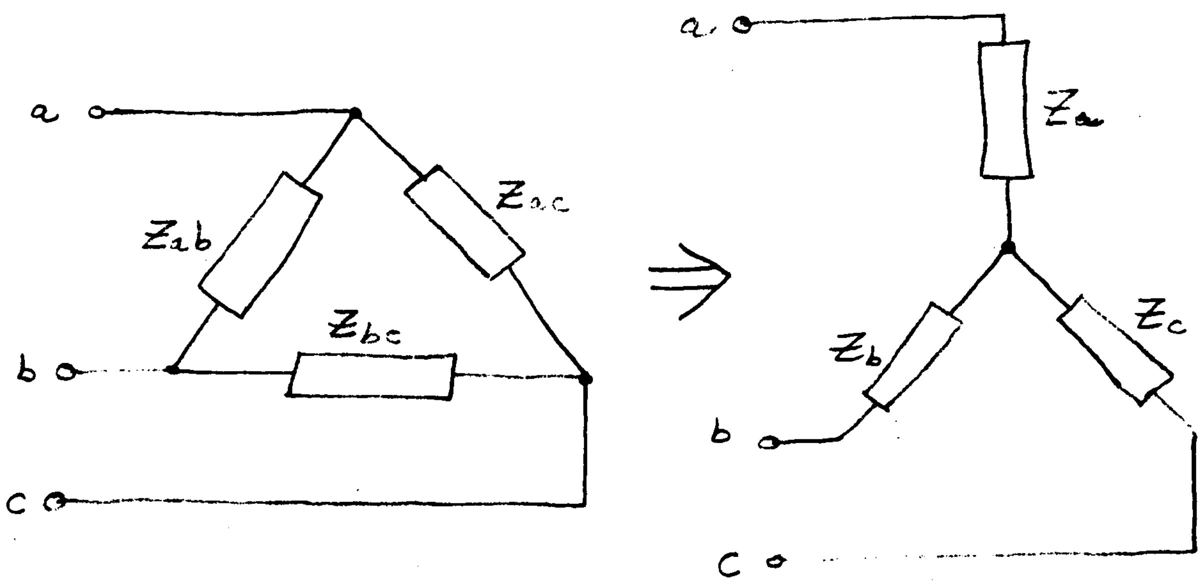


Fig. 9 Delta to Y-connection conversion

$$Z_a = \frac{Z_{ab} Z_{ac}}{Z_{ab} + Z_{bc} + Z_{ac}} \qquad (46)$$

$$Z_b = \frac{Z_{ab} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ac}} \qquad (47)$$

$$Z_c = \frac{Z_{ac} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ac}} \qquad (48)$$

7. Load

(a) Balanced load -

Balanced load can also be resolved into two-axis, direct and quadratic components. Only the resistive and inductive load are considered.

$$e_d = R_L i_d + L_L \frac{di_d}{dt} - L_L i_q \omega_g \qquad (49)$$

$$e_q = R_L i_q + L_L \frac{di_q}{dt} + L_L i_d \omega_g \qquad (50)$$

The load can be expressed in another form.

$$i_d = \frac{\cos\theta}{|Z_L|} e_d + \frac{\sin\theta}{|Z_L|} e_q \qquad (51)$$

$$i_q = \frac{-\sin\theta}{|Z_L|} e_d + \frac{\cos\theta}{|Z_L|} e_q \qquad (52)$$

$$|Z_L| = \sqrt{R_L^2 + X_L^2} \qquad (53)$$

$$\theta = \tan^{-1}(X_L/R_L) \qquad (54)$$

Thus, the load is governed by the power factor, or vice versa.

(b) Unbalanced load -

Again, only resistive and inductive load are considered. However, mutual inductances among the loads are included.

$$e_a = R_A i_a + L_A \frac{di_a}{dt} - M_{ab} \frac{di_b}{dt} - M_{ca} \frac{di_c}{dt} \qquad (55)$$

$$e_b = R_B i_b + L_B \frac{di_b}{dt} - M_{ab} \frac{di_a}{dt} - M_{bc} \frac{di_c}{dt} \qquad (55a)$$

$$e_c = R_C i_b + L_C \frac{di_c}{dt} - M_{bc} \frac{di_b}{dt} - M_{ca} \frac{di_a}{dt} \qquad (55b)$$

8. Parameter

All the machine parameters are practically time invariant. Their derivations can be found in the enclosed design manual or other standard texts on synchronous generator. Usually inductive reactance are given. To obtain the absolute inductive value, divide the reactance by the rated generator frequency. The unit of frequency should be in radians per second. The direct and the quadratic reactances computed from the design manual have taken care of whether the armature winding is Y or delta-connected as well as the number of pole pairs.

9. Time Constants

Direct-axis open-circuit transient time constant

$$T'_{do} = \frac{L_{fl} + L_{md}}{R_f} \qquad (56)$$

Direct-axis short-circuit time constant

$$T'_d = T'_{do} \, \frac{L'_d}{L_d} \qquad (56a)$$

where

$$L_d = L_{md} + L_{al} \qquad (56b)$$

$$L'_d = L_{al} + \frac{L_{md} \cdot L_{fl}}{L_{md}\, L_{fl}} \qquad (56c)$$

With external inductive load, the direct-axis short-circuit time constant is adjusted to

$$T'_{de} = T'_d \, \frac{L'_d + L_L}{L_d + L_L} \cdot \frac{L_d}{L'_d} \qquad (57)$$

There is no definite formula to compute the direct-axis short-circuit subtransient time constant. Usually it is obtained from measurement.

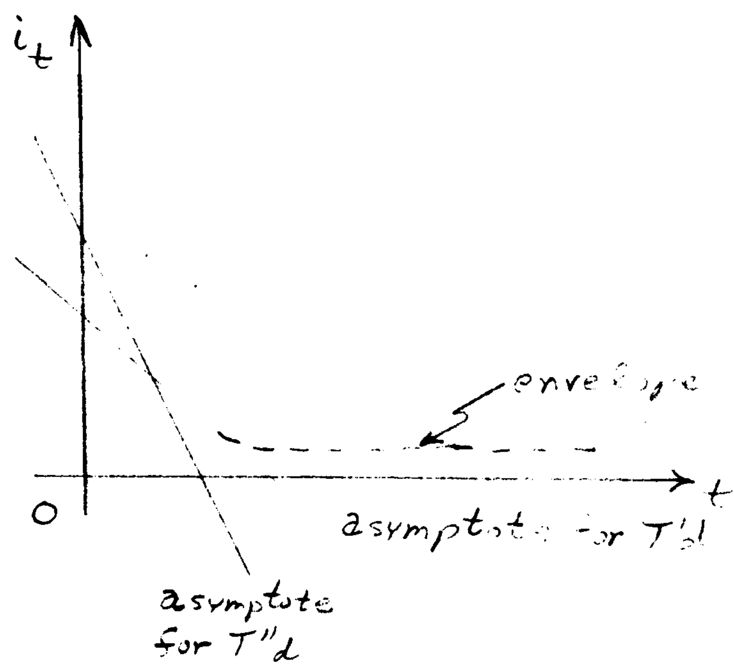


Fig. 10 Short circuit transient time constants measurement

10. Non-Linear Elements

(a) The basic equations -

$$e_d = -R_a i_d + \frac{d}{dt} \psi_d - \psi_q \omega_q \qquad (3)$$

etc., are non-linear. The non-linear term $\psi_q \omega_q$ in this equation is introduced because of the transformation from holonomic reference frames into non-holonomic and so as the other analogous.

(b) Magnetic saturation -

It is an inherent property of magnetic material. Usually for the design of generators a steel of low retentivity is used. The hysteresis loop is narrow and thus its effect can be neglected. An average saturation curve can be used to describe the characteristics of the magnetic path.

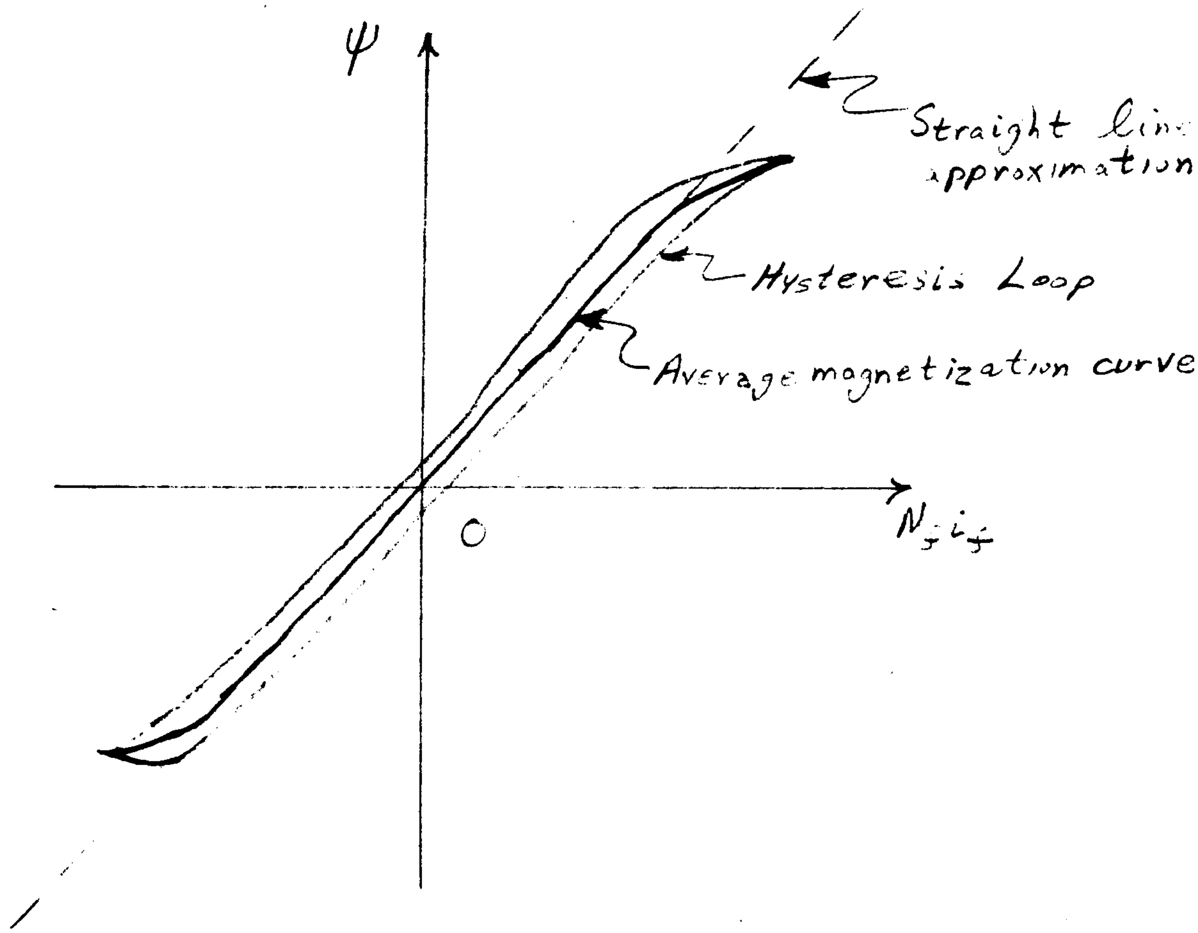


Fig. 11a Magnetization curve

The average magnetization curve can also be expressed in terms of and

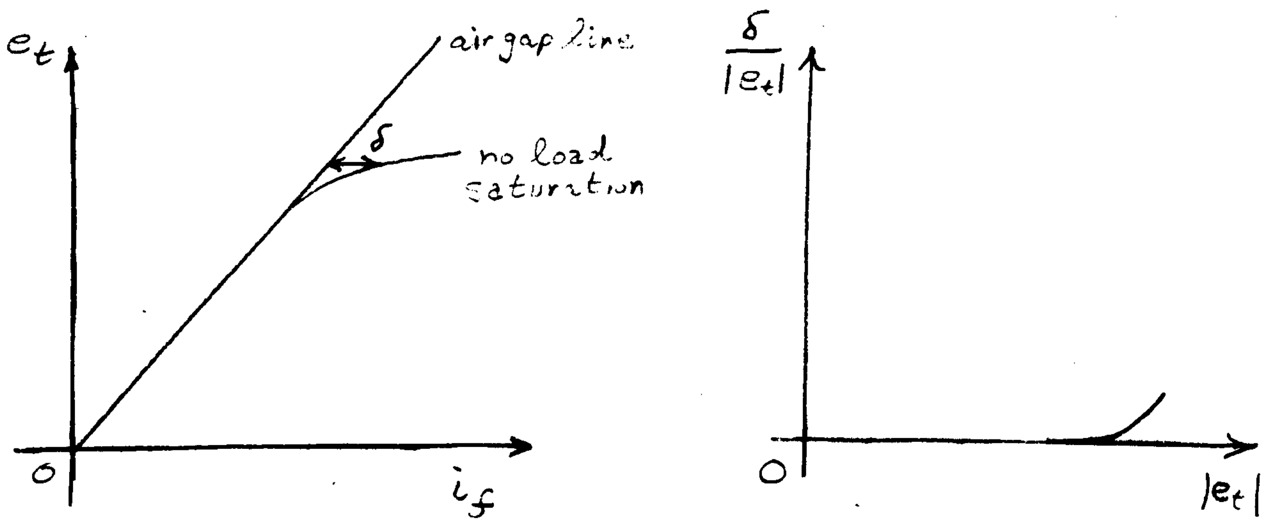


Fig. 11b Magnetic saturation approximation

The compensation δ versus the terminal voltage e_t is derived from the difference between the air gap line and the no-load saturation curve.

Further approximation can be developed by assuming X_{mq} independent or saturation (corresponds to path mostly in air). Only X_{md} varies with the flux. As the generator starts to saturate, X_{md} changes accordingly. This can be approximated by adding a factor to i_f by the amount proportional to the difference between the air gap line and the no-load saturation curve. If the operating point is below the knee of the curve, a linear relation can be assumed.

(c) Mechanical elements -

As in the more detail simulation, the mechanical relation between the prime mover and the generator is included.

(i) If gear is used for coupling, there will be backlash.

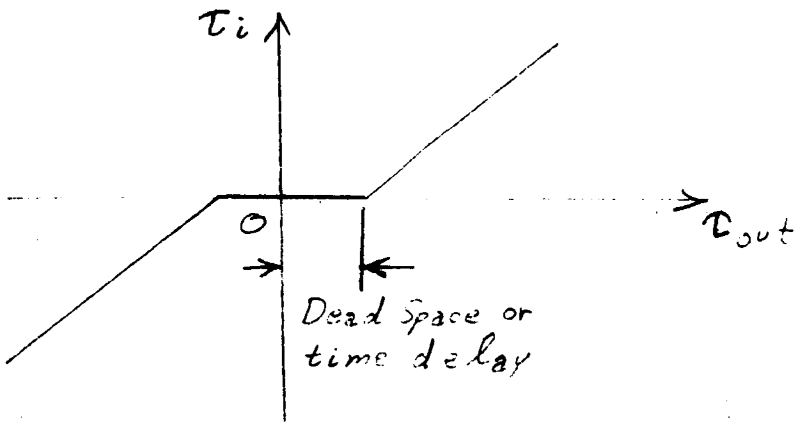


Fig. 12 Backlash

(ii) Sometimes a mechanical damper is used to eliminate the mechanical resonance near the low speed end.

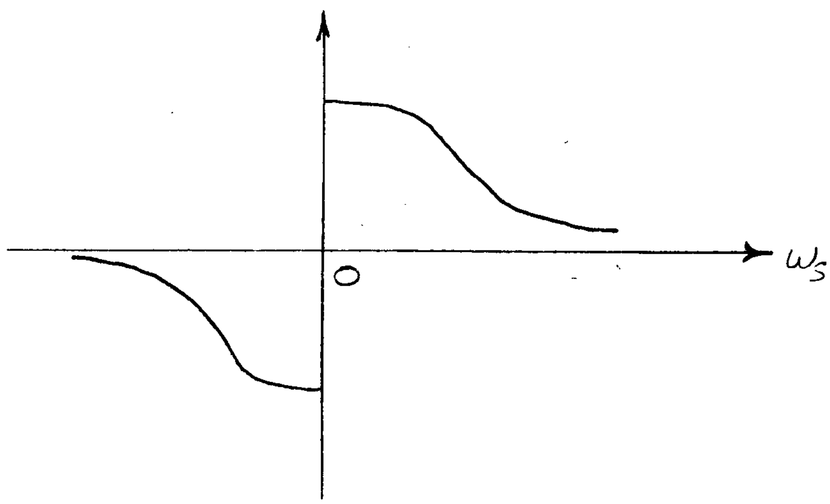


Fig. 13 Mechanical Damper *Response*

(iii) The windage and friction loss is proportional to the square of the shaft speed.

Analog simulations of

(i) Magnetic saturation - (approximated)

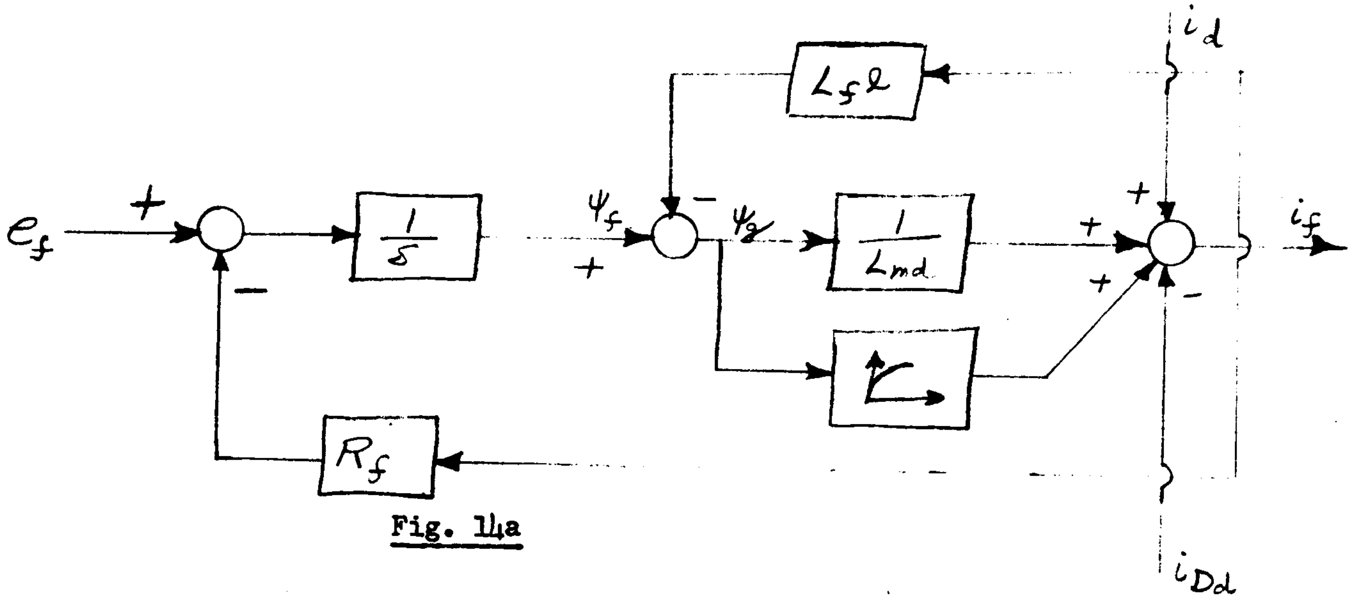


Fig. 14a

(ii) Mechanical relation -

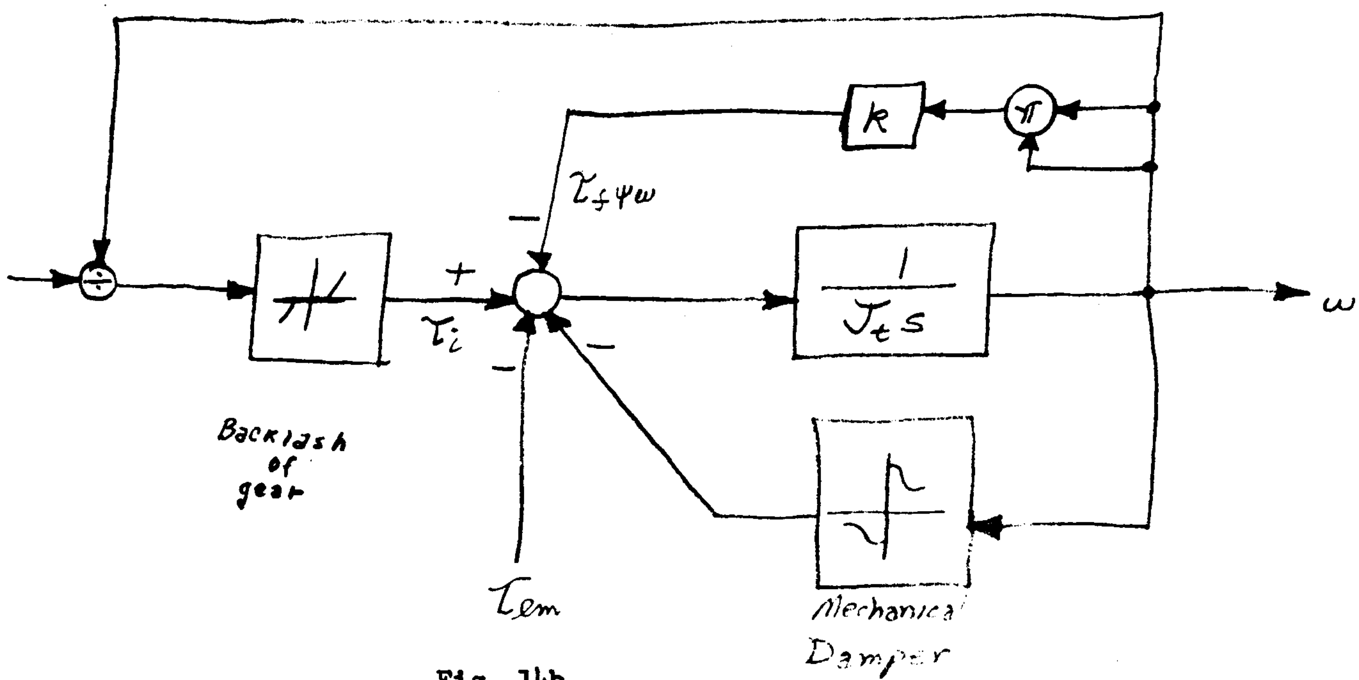


Fig. 14b

11. Linearization

In the basic equation

$$e_d = -R_a i_d + \frac{d}{dt}\psi_d - \omega_g \psi_q$$

the non-linearity is introduced by the product of two time varying functions ω_g and ψ_q. Since all the variables are continuous functions of time and are likely to be monotonic, linearization is possible. For small increment of change, the equation can be written in a linear form. (Derivation is in section II-2. $\overline{\omega}_g$, etc., are steady state values.)

$$\Delta e_d = -R_a(\Delta i_d) + \frac{d}{dt}(\Delta \psi_d) - \overline{\psi}_q(\Delta \omega_g) - \overline{\omega}_g(\Delta \psi_q) \qquad (58)$$

For constant drive generator, $\Delta \omega_g = 0$

$$\Delta e_d = -R_a(\Delta i_d) + \frac{d}{dt}(\Delta \psi_d) - \overline{\omega}_g(\Delta \psi_q) \qquad (59)$$

To compare with the original equation by setting $\omega_g = \overline{\omega}_g$, the choice of magnitude of the increments for accuracy becomes obvious. Indeed they can be simply expressed as -

$$e_d = -R_a i_d + \frac{d}{dt}\psi_d - \overline{\omega}_g \psi_q \qquad (60)$$

Another alternative is that the flux linkages are kept constant. Thus all currents are invariant. Neglecting R_a,

$$e_d = -\overline{\psi}_q \omega_g \qquad (61)$$

the voltage will be directly proportional to the generation frequency.

However, when the change $\Delta \omega_g$ and $\Delta \psi_q$ are considered simultaneously, the constraints of the increments are imposed. A larger value of increment will sacrifice the accuracy. Since a steady state value of $\overline{\omega}_g$ has been chosen as the coefficient of $\Delta \psi_q$, on the other hand, $\Delta \omega_g$ is time varying and its relatively large change will make $\overline{\omega}_g$ invalid. Similar argument applies to the term $\overline{\psi}_q(\Delta \omega_g)$ and other related equations.

The linear transfer relations are:

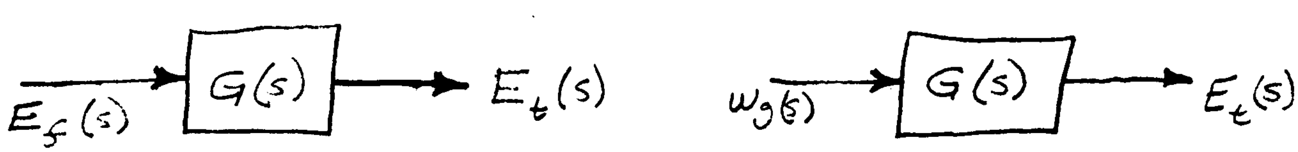


Fig. 15

(a) Constant speed

(b) Constant flux linkage

12. Per unit system

It may be convenient for some individuals to use per unit system instead of absolute value.

$$\text{Quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}} \qquad (62)$$

$$p_{base} = e_{base}\, i_{base} \qquad (63)$$

$$R_{base},\ X_{base},\ Z_{base} = \frac{e_{base}}{i_{base}} \qquad (64)$$

$$T_{base} = \frac{p_{base}}{\omega_{base}} \qquad (65)$$

13. Power density spectrum

If the input is in power density spectrum form and the generator is linearized and expressed in frequency domain as G(S) and its conjugate G(-S)

$$\Phi_{oo}(s) = G(s)\,G(-s)\,\Phi_{ii}(s) \qquad (66)$$

assuming the input and output spectrums are autocorrelated. The output can be converted into mean square value, say of e_t .

$$\overline{e_t(t)^2} = \phi_{oo}(0)$$

$$= \int_{-\infty}^{\infty} \Phi_{oo}(s)\, e^{st}\, ds \Big|_{t=0}$$

$$= \int_{-\infty}^{\infty} G(s)\,G(-s)\,\Phi_{ii}(s)\, ds \qquad (67)$$

The evaluation can be implemented analogously or by using the table of integrals which can be found in many advanced control engineering texts.

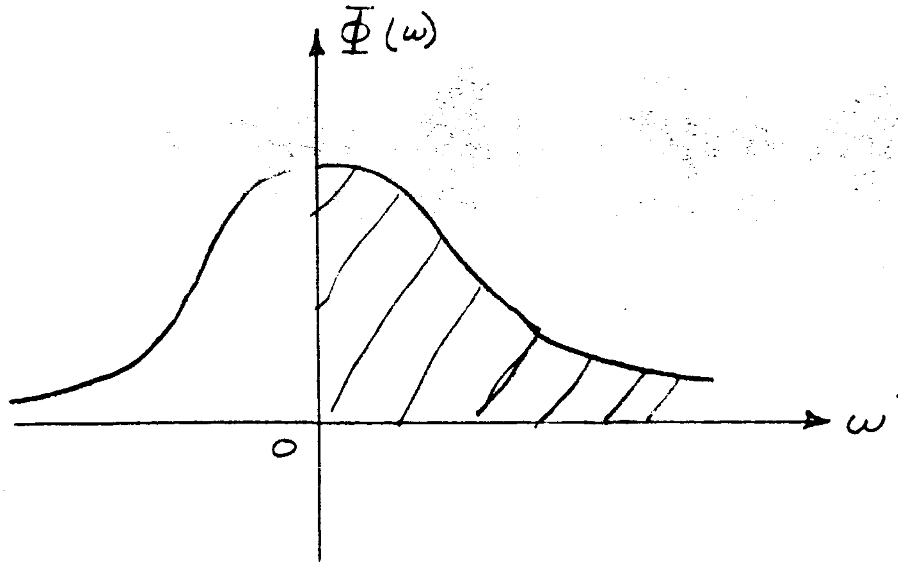


Fig. 16a Power density spectrum

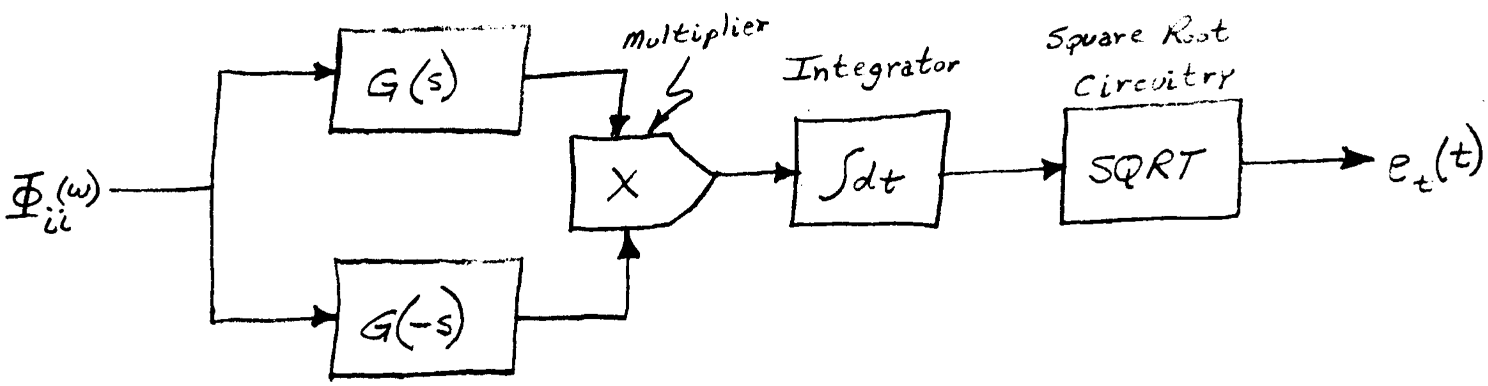


Fig. 16b Analog simulation

14. Faults

Faults are restricted to the load. They may be line to line, line to neutral, etc. In the unbalanced load simulation, the faults can be pictured simply by appropriate arrangement or by adjusting the load parameters of the corresponding phase. For example, if phase a is shorted to neutral, set $R_A = 0$; $L_A = 0$.

When it is open, theoretically R_A and L_A become infinity. In computer practice they can be set many orders larger than the normal value. Use the same tactic as in cases like $1/L_A$, while L_A is zero.

15. Converter

In the unbalanced load simulation, converters are required to generate $\cos \omega_g t$ and $\sin \omega_g t$ as functions of ω_g. (Refer to eqs. (39) - (43)) By Laplace transformation

$$\mathcal{L}\left[\cos \omega_g t\right] = \frac{s}{s^2 + \omega_g^2} = A \qquad (68a)$$

$$\mathcal{L}\left[\sin \omega_g t\right] = \frac{\omega_g}{s^2 + \omega_g^2} = B \qquad (68b)$$

$$A = \frac{B}{\omega_g} \cdot S \qquad (68c)$$

The analog simulation -

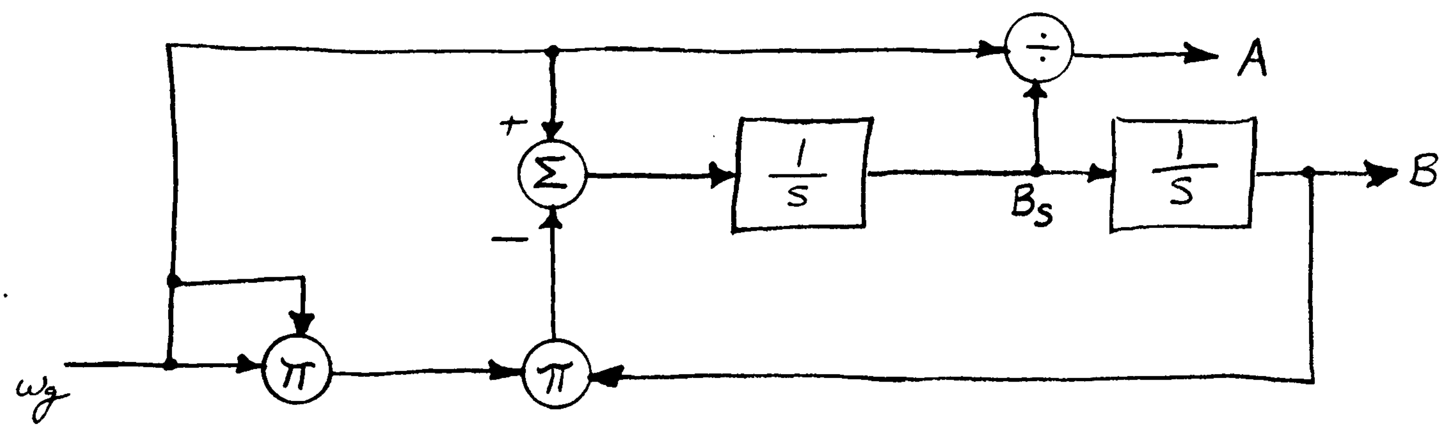


Fig. 17 DC to AC converter analog

16. Minimum Time Starting

It has been proved theoretically that switching control can achieve the minimum time for a system to reach its steady state value after a step disturbance. Due to the inherent defect of physical components like dead-band and frictions, dual-mode control is suggested. That is, the switching control takes care of large error signal while the linear control takes care of the small error signal in the feedback control loop to generate the manipulated input, say excitation voltage e_f for the synchronous generator. (Constant shaft drive)

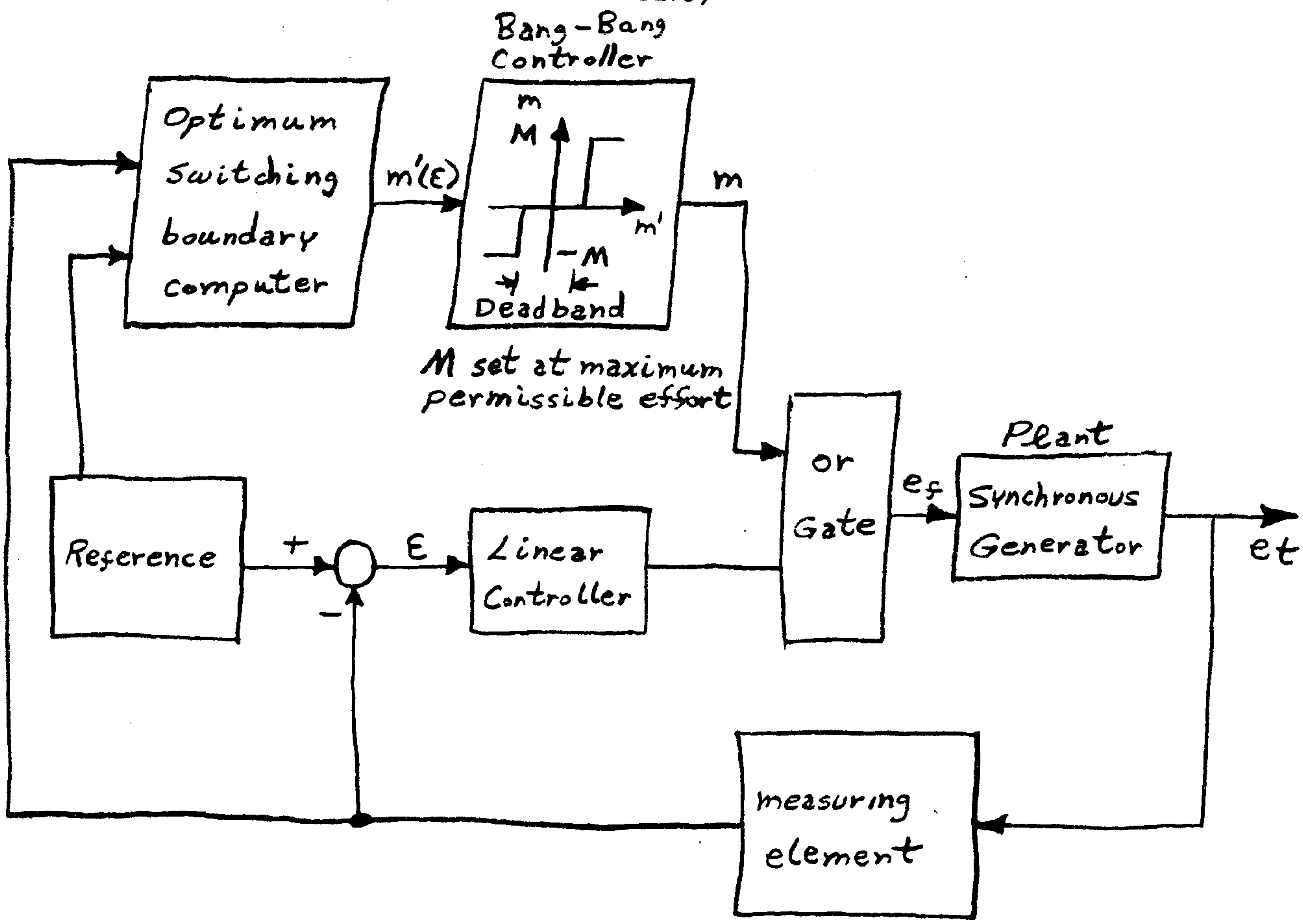


Fig. 18 Dual-mode control for minimum time starting

To start a synchronous generator, considering the shaft drive has assumed its constant speed, the reference as a step function is applied. The optimum switching boundary computer recognizes the zero initial state and the final state from the reference signal and decides the switching points according to the orders of dynamics of the plant. (For an n^{th} order linear time invariant controllable system, with poles real and non-positive, requires no more than n -1 switchings and an initial-on and a final-off operation to reach final steady state in minimum time.) When the error signal falls within the dead-band of bang-bang controller, the linear control takes over.

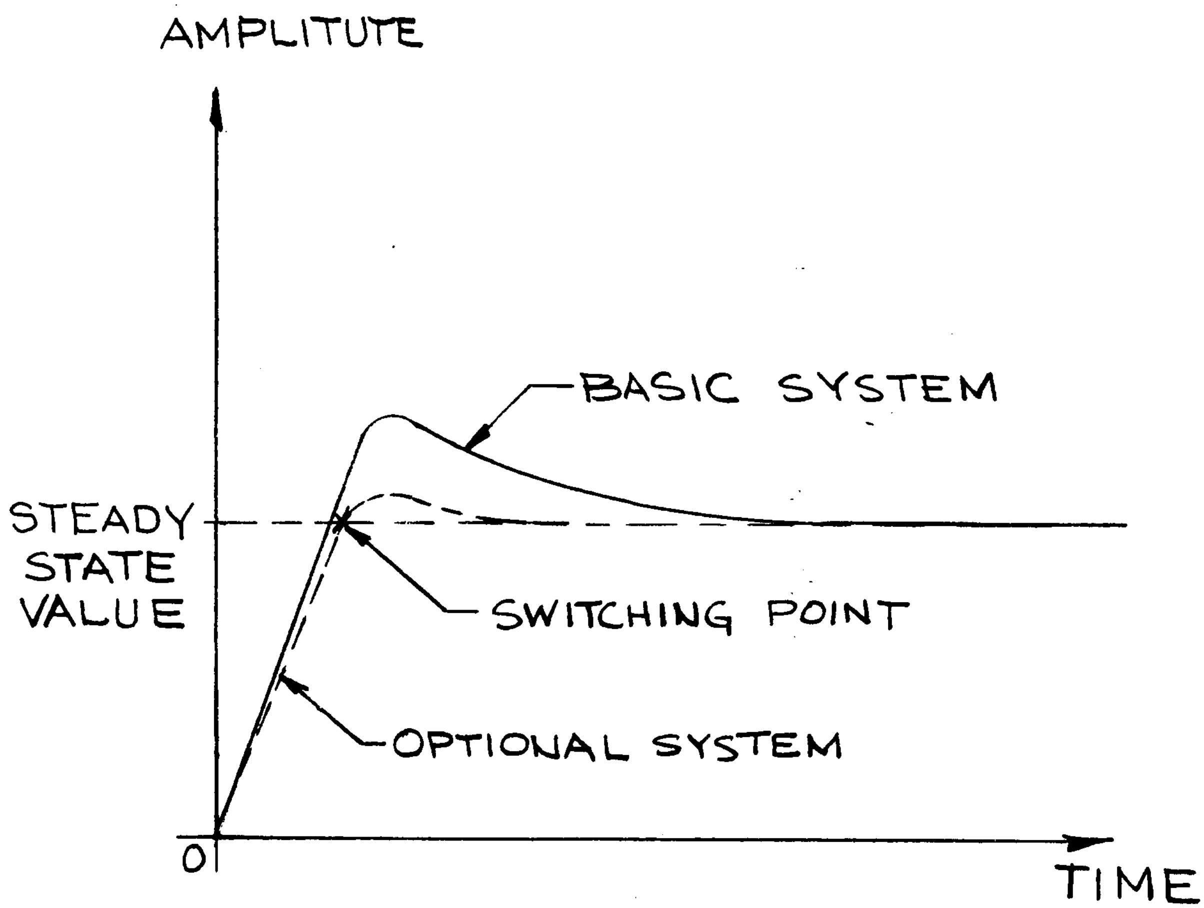


Fig. 18a   Second order system step function response

17. Modern Control Formulation

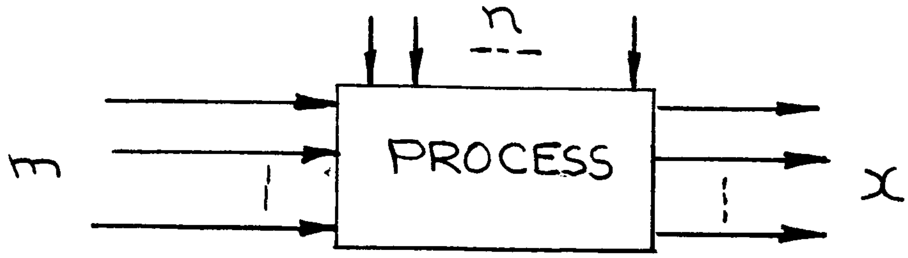


Fig. 19a Multivariable process

$\underline{x}$ - state vector

$\underline{m}$ - control vector

$\underline{n}$ - disturbance vector

For a stationary process, the dynamic characteristics are -

$$\dot{\underline{x}}(t) = A\,\underline{x}(t) + B\,\underline{m}(t) + \underline{n}(t) \tag{69a}$$

$\dot{\underline{x}}$ - differential state vector

A - coefficient matrix

B - driving matrix

The solution will be - (from initial state at time t_o to final state at t)

$$\underline{x}(t) = \text{Exp}\,A(t-t_o)\,\underline{x}(t_o) + \int_{t_o}^{t} [\text{Exp}\,A(t-\tau)][B\,\underline{m}(\tau) + \underline{n}(\tau)]\,d\tau \tag{69b}$$

Exp: Exponential

II BALANCED LOAD

1. Analog Simulation

Use the basic synchronous generator dynamic eqs. (3) to (13), (16) and balanced load eqs. (51) to (54). The operating frequency is absorbed into the reactances such as $X_{md} = \omega_r L_{md}$ where ω_r is the rated frequency. Per unit system is used. After s me manipulation, a block diagram is concluded in Fig. I where

$$T_{Dd} = \frac{X_{md} + X_{Dd}}{R_{Dd}} \qquad (71a)$$

$$T_{Dq} = \frac{X_{mq}}{R_{Dq}} \qquad (71b)$$

For the sake of convenience, Laplace operator S is used for differentiation while 1/S, for integration with initial condition, equals to zero. Magnetic saturation is approximated.

The inputs to the generator are frequency ω_q and excitation voltage e_f.

The transfer functions appear in the block diagram.

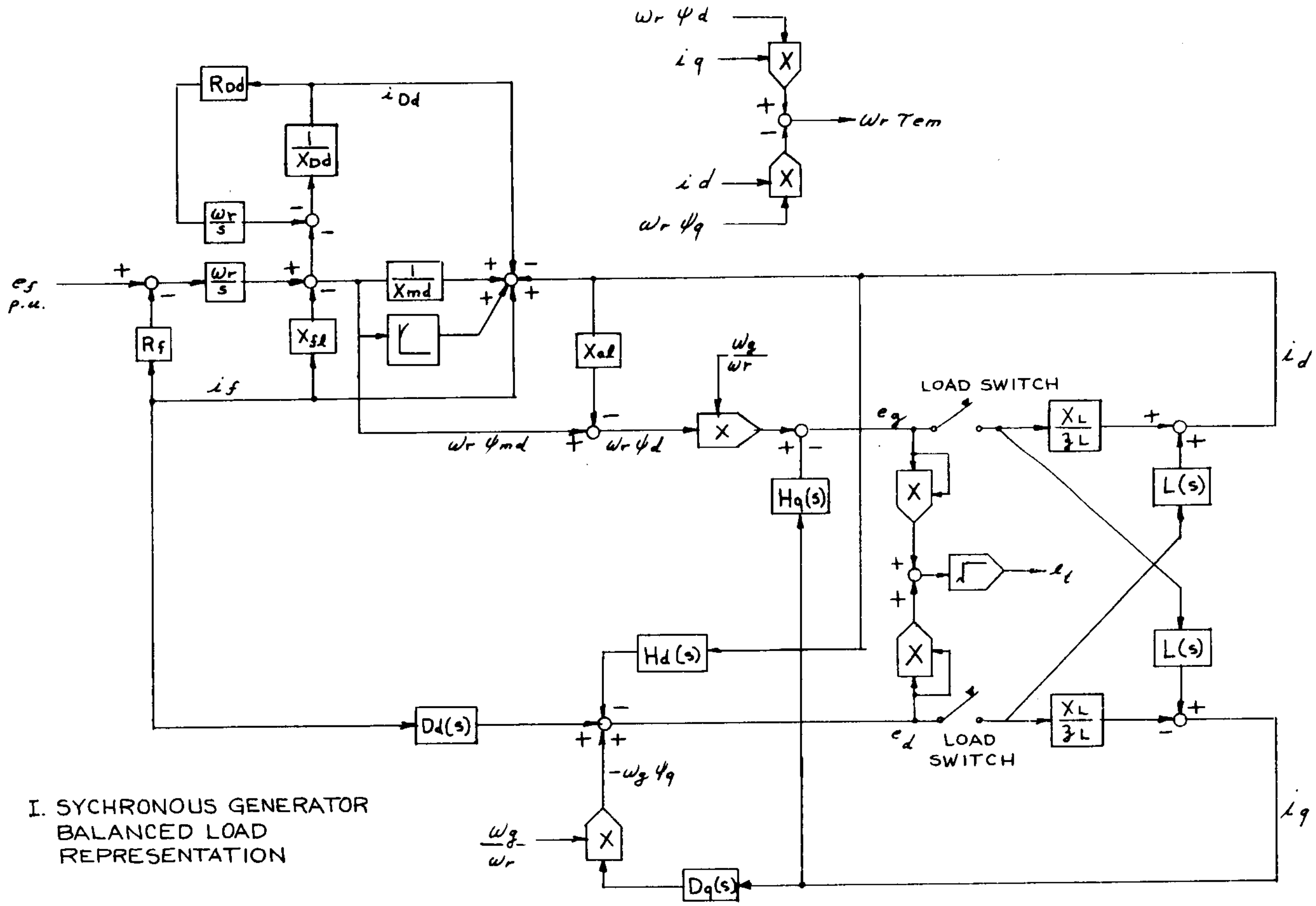
$$H_d(S) = R_a + \frac{S}{\omega_r}\left[X_{al} + \frac{X_{md}X_{Dd}}{X_{md}+X_{Dd}}\right] + \frac{X^2_{md}}{X_{md}+X_{Dd}} \cdot \frac{S/\omega_r}{1+T_{Dd}\cdot S/\omega_r} \qquad (72a)$$

$$H_q(S) = R_a + \frac{S}{\omega_r}\left[X_{al} + \frac{X_{mq}X_{Dq}}{X_{mq}+X_{Dq}}\right] + \frac{X^2_{mq}}{X_{md}+X_{Dq}} \cdot \frac{S/\omega_r}{1+T_{Dq}\cdot S/\omega_r} \qquad (72b)$$

$$D_d(S) = \frac{S}{\omega_r} \cdot \frac{X_{mq}X_{Dq}}{X_{md}+X_{Dd}} + \frac{X^2_{md}}{X_{md}+X_{Dd}} \cdot \frac{S/\omega_r}{1+T_{Dd}\cdot S/\omega_r} \qquad (72c)$$

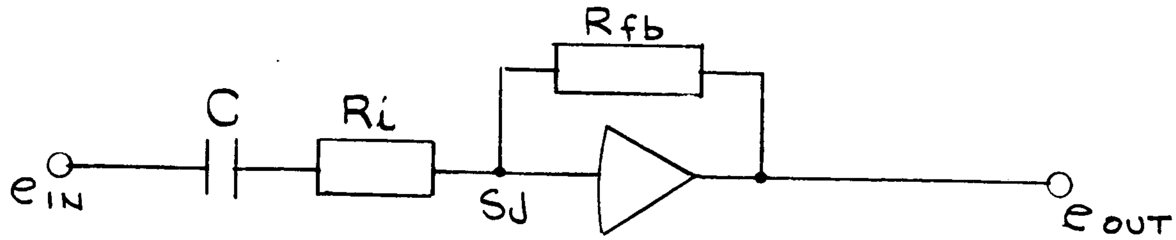
$$D_q(S) = X_{al} + \frac{X_{mq}X_{Dq}}{X_{mq}+X_{Dq}} + \frac{X^2_{mq}}{X_{mq}+X_{Dq}} \cdot \frac{1}{1+T_{Dq}\cdot S/\omega_r} \qquad (72d)$$

$$L(S) = \frac{R_L}{Z^2_L} + \frac{S}{\omega_r} \cdot \frac{X_L}{Z^2_L} \qquad (72e)$$



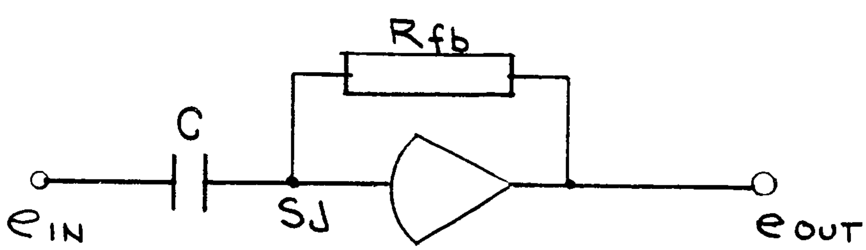
I. SYCHRONOUS GENERATOR
BALANCED LOAD
REPRESENTATION

The analog computer simulation is in Fig. II. Notice the difference between the circuit representing the first order transfer function and the differentiator.



$$\frac{e_{OUT}}{e_{IN}} = -\frac{(R_{fb}C)s}{(R_iC)s+1} \tag{73}$$

Fig. 20a First order transfer function



$$\frac{e_{OUT}}{e_{IN}} = -(R_{fb}C)s$$

Fig. 20b Differentiator

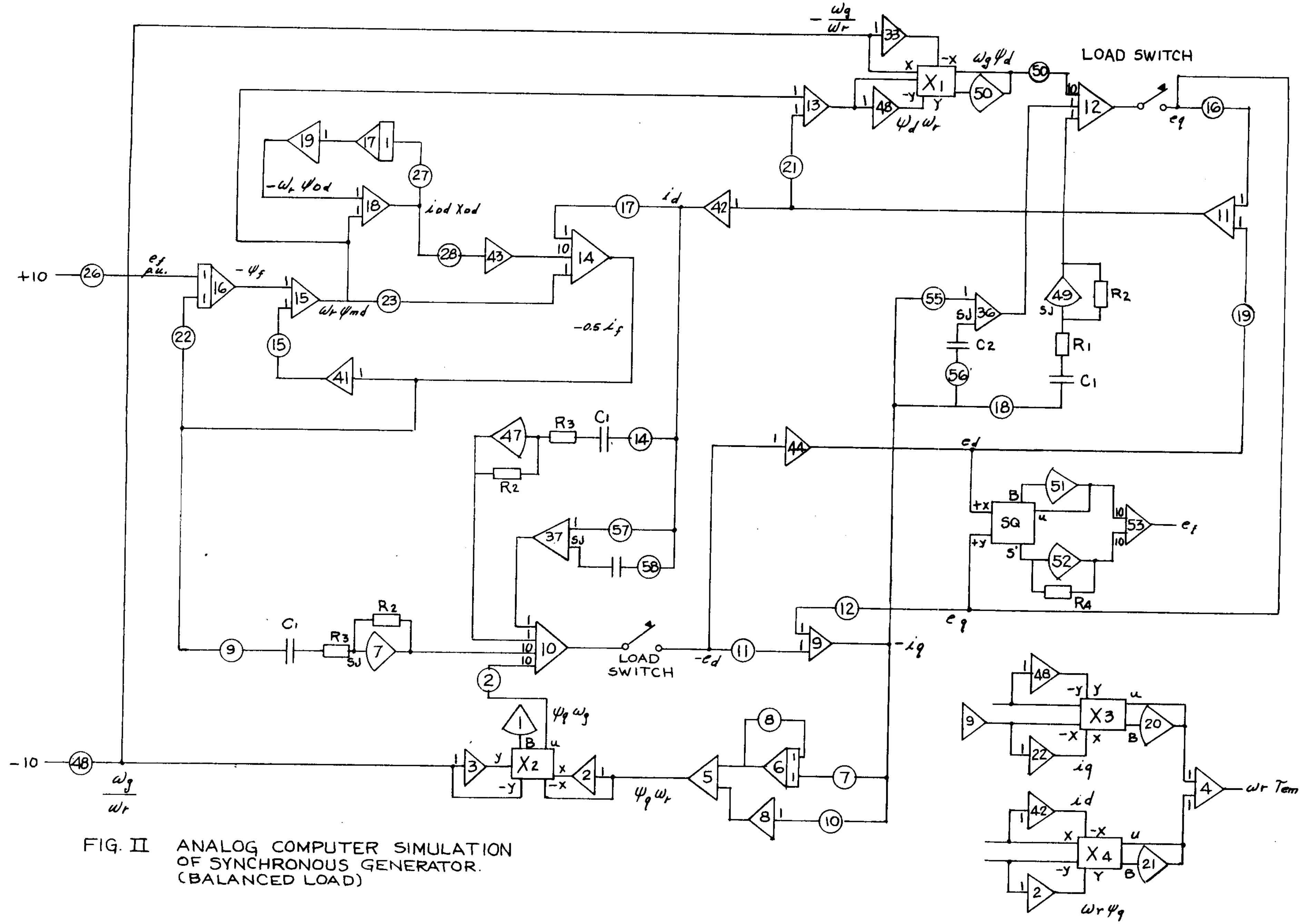
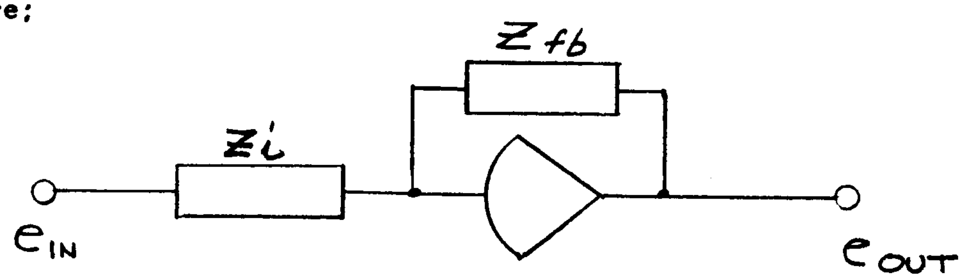


FIG. II ANALOG COMPUTER SIMULATION OF SYNCHRONOUS GENERATOR. (BALANCED LOAD)

The basic values of the components in the presenting analog computer are;



$$\frac{e_{OUT}}{e_{IN}} = -\frac{Z_{fb}}{Z_i} \qquad (75)$$

<u>Fig. 21</u>

Z_{fb} = 100 K resistor

Z_i = 100 K resistor

for an operational amplifier with unity gain. While

Z_{fb} = 10 microfarad capacitor

Z_i = 100 K resistor

for an integrator with unity gain and a time constant of one sec.

Pot. No.	Variable	Setting
2	Scaling constant	0.2
7	$\frac{X_{mq}^2}{X_{mq}+X_{Dq}} \cdot \frac{10^{-2}}{T_{Dq}}$	0.253
8	$\frac{10^{-2}}{T_{Dq}}$	0.318
9	$\frac{(1/\omega_r)\, X_{md}^2}{10^3 (X_{md}+X_{Dd})}$	0.054
10	$X_{al} + \frac{X_{mq} X_{Dq}}{X_{mq}+X_{Dq}}$	0.137
11	$\frac{X_L}{\lvert Z_L \rvert}$ AT 0.75 P.F.	0.661
12	$\frac{R_L}{\lvert Z_L \rvert}$ AT 0.75 P.F.	0.75
14	$\frac{X_{m^2d}}{\omega_r (X_{md}+X_{Dd})}$	0.536
15	$2\,\omega_g\, X_{fl}$	0.248

Pot. No.	Variable	Setting
16	$\frac{X_L}{\lvert Z_L \rvert}$ AT 0.75 P.F.	0.661
17	Scaling constant	0.5
18	$\frac{X_{mq}^2}{\omega_r (X_{mq} + X_{Dq})}$	0.317
19	$\frac{R_L}{\lvert Z_L \rvert}$ AT 0.75 P.F.	0.75
21	X_{al}	0.083
22	$2 w_g R_f \cdot 10^{-2}$	0.173
23	$1/2\ X_{md}$	0.361
26	$\frac{50\ \omega_r}{100 (e_f)_{base}}$	0.27
28	$\frac{1}{20\ X_{Dd}}$	0.98

Pot. No.	Variable	Setting
48	ω/ω_r	1.0
50	Scaling constant	0.2
55	R_a	0.205
56	$\frac{1}{\omega_r}\left(X_{al} + \frac{X_{mq} X_{Dq}}{X_{mq} + X_{Dq}}\right)\cdot 10^{-1}$	0.055
57	R_a	0.205
58	$\frac{1}{\omega_r}\left(X_{al} + \frac{X_{md} X_{Dd}}{X_{md} + X_{Dd}}\right)\cdot 10^{-1}$	0.053

Component	Value	Component	Value
R_1	306 K Ω	C_1	10 uf
R_2	10 K Ω	C_2	1 uf
R_3	185 K Ω		
R_4	100 K Ω		

Time scale: Real time: Computer time = 100:1

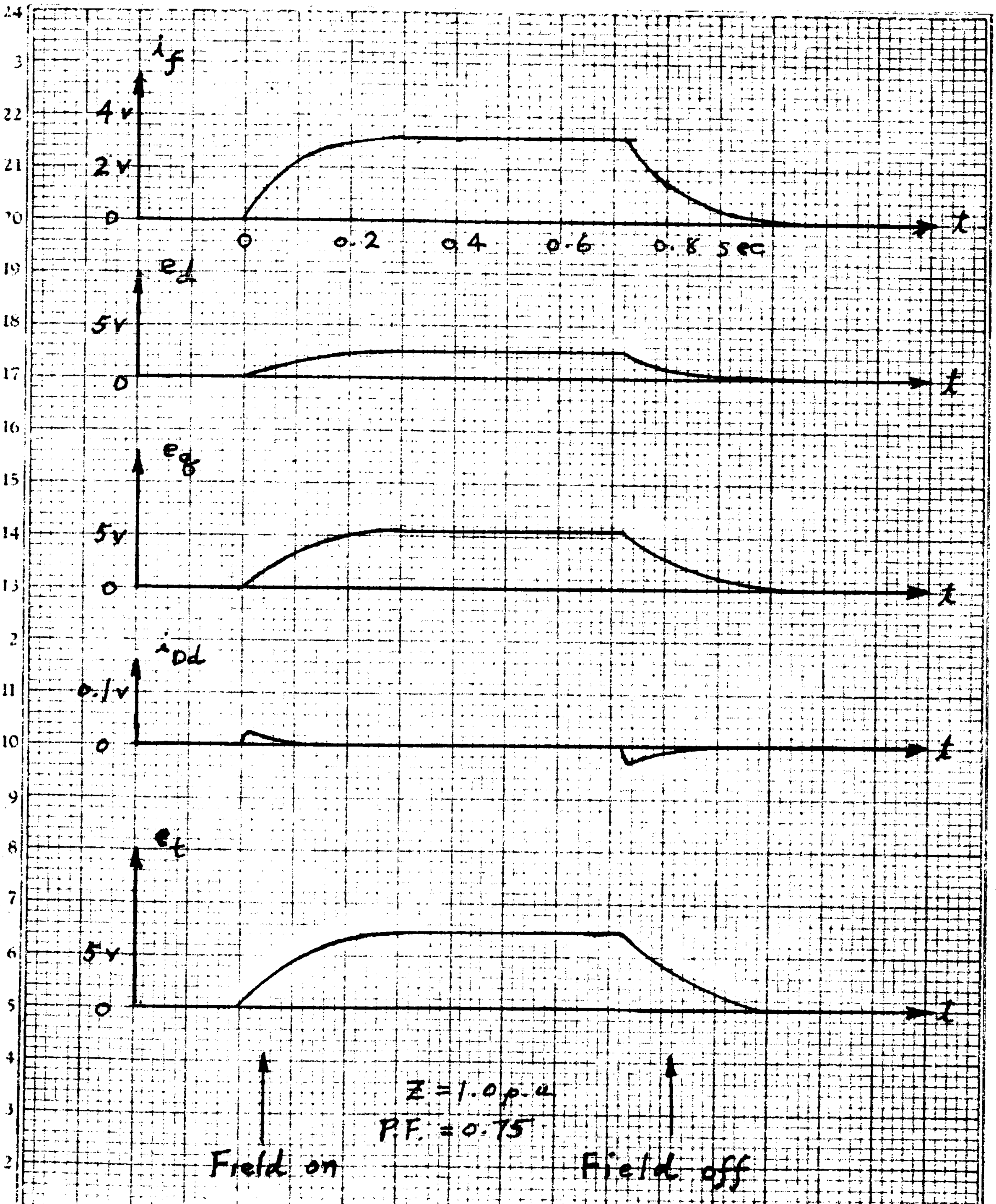


FIG III TRANSIENT RESPONSE OF SYNCHRONOUS GENERATOR TO STEP CHANGE OF FIELD VOLTAGE

2. Digital Computation

(i) Linearization:

For $$e_d = -R_a i_d + \frac{d}{dt}\psi_d - \psi_q \omega_g$$

Let
$$\begin{aligned} e_d &= \bar{e}_d + \Delta e_d \\ i_d &= \bar{I}_d + \Delta i_d \\ \psi_d &= \bar{\Psi}_d + \Delta \psi_d \\ \psi_q &= \bar{\Psi}_q + \Delta \psi_q \\ \omega_g &= \bar{\omega}_g + \Delta \omega_g \end{aligned}$$

where $\bar{e}_d$ is the steady state value and Δe_d, a small increment of change. The same definition is applied to other variables.

Let
$$x = \psi_q \omega_g$$
$$\Delta x = \frac{\partial x}{\partial \psi_q}\Delta \psi_q + \frac{\partial x}{\partial \omega_g}\Delta \omega_g$$
$$\frac{\partial x}{\partial \psi_q} \triangleq \omega_g$$
$$\frac{\partial x}{\partial \omega_g} \triangleq \bar{\Psi}_q$$

Substitute the relations into the original equation.

$$\Delta e_d = -R_a \Delta i_d + \frac{d}{dt}\Delta \psi_d - (\omega_g \Delta \psi_q + \bar{\Psi}_q \Delta \omega_g)$$

Similar procedure is applied to the other basic equations. The results are expressed in matrix.

Load:

$$\begin{bmatrix} \Delta e_d \\ \Delta e_q \end{bmatrix} = \begin{bmatrix} R_L + SL_L & -\bar{\omega}_g L_L \\ \bar{\omega}_g & R_L + SL_L \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} + \begin{bmatrix} -L_L i_q \\ L_L \bar{I}_d \end{bmatrix} [\Delta \omega_g] \quad (76)$$

Flux linkages:

$$\begin{bmatrix} \Delta\psi_d \\ \Delta\psi_q \\ \Delta\psi_f \\ \Delta\psi_{Dd} \\ \Delta\psi_{Dq} \end{bmatrix} = \begin{bmatrix} -L_{sd} & 0 & L_{md} & L_{md} & 0 \\ 0 & -L_{sq} & 0 & 0 & L_{mq} \\ -L_{md} & 0 & L_{md}+L_{fl} & L_{md} & 0 \\ -L_{md} & 0 & L_{md} & L_{Dd} & 0 \\ 0 & -L_{mq} & 0 & 0 & L_{Dq} \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \\ \Delta i_f \\ \Delta i_{Dd} \\ \Delta i_{Dq} \end{bmatrix} \tag{77}$$

Voltages:

$$\begin{bmatrix} \Delta e_d \\ \Delta e_q \\ \Delta e_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} s & -\bar{\omega}_g & 0 & 0 & 0 \\ \bar{\omega}_g & s & 0 & 0 & 0 \\ 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & s & 0 \\ 0 & 0 & 0 & 0 & s \end{bmatrix} \begin{bmatrix} \Delta\psi_d \\ \Delta\psi_q \\ \Delta\psi_f \\ \Delta\psi_{Dd} \\ \Delta\psi_{Dq} \end{bmatrix}$$

$$+ \begin{bmatrix} -R_a & 0 & 0 & 0 & 0 \\ 0 & -R_a & 0 & 0 & 0 \\ 0 & 0 & R_f & 0 & 0 \\ 0 & 0 & 0 & R_{Dd} & 0 \\ 0 & 0 & 0 & 0 & R_{Dq} \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \\ \Delta i_f \\ \Delta i_{Dd} \\ \Delta i_{Dq} \end{bmatrix} + \begin{bmatrix} -\bar{\psi}_q \\ \bar{\psi}_d \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \Delta\omega_g \end{bmatrix} \tag{78}$$

Laplace transformation has been applied with initial conditions equal to zero. In order to simplify the problem, neglect damper bar, armature resistance, magnetic saturation, armature and field leakage inductances. Eqs. (76) to (78) and the balanced load equations become:

$$\begin{bmatrix} \Delta e_d \\ \Delta e_q \end{bmatrix} = \begin{bmatrix} R_L + SL_L & -L_L \bar{\omega}_g \\ L_L \bar{\omega}_g & R_L + SL_L \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} + \begin{bmatrix} -L_L \bar{i}_q \\ L_L \bar{i}_d \end{bmatrix} \begin{bmatrix} \Delta \omega_g \end{bmatrix} \qquad (79)$$

$$\begin{bmatrix} \Delta e_f \\ \Delta e_d \\ \Delta e_q \end{bmatrix} = \begin{bmatrix} S & 0 & 0 \\ 0 & S & -\bar{\omega}_g \\ 0 & \bar{\omega}_g & S \end{bmatrix} \begin{bmatrix} \Delta \psi_f \\ \Delta \psi_d \\ \Delta \psi_q \end{bmatrix} + \begin{bmatrix} 0 \\ -\bar{\psi}_q \\ \bar{\psi}_d \end{bmatrix} \begin{bmatrix} \Delta \omega_g \end{bmatrix}$$

$$+ \begin{bmatrix} R_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_f \\ \Delta i_d \\ \Delta i_q \end{bmatrix} \qquad (80)$$

$$\begin{bmatrix} \Delta \phi_f \\ \Delta \phi_d \\ \Delta \phi_q \end{bmatrix} = \begin{bmatrix} L_{md} & -L_{md} & 0 \\ L_{md} & -L_{md} & 0 \\ 0 & 0 & -L_{mq} \end{bmatrix} \begin{bmatrix} \Delta i_f \\ \Delta i_d \\ \Delta i_q \end{bmatrix} \qquad (81)$$

$$\begin{bmatrix} \Delta e_f \\ \Delta e_d \\ \Delta e_q \end{bmatrix} = \begin{bmatrix} R_f + SL_{md} & -SL_{md} & 0 \\ SL_{md} & -SL_{md} & \bar{\omega}_g L_{mq} \\ \bar{\omega}_g L_{md} & \bar{\omega}_g L_{md} & -SL_{mq} \end{bmatrix} \begin{bmatrix} \Delta i_f \\ \Delta i_d \\ \Delta i_q \end{bmatrix} \qquad (82)$$

Assume constant generator frequency.

That is $\Delta\omega_g = 0$. From eqs. (79) to (82). First solve for Δi_d and Δi_q.

$$\begin{bmatrix}\Delta e_f\end{bmatrix} = \begin{bmatrix}R_f + sL_{md}\end{bmatrix}\begin{bmatrix}\Delta i_f\end{bmatrix} - \begin{bmatrix}sL_{md} & 0\end{bmatrix}\begin{bmatrix}\Delta i_d \\ \Delta i_q\end{bmatrix} \qquad (83)$$

$$\begin{bmatrix}\Delta e_d \\ \Delta e_q\end{bmatrix} = \begin{bmatrix}sL_{md} \\ \bar{\omega}_g L_{md}\end{bmatrix}\begin{bmatrix}\Delta i_f\end{bmatrix} + \begin{bmatrix}-sL_{md} & \bar{\omega}_g L_{mq} \\ \bar{\omega}_g L_{md} & -sL_{mq}\end{bmatrix}\begin{bmatrix}\Delta i_d \\ \Delta i_q\end{bmatrix} \qquad (84)$$

$$\begin{bmatrix}R_f + sL_{md}\end{bmatrix}\begin{bmatrix}\Delta e_d \\ \Delta e_q\end{bmatrix} = \begin{bmatrix}sL_{md} \\ \bar{\omega}_g L_{md}\end{bmatrix}\begin{bmatrix}\Delta e_f\end{bmatrix}$$

$$+ \begin{bmatrix}-sL_{md}R_f & s\omega_g L_{md}L_{mq} + \bar{\omega}_g L_{mq} R_f \\ -\bar{\omega}_g L_{md} R_f & -s^2 L_{md} L_{mq} - sL_{mq} R_f\end{bmatrix}\begin{bmatrix}\Delta i_d \\ \Delta i_q\end{bmatrix} \qquad (85)$$

$$\left[\begin{matrix}s^2 L_{md} L_L + s(L_{md} R_f + L_{md} R_L + L_L R_f) + R_f R_L \\ s\,\bar{\omega}_g L_{md} L_L + \bar{\omega}_g R_f (L_{md} + L_L\end{matrix}\right.$$

$$\left.\begin{matrix}-s\bar{\omega}_g L_{md}(L_{mq} + L_L) - \bar{\omega}_g R_f (L_{mq} + L_L) \\ s^2 L_{md}(L_{mq} + L_L) + s(L_{md} R_L + L_L R_f + L_{mq} R_f) + R_f R_L\end{matrix}\right]\begin{bmatrix}\Delta i_d \\ \Delta i_q\end{bmatrix}$$

$$= \begin{bmatrix}sL_{md} \\ \bar{\omega}_g L_{md}\end{bmatrix}\begin{bmatrix}\Delta e_f\end{bmatrix} \qquad (86)$$

i.e.

$$[A]\begin{bmatrix}\Delta i_d\\ \Delta i_q\end{bmatrix} = \begin{bmatrix} sL_{md}\\ \bar{\omega}_g L_{md}\end{bmatrix}[\Delta e_f] \qquad (87)$$

$$\therefore \begin{bmatrix}\Delta i_d\\ \Delta i_q\end{bmatrix} = [A]^{-1}\begin{bmatrix} sL_{md}\\ \bar{\omega}_g L_{md}\end{bmatrix}[\Delta e_f]$$

$$= \begin{bmatrix} \dfrac{K_1(s^3 + C_{11}s^2 + b_{11}s + a_{11})}{s^4 + d_p s^3 + C_p s^2 + b_p s + a_p} \\[2ex] \dfrac{K_2(s^2 + b_{22}s + a_{22})}{s^4 + d_p s^3 + C_p s^2 + b_p s + a_p} \end{bmatrix}[\Delta e_f]$$

$$= \begin{bmatrix} G_1(s)\\ G_2(s)\end{bmatrix}[\Delta e_f] \qquad (88)$$

$$K_1 = \frac{1}{L_L} \qquad (89a)$$

$$K_2 = \frac{\bar{\omega}_g(L_{mq} + Z L_L)}{L_L(L_{mq} + L_L)} \qquad (89b)$$

$$a_p = \frac{R_f^2 R_L^2}{L^2_{md} L_L (L_{mq} + L_L)} + \frac{R_f^2 \bar{\omega}_g^2 (L_{md} + L_L)}{L_{md}^2 L_L} \qquad (89c)$$

$$b_p = \frac{R_f R_L \left[2 L_{md} R_L + 2 L_L R_f + L_{mq} R_f + L_{md} R_f \right]}{L_{md}^2 L_L (L_{mq} + L_L)} + \frac{\bar{\omega}_g^2 R_f (L_{md} + 2 L_L)}{L_{md} L_L} \tag{89d}$$

$$c_p = \frac{R_f R_L}{L_{md} L_L} + \frac{R_f R_L}{L_{md}(L_{mq} + L_L)} + \bar{\omega}_g^2 + \frac{(L_{md} R_f + L_{md} R_L + L_L R_f)(L_{md} R_L + L_L R_f + L_{mq} R_f)}{L_{md}^2 L_L (L_{mq} + L_L)} \tag{89e}$$

$$d_p = \frac{L_{md} R_f + L_{md} R_L + L_L R_f}{L_{md} L_L} + \frac{L_{md} R_L + L_L R_f + L_{mq} R_f}{L_{md}(L_{mq} + L_L)} \tag{89f}$$

$$a_{11} = \frac{\bar{\omega}_g^2 R_f (L_{md} + L_L)}{L_{md} (L_{mq} + L_L)} \tag{89g}$$

$$b_{11} = \frac{R_f R_L}{L_{md} (L_{mq} + L_L)} - \frac{\bar{\omega}_g^2 L_L}{L_{mq} + L_L} \tag{89h}$$

$$C_{11} = \frac{R_L}{L_{mq} + L_L} + \frac{R_f}{L_{md}} \tag{89i}$$

$$a_{22} = \frac{R_f R_L}{L_{md}(L_{mq} + 2L_L)} \tag{89j}$$

$$b_{22} = \frac{R_f + R_L}{L_{mq} + 2L_L} + \frac{R_f}{L_{md}} \tag{89k}$$

Generally, $\bar{\omega}_g L_{md}, \bar{\omega}_g L_{mq} >> \bar{\omega}_g L_L, R_f, R_L$
AND $R_f, R_L >> L_{md}, L_{mq}, L_L$

The coefficients can be approximated.

$$K_1 = \frac{1}{L_L} \tag{90a}$$

$$K_2 = \frac{\bar{\omega}_g}{L_L} \tag{90b}$$

$$a_p = \frac{R_f^2}{L_{md} L_L} \left(\frac{R_L^2}{L_{md} L_{mq}} + \bar{\omega}_g^2 \right) \tag{90c}$$

$$b_p = \frac{R_f}{L_L}\left[\frac{R_L(2L_{md}R_L + L_{mq}R_f + L_{md}R_f)}{L_{md}^2 L_{mq}} + \bar{\omega}_g^2\right] \quad (90d)$$

$$c_p = \frac{R_f R_L}{L_{md} L_L} + \frac{(R_L + R_f)(L_{md} R_L + L_{mq} R_f)}{L_{md} L_{mq} L_L} + \bar{\omega}_g^2 \quad (90e)$$

$$d_p = \frac{R_f + R_L}{L_L} \quad (90f)$$

$$a_{11} = \frac{\bar{\omega}_g^2 R_f}{L_{mq}} \quad (90g)$$

$$b_{11} = \frac{1}{L_{mq}}\left(\frac{R_f R_L}{L_{md}} - \bar{\omega}_g^2 L_L\right) \quad (90h)$$

$$C_{11} = \frac{R_L}{L_{mq}} + \frac{R_f}{L_{md}} \quad (90i)$$

$$a_{22} = \frac{R_f R_L}{L_{md} L_{mq}} \quad (90j)$$

$$b_{22} = \frac{R_f R_L}{L_{mq}} + \frac{R_f}{L_{md}} \quad (90k)$$

Thus, solve for Δe_d, Δe_q.

$$\begin{bmatrix} \Delta e_d \\ \Delta e_q \end{bmatrix} = \begin{bmatrix} R_L + sL_L & -\bar{\omega}_g L_L \\ \bar{\omega}_g L_L & R_L + sL_L \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix}$$

$$= \begin{bmatrix} (R_L + sL_L) G_1(s) - \bar{\omega}_g L_L G_2(s) \\ \bar{\omega}_g L_L G_1(s) + (R_L + sL_L) G_2(s) \end{bmatrix} \left[\Delta e_f \right]$$

$$= \begin{bmatrix} \dfrac{K_3 (s^4 + d_{33} s^3 + c_{33} s^2 + b_{33} s + a_{33})}{s^4 + d_p s^3 + c_p s^2 + b_p s + a_p} \\[2ex] \dfrac{K_4 (s^3 + c_{44} s^2 + b_{44} s + a_{44})}{s^4 + d_p s^3 + c_p s^2 + b_p s + a_p} \end{bmatrix} \left[\Delta e_f \right]$$

$$= \begin{bmatrix} G_3(s) \\ G_4(s) \end{bmatrix} \left[\Delta e_f \right] \qquad (91)$$

$$K_3 = 1 \qquad (92a)$$

$$a_{33} = \frac{R_L}{L_L} a_{11} - \bar{\omega}_g^2 a_{22} \tag{92b}$$

$$b_{33} = a_{11} + \frac{R_L}{L_L} b_{11} - \omega_g^2 b_{22} \tag{92c}$$

$$c_{33} = \frac{R_L}{L_L} c_{11} + b_{11} - \bar{\omega}_g^2 \tag{92d}$$

$$d_{33} = \frac{R_L}{L_L} + c_{11} \tag{92e}$$

$$K_4 = 2\,\bar{\omega}_g \tag{92f}$$

$$a_{44} = \frac{1}{2}\left(a_{11} + \frac{R_L}{L_L} a_{22}\right) \tag{92g}$$

$$b_{44} = \frac{1}{2}\left(b_{11} + \frac{R_L}{L_L} b_{22} + a_{22}\right) \tag{92h}$$

$$c_{44} = \frac{1}{2}\left(c_{11} + b_{22} + \frac{R_L}{L_L}\right) \tag{92i}$$

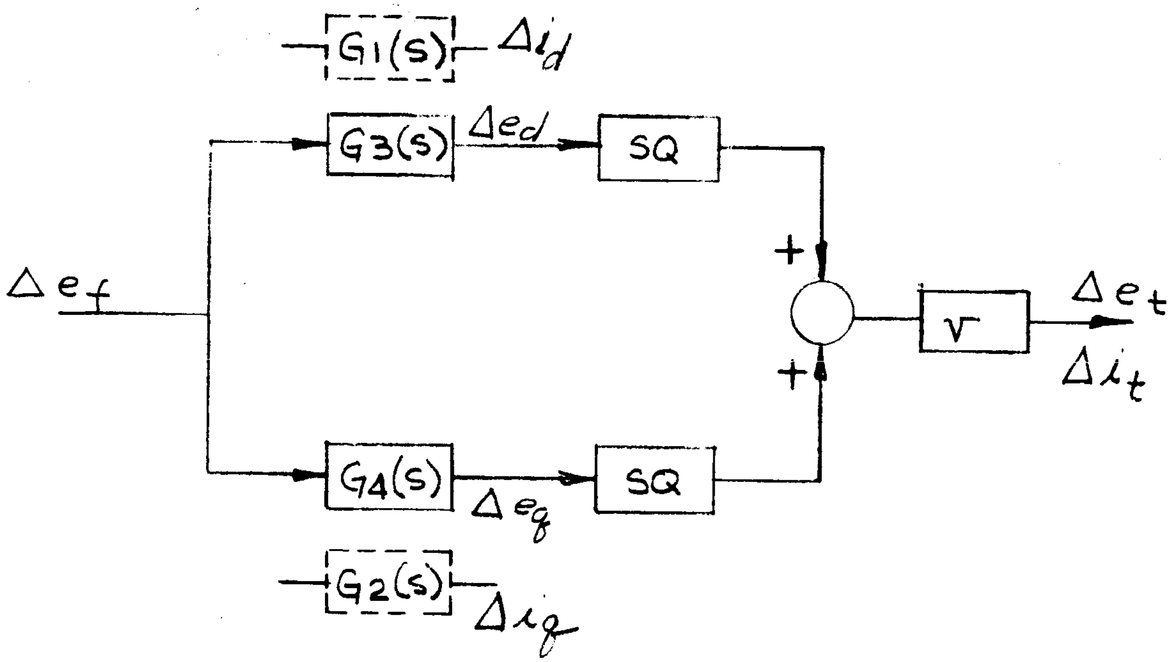


Fig. 22

$G_1(s)$, $G_2(s)$, $G_3(s)$ and $G_4(s)$ are linear filters. They can be implemented on an analog computer. The coefficient of the filters can be tabulated by digital computer so that a new set of values can readily be obtained when the machine and/or load parameters are changed while this implies to change of potentiometer settings.of the analog computer. However, this section will emphasize on theoretical analysis of the equivalent filters. Different kinds of stability analysis methods are used to interpret the relative stability, transient and other concerns. Numerical examples are given along with the discussion. Digital computer is used for the computations.

(ii) First, the characteristics of the transfer functions of the models $G_1(s)$, $G_2(s)$, $G_3(s)$ and $G_4(s)$ are investigated. The denominator is a fourth order polynomial with all the coefficients positive. There will be four poles. Their locations depend on the generator and load parameters and the generator frequency which has been assumed constant. For the system to be stable, all these poles of the closed loop system must lie on the left half of the complex plane so as to ensure convergence. The closed loop system is assumed to be: (with constant speed drive)

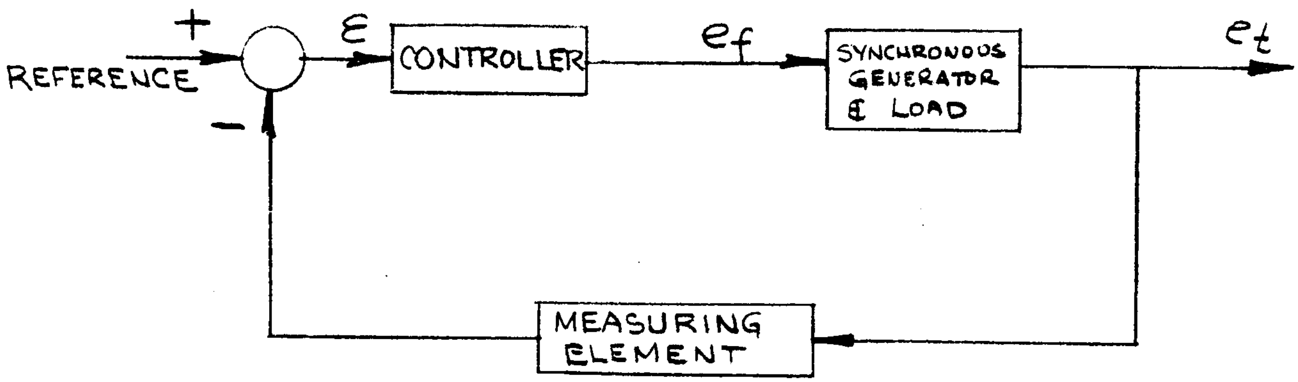


Fig. 23
Closed-Loop System

Thus, the synchronous generator and the load can be considered as an open-loop plant.

A synchronous generator used as a sample throughout the following discussion is rated at 120 volt/111 amp line to neutral with a power factor of 0.75.

$\bar{\omega}_g$ = 2500 radians/second

L_{md} = 0.068 henries

L_{mq} = 0.044 henries

R_f = 1.0 ohms

R_L = 0.8 ohms

L_L = 0.0003 henries

From the previous argument and derivation

$$G_3(s) = \frac{E_d(s)}{E_f(s)} \tag{93a}$$

$$= \frac{s^4 + 2.73\times10^3 s^3 - 6.2\times10^6 s^2 - 6\times10^8 s - 3.84\times10^{11}}{s^4 + 6\times10^3 s^3 + 6.5\times10^6 s^2 + 1.37\times10^{10} s + 3.08\times10^{11}}$$

$$G_4(s) = \frac{E_q(s)}{E_f(s)}$$

$$= \frac{5\times10^3(s^3 + 1.4\times10^3 s^2 - 1.97\times10^4 s - 7.07\times10^7}{s^4 + 6\times10^3 s^3 + 6.5\times10^6 s^2 + 1.37\times10^{10} s + 3.08\times10^{11}} \tag{93b}$$

The steady state gains are -

$$\lim_{s\to 0} \left| G_3(s) \right| = 1.25$$

$$\lim_{s\to 0} \left| G_4(s) \right| = 1.15$$

Factorize $G_3(s)$ and $G_4(s)$

$$G_3(s) = \frac{5\times10^3(s-215)(s+238)(s+1377)}{(s+23)(s+5300)(s+360 \pm j\,1560}$$

$$G_4(s) = \frac{(s-1570)(s+4300)(s+57 \pm j235)}{(s+23)(s+5300)(s+360 \pm j1560)}$$

The denominator determines the locations of the open-loop poles while the numerator determines the open-loop zeros. The poles are the starting points of the root locus which terminate at the corresponding zeros as the gain approaches to infinity. Observe both $G_3(s)$ and $G_4(s)$ have the same denominator and the poles are all in the left half plane, therefore, the open loop plant is a stable one. Only $G_4(s)$ is plotted on the complex plane.

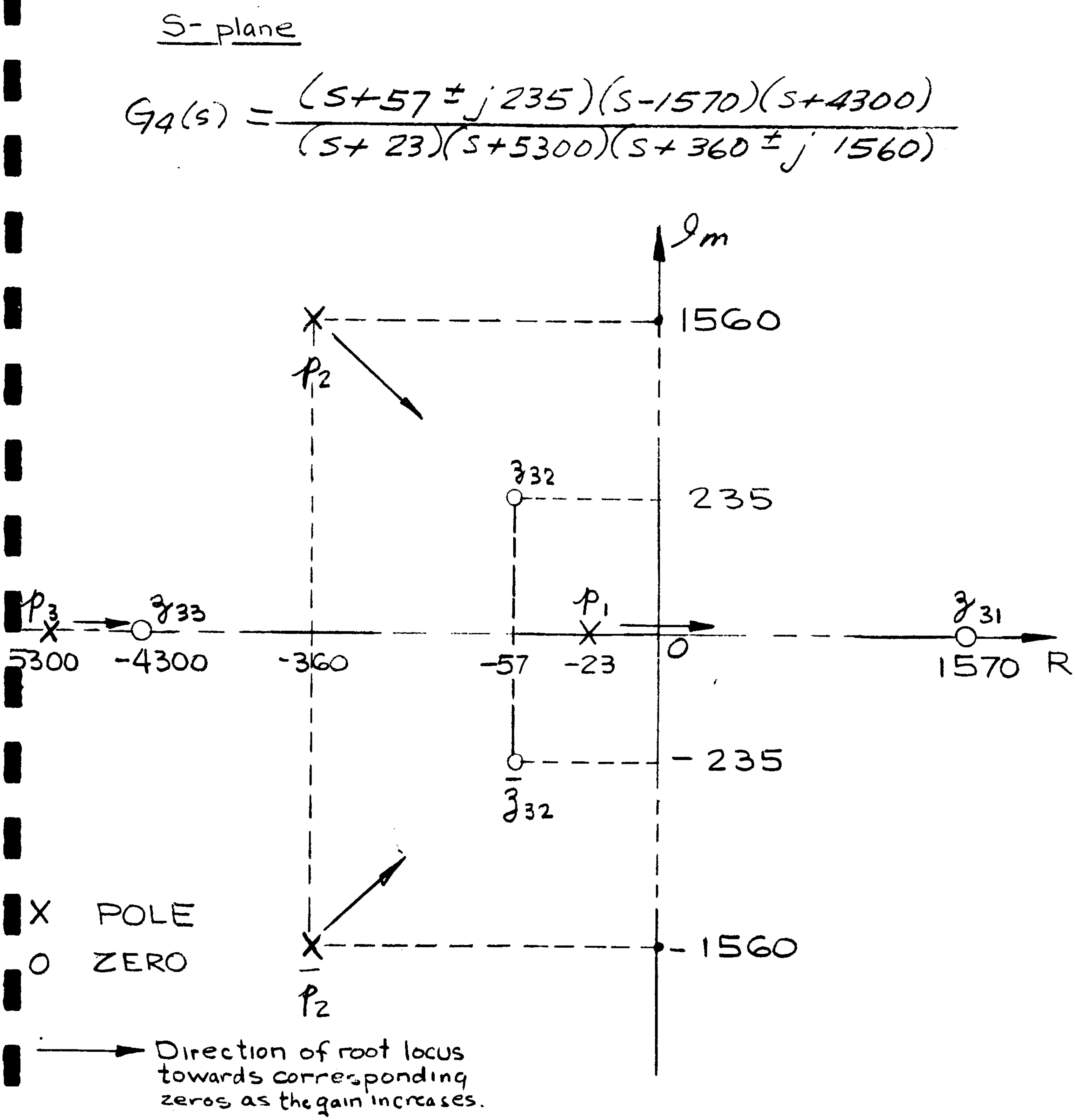


Fig. 24 Root locus of G4(s)

$G_3(s)$ and $G_4(s)$ have poles located at -23, -5300, and -360. The latter is taken from the real part of the pair of complex root. The correspondent time constants are

$$T_1 = \frac{1}{23} = 0.0435 \text{ sec.} \tag{96a}$$

$$T_2 = \frac{1}{360} = 0.00277 \text{ sec.} \tag{96b}$$

$$T_3 = \frac{1}{5300} \; 2 \; 0.000189 \text{ sec.} \tag{96c}$$

The last two are comparatively insignificant. Thus, for a rough estimate, the synchronous generator with excitation voltage e_f as the only feed forward control effort, can be approximated as a first order system with a time constant of T_1. Generally, T_2 can be included as subtransient time while T_1 as transient time constant. From eqs. (94a) and 94b), steady state gain of terminal voltage e_t over excitation voltage e_f can be derived.

$$\lim_{s \to 0} \frac{E_t(s)}{E_f(s)} = \lim_{s \to 0} \left\{ \left[G_3(s)\right]^2 + \left[G_4(s)\right]^2 \right\}^{\frac{1}{2}}$$

$$= (1.25^2 + 1.15^2)^{\frac{1}{2}}$$

$$= 1.7 \tag{97}$$

Therefore, the approximated linear transfer function of e_t/e_f can be written as:

$$\frac{E_t(s)}{E_f(s)} = \frac{K_t}{(1+T_1 s)(1+T_2 s)}$$

$$= \frac{1.7}{(1+0.0435s)(1+0.00277s)} \tag{98}$$

(iii) Frequency domain plot:

To plot $G_3(s)$ and $G_4(s)$ in the frequency domain, let

$$s = j\omega$$

$$D(s) = s^4 + dp\,s^3 + cp\,s^2 + bp\,s + ap \qquad (99a)$$

$$D(j\omega) = (ap - cp\,\omega^2 + \omega^4) + j\omega(bp - dp\,\omega^2)$$
$$= |D(j\omega)| \angle\theta_D \qquad (99b)$$

where

$$|D(j\omega)| = \left[(ap - cp\,\omega^2 + \omega^4)^2 + \omega^2(bp - dp\,\omega^2)^2\right]^{\frac{1}{2}} \qquad (99c)$$

$$\theta_D = \tan^{-1}\left[\frac{\omega(bp - dp\,\omega^2)}{ap - cp\,\omega^2 + \omega^4}\right] \qquad (99d)$$

Similarly:

$$N_3(s) = K_3(s^4 + d_{33}s^3 + c_3 s^2 + b_{33}s + a_{33} \qquad (100a)$$

$$N_3(j\omega) = |N_3(j\omega)| \angle\theta_3 \qquad (100b)$$

where

$$|N_3(j\omega)| = K_3\left[(a_{33} - c_{33}\omega^2 + \omega^4)^2 + \omega^2(b_{33} - d_{33}\omega^2)^2\right]^{\frac{1}{2}} \qquad (100c)$$

$$\theta_3 = \tan^{-1}\left[\frac{\omega(b_{33} - d_{33}\omega^2)}{a_{33} - c_{33}\omega^2 + \omega^4}\right] \qquad (100d)$$

$$N_4(s) = K_4(s^3 + c_{44}s^2 + b_{44}s + a_{44}) \qquad (101a)$$

$$N_4(j\omega) = |N_4(j\omega)|\,\theta_4 \qquad (101b)$$

where

$$|N_4(j\omega)| = \left[K_4 \quad (a_{44} - c_{44}\omega^2)^2 + \omega^2(b_{44} - \omega^2)^2\right]^{\frac{1}{2}} \qquad (101c)$$

$$\theta_4 = \tan^{-1}\left[\frac{\omega(b_{44} - \omega^2)}{a_{44} - c_{44}\omega^2}\right] \qquad (101a$$

$$e_f \longrightarrow \boxed{\frac{K_i N_i(s)}{D(s)}} \longrightarrow \begin{array}{l} e_d\ (i=3) \\ e_g\ (i=4) \end{array}$$

Fig. 25

Use the same data for the synchronous generator and impose the same assumptions as in the previous example. Plot the transfer functions with respect to frequency in Fig. IV. Consider $G_3(s)$, the zero cross-over of the amplitude curve corresponds to a phase lag of 35°. That is a phase margin of 145°. $G_3(s)$ is far from unstable. One must know that not all the poles and zeros are in the left half of the complex plane. The non-minimum phase characteristics prevent the direct approximation of the phase angle derived from the asymptotic plot of the amplitude curve.

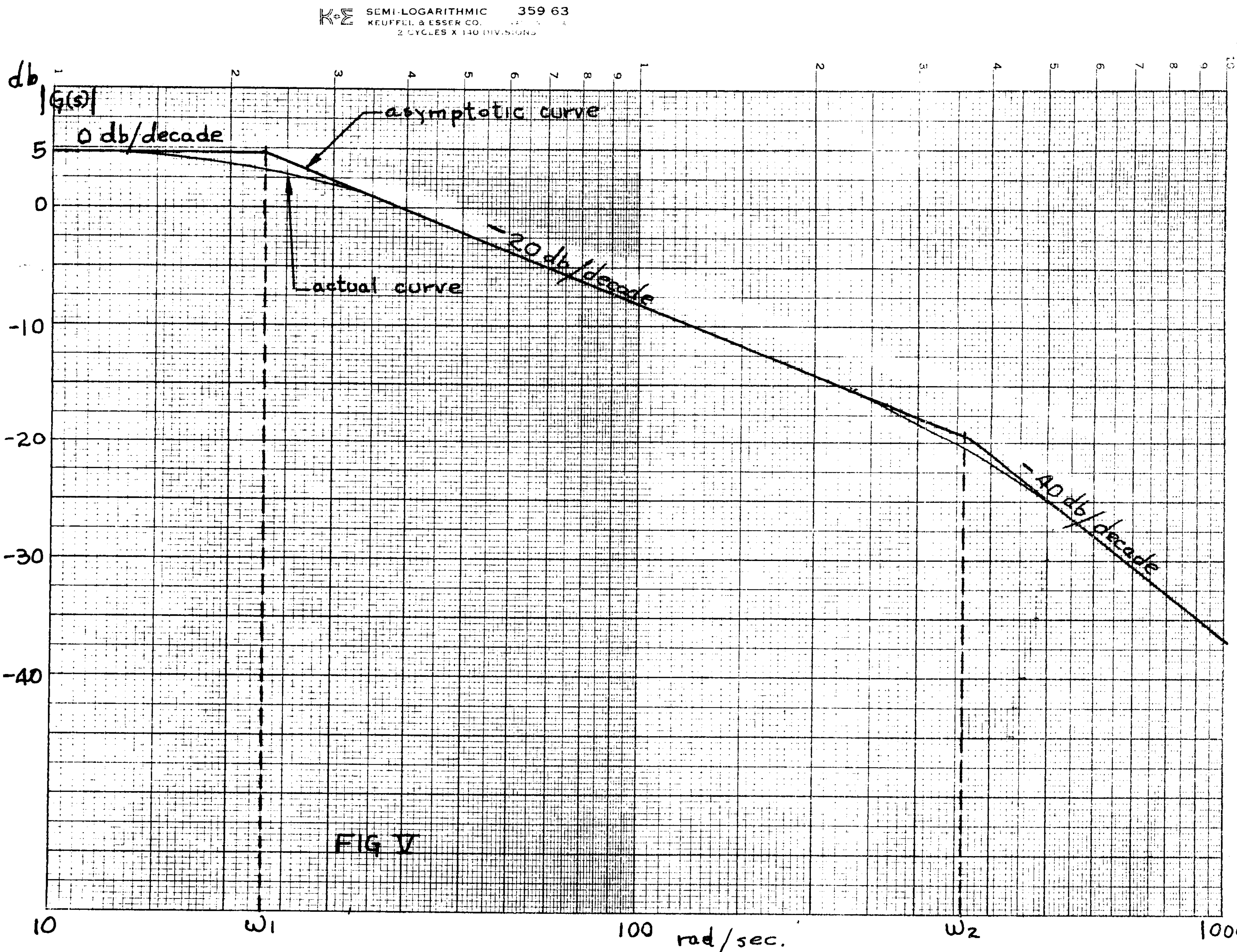

K+E SEMI-LOGARITHMIC 359 63
KEUFFEL & ESSER CO.
2 CYCLES X 140 DIVISIONS
db
|G(s)|
0 db/decade
asymptotic curve
actual curve
-20 db/decade
-40 db/decade
5
0
-10
-20
-30
-40
10
ω1
100
rad/sec.
ω2
1000

FIG V

(iv) Transfer function derivation from laboratory data:

Conversely, if a transient response curve is in hand, a transfer function can be derived from asymptotic plot in a frequency domain curve. The break-away points of two asymptotes with 20 db/decade decay difference determines the time constants. The order of the transfer function depends on the need of accuracy in describing the characteristics. It must be noted that a time domain plot which is the usual case of laboratory data, should be transformed into frequency domain plot before applying the approximation technique. The abscissa should be the ratio of output versus input in decibel while the ordinate, frequency on radians per second. Suppose an actual curve is plotted in Fig. D. Three asymptotic lines are approximated. The zero db/decade line is at 4.6 db which determines the gain of the transfer function while the two break-away points at

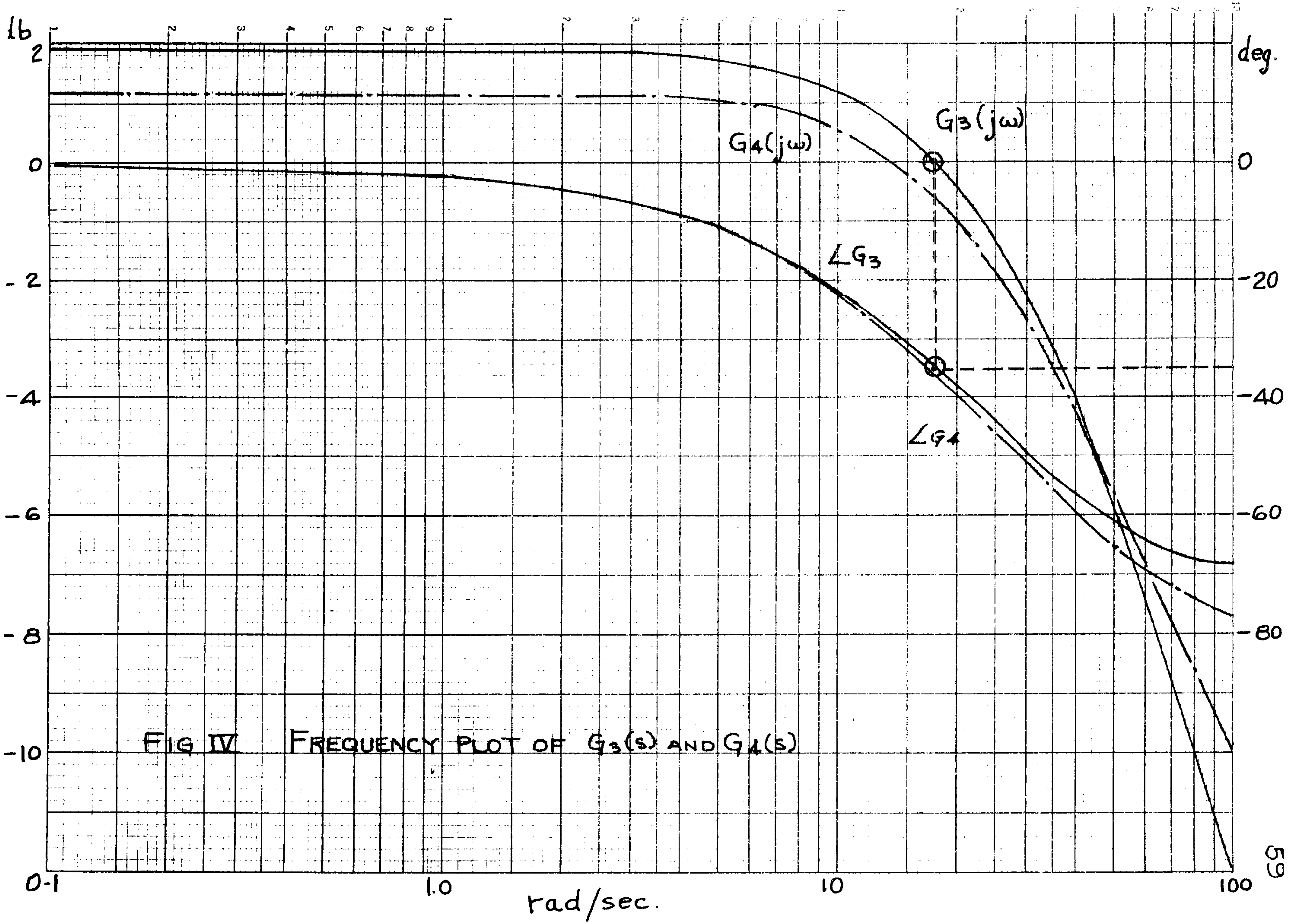


FIG IV FREQUENCY PLOT OF $G_3(s)$ AND $G_4(s)$

$\omega_1 = 23$ rad/sec AND $\omega_2 = 360$ rad/sec

The transfer function becomes -

$$G(s) = \frac{K}{(1 + T_1 s)(1 + T_2 s)}$$

$$= \frac{\frac{1}{20} \text{ antilog}_{10} (4.6)}{(1 + \frac{1}{23} s)(1 + \frac{1}{360} s)}$$

$$= \frac{1.7}{(1 + 0.0435 s)(1 + 0.00277 s)} \qquad (98a)$$

(v) Two manipulated variables:

If both Δw and Δe_f are considered simultaneously,

$$\begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} = \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} \begin{bmatrix} \Delta e_f \end{bmatrix} + \begin{bmatrix} A \end{bmatrix}^{-1}$$

$$\begin{bmatrix} R_f \bar{i}_q (L_{mq} + L_L) + s L_{md} \bar{i}_q (L_{mq} + L_L) \\ R_f \left\{ L_{md} \bar{i}_f - \bar{i}_d (L_{md} + L_L) \right\} + s L_{md} \left\{ L_{md} \bar{i}_f - \bar{i}_d (L_{md} + L_L) \right\} \end{bmatrix} \begin{bmatrix} \Delta \omega \end{bmatrix}$$

$$= \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} \begin{bmatrix} \Delta e_f \end{bmatrix} + \begin{bmatrix} G_5(s) \\ G_6(s) \end{bmatrix} \begin{bmatrix} \Delta \omega \end{bmatrix} \tag{102}$$

where

$$G_5(s) = \frac{s^3 d_5 + s^2 c_5 + s b_5 + a_5}{s^4 e_p + s^3 dp + s^2 cp + s bp + ap} \tag{102a}$$

$$G_6(s) = \frac{s^3 d_6 + s^2 c_6 + s b_6 + a_6}{s^4 ep + s^3 dp + s^2 cp + s bp + ap} \tag{102b}$$

$$\begin{bmatrix} \Delta e_d \\ \Delta e_q \end{bmatrix} = \begin{bmatrix} G_3(s) \\ G_4(s) \end{bmatrix} \begin{bmatrix} \Delta e_f \end{bmatrix} + \begin{bmatrix} G_7(s) \\ G_8(s) \end{bmatrix} \begin{bmatrix} \Delta \omega \end{bmatrix} \tag{103}$$

where

$$G_7(s) = \frac{s^4 e_7 + s^3 d_7 + s^2 c_7 + s b_7 + a_7}{s^4 ep + s^3 dp + s^2 cp + s bp + ap} \tag{103a}$$

$$G_8(s) = \frac{s^4 e_8 + s^3 d_8 + s^2 c_8 + s b_8 + a_8}{s^4 ep + s^3 dp + s^2 cp + s bp + ap} \tag{103b}$$

Again, approximate the coefficients by assuming

$L_{mq}, L_{md} >> L_L$

$$d_5 = L^2_{md} L^2_{mq} \bar{i}_q \qquad (104a)$$

$$c_5 = \left[L_{md} (L_{md} R_L + 2 L_{mq} R_f) \bar{i}_q + \bar{\omega}_g L_L L^2_{md} (\bar{i}_f - \bar{i}_d) \right] \qquad (104b)$$

$$b_5 = R_f \left[L_{mq} (2 L_{md} R_L + L_{mq} R_f) i_q + \bar{\omega}_g L^3_{md} (\bar{i}_f - \bar{i}_d) \right] \qquad (104c)$$

$$a_5 = R_f^2 \left[R_L L_{mq} \bar{i}_q + \bar{\omega}_g L^2_{md} (\bar{i}_f - \bar{i}_d) \right] \qquad (104d)$$

$$d_6 = L_L L^3_{md} (\bar{i}_f - \bar{i}_d) \qquad (104e)$$

$$c_6 = L^3_{md} (\bar{i}_f - \bar{i}_d)(R_L + R_f) - \bar{\omega}_g L^2_{md} L^2_{mq} \bar{i}_q \qquad (104f)$$

$$b_6 = R_f L_{md} \left[L_{md} (2 R_L + R_f)(\bar{i}_f - \bar{i}_d) - 2 \bar{\omega}_g L^2_{mq} \bar{i}_q \right] \qquad (104g)$$

$$a_6 = R_f^2 \left[R_L L_{md} (\bar{i}_f - \bar{i}_d) - \bar{\omega}_g L^2_{mq} \bar{i}_q \right] \qquad (104h)$$

$$e_7 = L_L (d_5 - e_p \bar{i}_q) \qquad (104i)$$

$$d_7 = R_L d_5 + L_L (c_5 - \bar{\omega}_g d_6 - d_p \bar{i}_q) \qquad (104j)$$

$$c_7 = R_L c_5 + L_L (b_5 - \bar{\omega}_g c_6 - c_p \bar{i}_q) \qquad (104k)$$

$$b_7 = R_L b_5 + L_L (a_5 - \bar{\omega}_g b_6 - b_p \bar{i}_q) \qquad (104l)$$

$$a_7 = R_L a_5 - L_L (\bar{\omega}_g a_6 + a_p \bar{i}_q) \qquad (104m)$$

$$e_8 = L_L (d_6 + e_p \bar{i}_d) \qquad (104n)$$

$$d_8 = R_L d_6 + L_L (c_6 + \bar{\omega}_g d_5 + d_p \bar{i}_d) \qquad (104o)$$

$$c_8 = R_L c_6 + L_L (b_6 + \bar{\omega}_g c_5 + c_p \bar{i}_d) \qquad (104p)$$

$$b_8 = R_L b_6 + L_L (a_6 + \bar{\omega}_g b_5 + a_p \bar{i}_d) \qquad (104q)$$

$$a_8 = R_L a_6 + L_L (\bar{\omega}_g a_5 + a_p \bar{i}_d) \qquad (104r)$$

$$e_p = L_{md}^2 L_{mq} L_L \qquad (104s)$$

$$d_p = L_{md} \left[L_{md} L_{mq} (R_f + R_L) + L_L (L_{md} R_L + L_{mq} R_f) \right] \qquad (104t)$$

$$c_p = L_{md} \left[L_{mq} (R_f R_L + \bar{\omega}_g^2 L_{md} L_L) + (R_f + R_L)(L_{md} R_L + L_{mq} R_f) \right] \qquad (104u)$$

$$b_p = R_f \left[R_L \left(2 L_{md} R_L + R_f L_{md} + L_{mq} R_f \right) + \bar{\omega}_g^2 L_{md}^2 L_{mq} \right] \qquad (104v)$$

$$a_p = R_f^2 \left(R_L^2 + \bar{\omega}_g^2 L_{md} L_{mq} \right) \qquad (104w)$$

The increments of the variables have to be small for the formulation to be valid. The result is an interacting system.

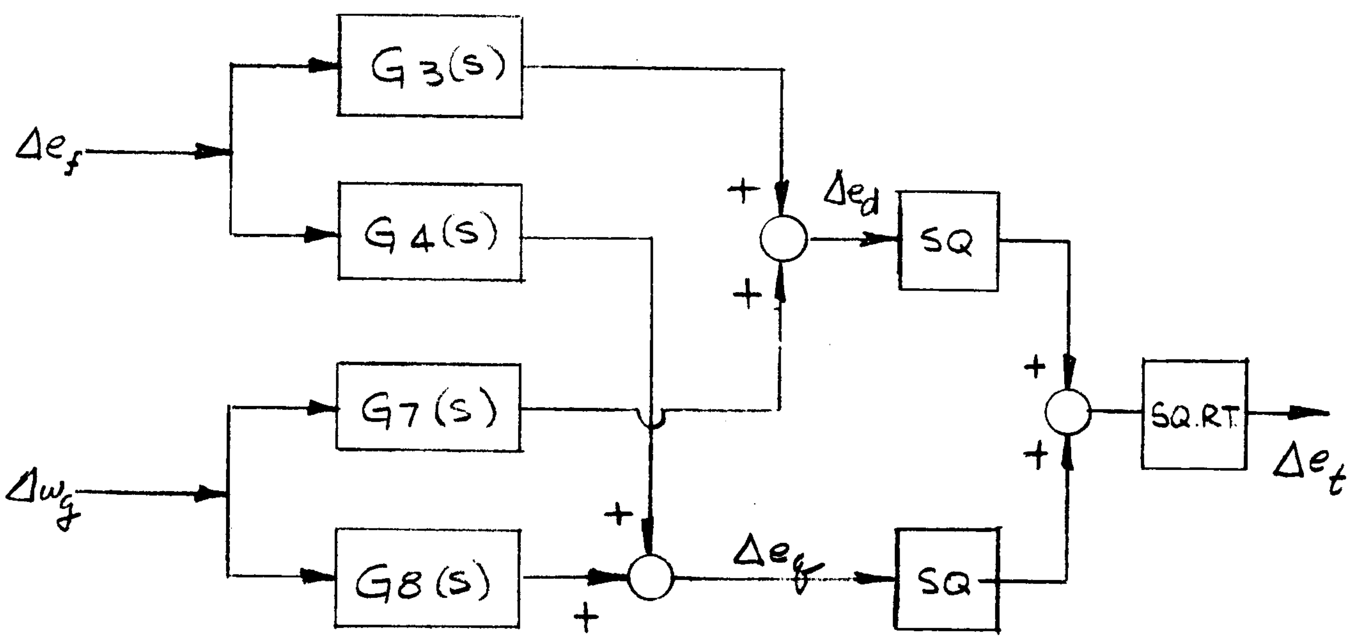
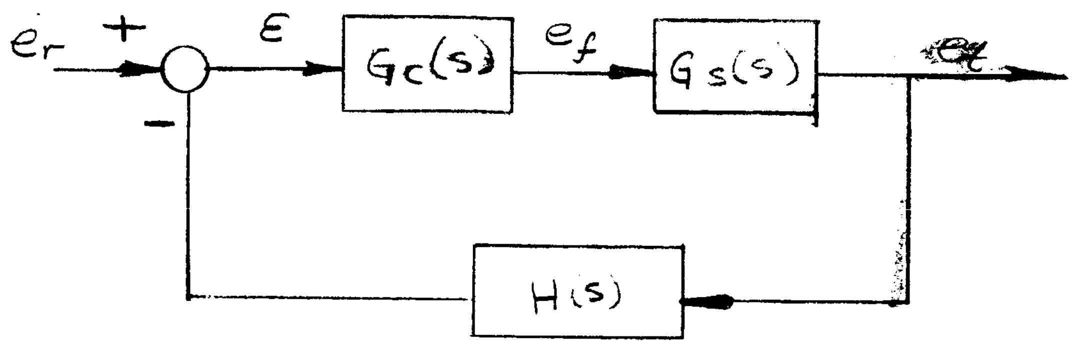


Fig. 26 Linearization of two control variables

(vi) <u>Closed loop control</u>:



$G_s(s)$ = transfer function of synchronous generator

$G_c(s)$ = controller

$H(s)$ = measuring elements

$T(s)$ = closed-loop transfer function

where

$$T(s) = \frac{G_c(s)\, G_s(s)}{1 + H(s)\, G_c(s)\, G_s(s)}$$

Assume:

$$G_s(s) = \frac{s^4 + 2.73\times10^3 s^3 - 6.2\times10^6 s^2 - 6\times10^8 s - 3.84\times10^{11}}{s^4 + 6\times10^3 s^3 + 6.5\times10^6 s^2 + 1.37\times10^{10} s + 3.08\times10^{1}}$$

$$H(s) = 1.0$$

$$G_c(s) = K$$

It is desired to find the maximum permissible gain K for a stable operation. Hurwitz criterion states that a characteristic equation

$$a_n s^n + a_{n-1} s^{n-1} + \text{------} \; a_2 s^2 + a_1 s + a_0 = 0$$

All the determinants

$$\begin{array}{c|c|c|c}
a_1 & a_0 & 0 & 0 \; --- \\
a_3 & a_2 & a_1 & 0 \; --- \\
a_5 & a_4 & a_3 & a_2 \; --- \\
a_7 & a_4 & ---- &
\end{array}$$

must be positive for a stable system. The characteristic equation of T(s) is

$$1 + H(S)\, G_c(S)\, G_s(s) = 0$$

i.e.,

$$(1+K)s^4 + (6-2.73K)10^7 s^3 + (6.5-6.2K)10^6 s^2 + (138-6K)10^8 s + (3.08-3.84K)10^{11} = 0$$

Set the determinants equal to zero for critical condition.

$$(138-6K)10^8 = 0$$

$$K_1 = 23$$

$$\begin{vmatrix} (138-6K)10^{8} & (3.08-3.84K)10^{11} \\ (6-2.73K)10^{3} & (6.5-6.2K)10^{6} \end{vmatrix} = 0$$

i.e., $K^2 - 32.6K + 33.3 = 0$

$$K = 31.6 \text{ or } 1.05$$

Since both values are valid, it is desirable to choose $K_2 = 31.6$

$$\begin{vmatrix} (1.38-6K)10^{8} & (3.08-3.84K)10^{11} & 0 \\ (6-2.73K)10^{3} & (6.5-6.2K)10^{6} & (1.38-6K)10^{8} \\ 0 & 1+K & (6-2.73K)10^{3} \end{vmatrix}$$

i.e.,

$$K^3 = 38.6K^2 + 135.5K - 48.6 = 0$$

$$K = 34.76,\ 3.38 \text{ OR } 48.6$$

It is desirable to have $K_3 = 34.76$

To compare with the Ks obtained from the three determinants, in order to satisfy the criterion, the smallest value should be chosen. That is $K = K_1 = 23$.

(vii) Sensitivity:

Since any component of the same kind may not be identical due to various reasons, it is beneficial to learn the variation of total performance with respect to the deviation of characteristics of a certain component. It can be the parameters of the plant, the gain of the amplifier or others. For instance, one would like to know the effect of K on T(s) in the last example. Define sensitivity as

$$S_K^T = \frac{d(\ln T)}{d(\ln K)}$$

$$= \frac{d\left[\ln\left(\frac{KG_s}{1+KG_s}\right)\right]}{d(\ln K)}$$

$$= \frac{1}{1+KG_s}$$

The smaller the value of S_K^T, the less effect of variation of K on T(s). However, in this example, the sensitivity is almost linearly related to K because $KG_s \gg 1$.

(viii) Degrees of Freedom:

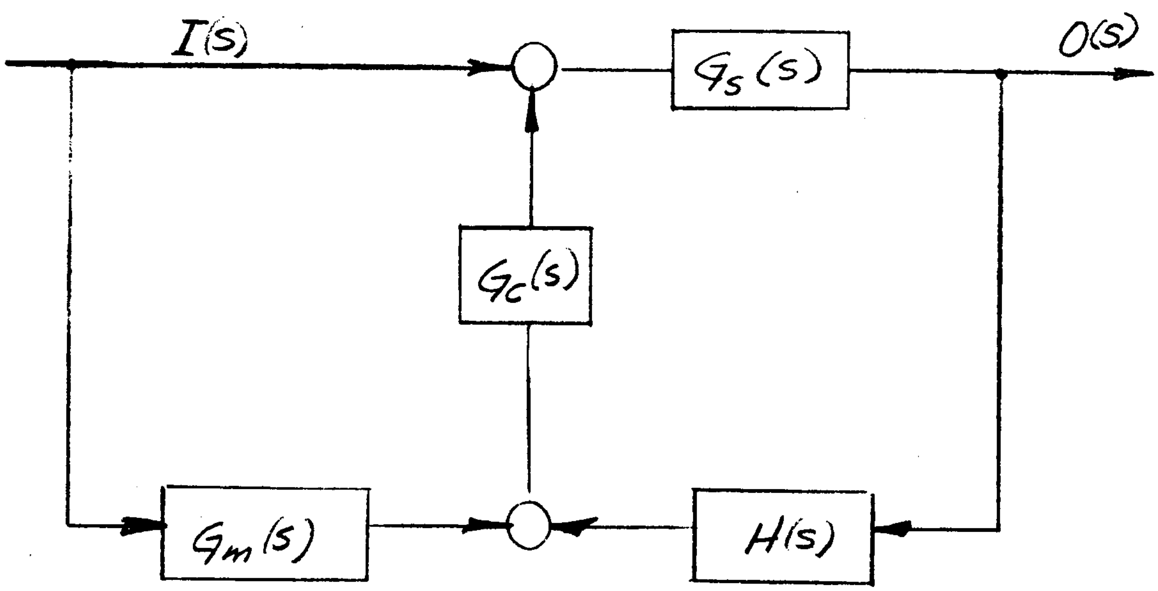
By investigating the closed-loop transfer function

$$T(s) \quad \frac{G_c(s)\,G_s(s)}{1 + H(s)\,G_c(s)\,G_s(s)}$$

assuming the plant $G_s(s)$ is fixed, one can adjust the controller $G_c(s)$ or feedback element $H(s)$ respectively to obtain a desired $T(s)$. Thus, there is only one degree of freedom. If $G_c(s)$ and $H(s)$ are adjusted simultaneously, there will be two degrees of freedom. The latter is more flexible and many a time the implementation is much easier to be realized.

(ix) Model Approach:

Another method to enforce a specified transient response of a synchronous generator is by introducing a model which describes the specification precisely. The block diagram will be as follows:



$$\frac{O}{I} = \frac{G_s(1 + G_c G_m)}{1 + G_c G_s H}$$

Let $H \cong 1$ and make

$|G_c| >> 1$

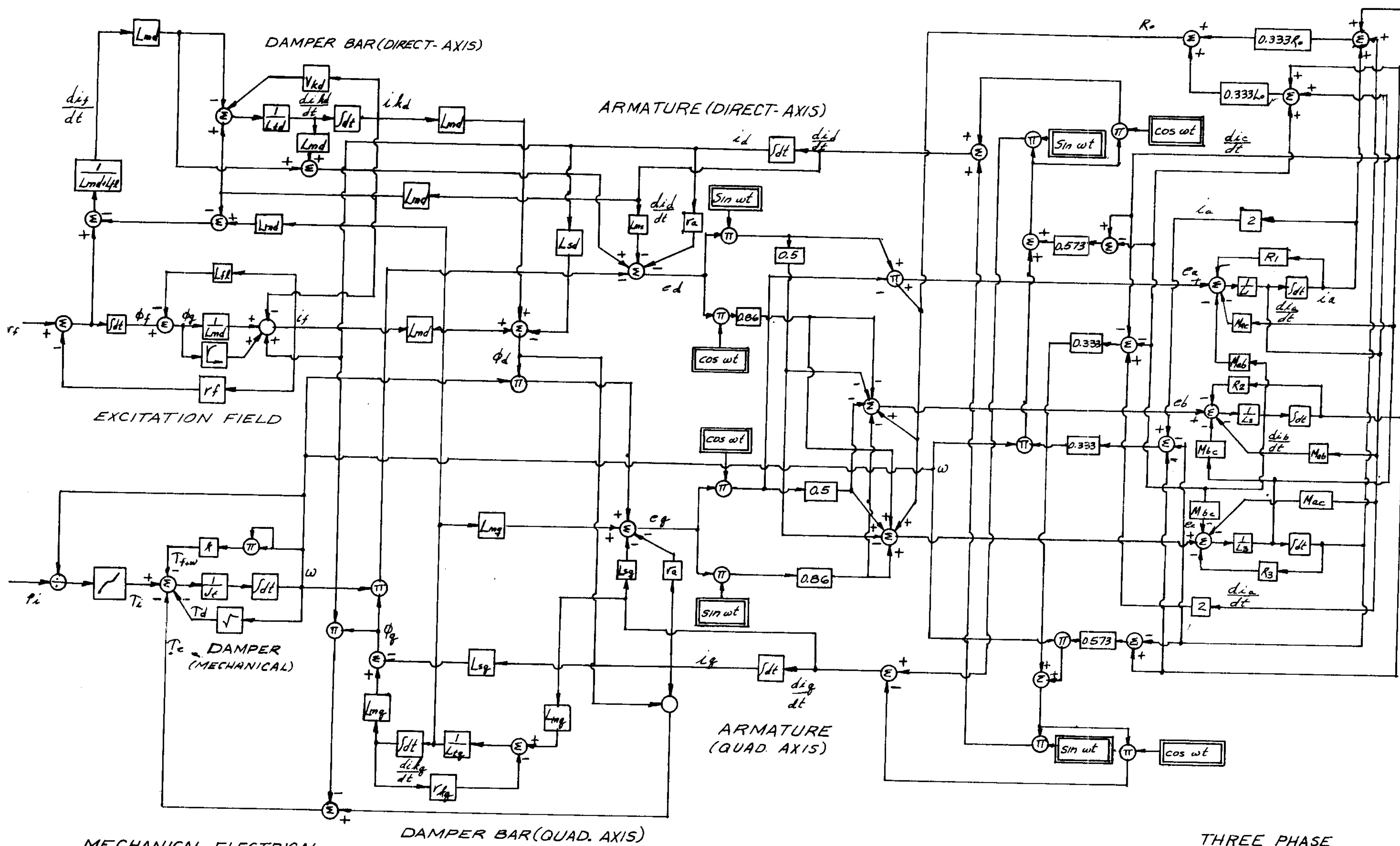
$$\frac{O}{I} = G_m$$

where

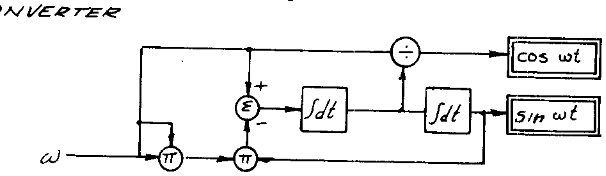
$O(s)$ = output

$I(s)$ = input

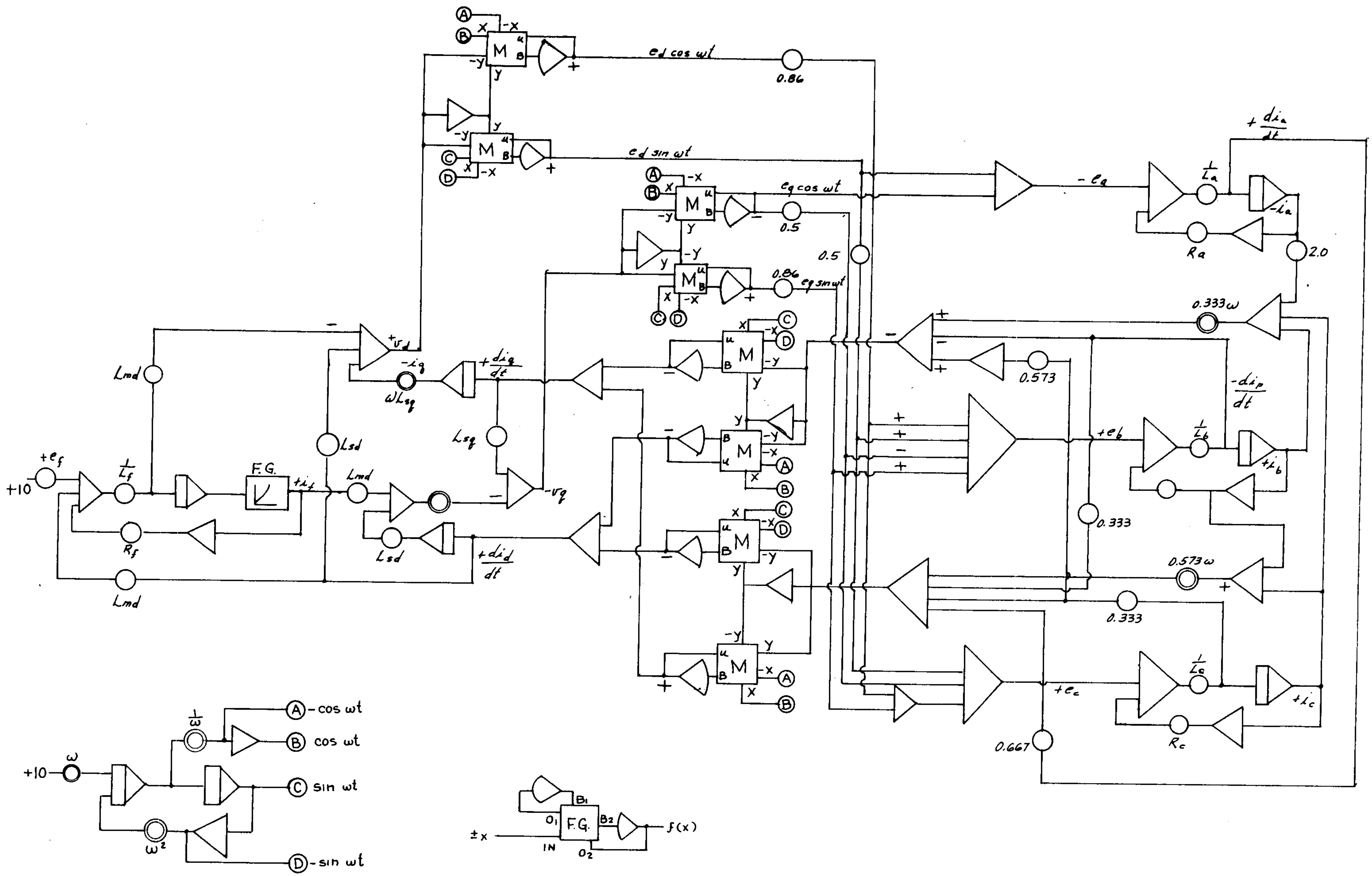
$G_m(s)$ = transfer function of model



TWO AXIS COMPONENTS TO THREE PHASE COMPONENTS CONVERTER



SYNCHRONOUS GENERATOR
DYNAMIC SIMULATION
(UNBALANCED LOAD)



ANALOG COMPUTER SIMULATION OF SYNCHRONOUS GENERATOR
WITH UNBALANCED LOAD